UC-NRLF
B 4 248 673

e
'e
ls
)-
n,
of
pe

tor
for

ic"
sed
ent,
, its
ants
on
can
om-
n of

D.

nville

349.

metic
ne. I
c—the
licious
in its
short,
ghness
ion to
wants
ne kind

RK.

RECOMMENDATIONS.

Whilst this work was passing through the press, the sheets were submitted for inspection to several *teachers* and gentlemen capable of judging of the merits of the work, and the following testimonials have been received. The reader will bear in mind that these notices are all from *practical Instructors,* who are now, or have been, until recently, engaged in teaching; and also, that the majority of them have been furnished *without solicitation,* and may therefore be regarded as in every sense of the word *impartial.*

From the Rev. J. H. Good, A. M., *of Columbus, formerly Rector of the Preparatory Department of Marshall College, Pa., and for many years a practical teacher.*

Mr. C. H. Mattoon—*Dear Sir:*

I have examined your "*Common Arithmetic*" with considerable care, having perused nearly the whole of it as it passed through the press. It is an excellent practical work. From its *arrangement,* its *fullness* of *Rules* and *Examples,* its *clearness* of *explanation*—above all, its *eminently practical character,* I regard it as peculiarly well adapted to the wants of that large class of persons, who usually pass through but one work on Arithmetic preparatory to their entrance upon public life. Every teacher can at once perceive, by examination, how well it is suited to the wants of our Common Schools. I can cordially recommend it to the careful examination of teachers.

Yours,

Respectfully,

J. H. GOOD.

From the Rev. E. W. Clark, *formerly an Agent for Granville College, Ohio.*

Genoa, Delaware Co., O., December 24th, 1849.

Mr. C. H. Mattoon—*Sir:*

I have examined those sheets of your Arithmetic which you sent me, and would say that I am well pleased with the same. I consider the definitions *accurate*—the arrangement *natural* and *scientific*—the introduction and application of the later improvements in this science *judicious*—the rejection of much old and useless matter, and the introduction in its stead of a large number of practical Rules, an *improvement*—and, in short, considering its *clearness* and *simplicity* of style, together with its *thoroughness* and *comprehensiveness* of subjects, and its full and extensive application to *practical questions,* I would say that I think it better adapted to meet the wants both of our Common and High Schools, than any other work of the kind with which I am acquainted.

"I am respectfully yours,

"E. W. CLARK.

Extract from a letter from Mr. BARTON MOORE, *a thorough practical instructor, near Sunbery, O.*

"I received the copies of your Arithmetic which you sent me, and in those I have examined, I believe the principles of numbers can be more easily understood by the youth, and the method of explanation is far superior to any with which I am acquainted.

"Yours, &c.,

"B. MOORE."

From MR. S. L. WALLACE, *a practical teacher near Kingston, Ross county, Ohio.*

KINGSTON, Ross Co., O., Dec. 28, 1849.

MR. C. H. MATTOON :—I received those sheets of your book, which you were kind enough to send me, and after a careful perusal of them, I am satisfied that the work is better adapted to the use of common schools, and academies, than any other work of the kind with which I am acquainted.

1st. Because it comprises more *useful* and *practical* matter than any other work on Arithmetic extant.

2nd. On account of its perfect *adaptation* to the purposes of instruction, together with its *thoroughly progressive* character; *one subject*, and *one difficulty* only, at a time, being presented, and that made *perfectly familiar* before proceeding to the next step.

3d. The style is clear and easy, thus fitting it for the lower as well as for the higher classes of learners.

4th. Although it is plain and simple, it is also full and compreh ensive, and the continual reference to preceding principles, &c., together with many original and important remarks interspersed throughout the work, form a material help to the learner.

5th. In the precision and accuracy of the definitions ; in the natural and scientific arrangement of subjects; in the careful and judicious introduction and application of the later improvements; in the clear and lucid explanation of hard and difficult subjects; and in the tersenesss and comprehensiveness of the rules, it far excels any other work upon this subject that has yet fallen under my observation, and supplies an important desideratum long wanted in our Arithmetics.

6th. The rejection of much old and useless matter, and the introduction of many rules applicable to *practical business*, which heretofore have receivd but little or no attention, speak well in its favor.

In short, I consider it a work of sterling merit, and superior excellence, and decidedly preferable to a majority of the trash that at this day is being crowded upon our schools. I remain, yours respectfully,

S. L. WALLACE.

From MR. WM. C. GILDERSLEEVE, *an old and experienced teacher near Chillicothe, Ohio.*

CHILLICOTHE, O., Dec. 29th, 1849.

MR. CHARLES H. MATTOON—*Sir:*—I have carefully examined your work entitled "COMMON ARITHMETIC" and unhesitatingly say, that in my opinion, it is far superior to any other work of the kind, I have yet perused. The *rules*, (some of which are new to me) are better calculated, and more nearly related to the nature and uses of mathematical computations, in all ordinary business, than those usually given. Therefore, on account of its *simplicity, comprehensiveness and intrinsic merit*, I would reccommend it to the Schools and Academies of our Western country: Yours respectfully,

WM. C. GILDERSLEEVE.

ANALYTIC SERIES, NO. 2.

COMMON ARITHMETIC,

UPON THE ANALYTIC METHOD OF INSTRUCTION.

ALSO: THE

PRINCIPLES OF CANCELATION,

AND OTHER MODERN IMPROVEMENTS.

ILLUSTRATED AND APPLIED.

The whole made simple and easy by numerous practical examples.

DESIGNED FOR THE USE OF SCHOOLS.

BY CHARLES H. MATTOON.

"There's nothing so hard, but search will find it out."

STEAM PRESS OF S. MEDARY.

1850.

M270538

Entered, according to Act of Congress, in the year 1849,
By CHARLES H. MATTOON,
In the Clerk's Office of the District Court of Ohio.

PREFACE.

In presenting this treatise to the public, the Author does not think it necessary either to give his reasons for so doing, or to apologise for its appearance. For the former, the student cares but little, and for the latter, still less.

Neither will he enumerate what he considers to be its excellencies being convinced that if it really possesses merit, its worth will be discovered by others, without his blazing it abroad.

He will, however, present a few of its most prominent characteristics for the consideration of the public, and leave it for them to decide whether they add to, or detract from the merits of the work.

1. Each section commences with a number of *Mental Exercises*, in order to give the pupil an idea of the subject before he is burdened with learning abstract rules and principles.

2. *Definitions* are then given, and the *principles* demonstrated and explained *separately;* the first being proved from *self-evident propositions*, and those following, by those already explained. In no case is one principle used to explain another, until it has itself been explained.

3. The principles are then summed up into a *general rule*, for reference and review. After the rule follow a number of *Exercises for the Slate*, and these, as well as the *Mental Exercises* are chiefly of a *practical* nature, and strictly confined to the principles already explained.

4. Those Rules most used in practical business have received more particular attention, than those of less importance.

5. Pounds, shillings and pence are rejected *entirely*, and their places supplied with *Federal Money*. If we have a national currency, why not adopt it exclusive of all others?

6. The tables of *Weights* and *Measures* have been prepared with direct reference to *existing laws* and *present uses*.

7. The subject of *Cancelation* is explained, and the method of applying its principles to business distinctly shown. Unlike some modern writers, however, the Author does not make it the chief of all rules. Other modern improvements are also illustrated and applied.

8. The Rules of the *Cu be Root, Progression, Alligation, &c.*, are omitted for three reasons: 1st. They seldom if ever occur in common business, and those who have occasion to use them in their particular pursuits, are generally men of science, who are acquainted with the higher departments of mathematics, in which they are explained more clearly and satisfactorily than they can be in an elementary work on Arithmetic. 2nd. Their place can be supplied with rules of more importance; that is, rules that frequently occur in ordinary transactions. 3d. They increase the size of a book, and the labors of the learner unnecessarily. The Author has yet to be convinced of the necessity of burdening the mind with abstruse and metaphysical questions, or tedious and complicated demonstrations, which in nine cases out of ten can never be of any practical utility to the pupil.

9. The rules in *Mensuration* are demonstrated, with but few exceptions. The language, however, is chosen more for perspicuity than for mathematical precision.

10. The language adopted throughout the work, is *simple, terse* and *definite*. Childishness and puerility are avoided on the one hand, as much as complication and mystery on the other. The aim has been to unite *brevity* with *perspicuity*.

11. The arrangement of subjects is such as seems the most consistent with the *natural order of the sciences*. Therefore *Fractions* follow immediately after the Fundamnetal Rules; because, 1st, they partake of the nature of *Division;* and 2nd. They are frequently used in *Compound Numbers;* hence, it is necessary to understand *Fractions* before the operations in *Compound Numbers* can be thoroughly understood. Also, *Federal Money* is placed with *Decimal Fractions*, because the same rules are applicable to both, since

they are based on the same scale of notation. Likewise, *Commission, Insurance, Interest, &c.* are placed under *Percentage,* upon the principles of which they are based.

12. The *analogies* and *relations* of numbers are clearly pointed out and illustrated ; and from these, the scholar is taught to *reason* —to *think* for himself. He is required to believe no theory without *proof* and to take no principle without *demonstration.* He is every where taught that there is no *guess work* about results—that all questions in Arithmetic can be solved upon *true principles*—that there is *no uncertainty* about mathematical conclusions—in short that every *theory* or *principle* connected either directly or indirectly with mathematics, that is not capable of demonstration, must be erroneous. Thus he is led to *investigate,* and *examine*—to exercise his *reasoning faculties*—to work *intellectually,* instead of *mechanically*—and to believe a theory or principle because it is *clearly* and *conclusively proven,* and not merely because *"the book says so."*

Such is a brief outline of the present work. We have endeavored to present every principle that a business man may have occasion to use, in a *clear* and *systematic* manner, and also, to prepare it in such a way as to be adapted to precede the study of Algebra, and the higher branches of mathematics. Our object has not been to produce an *abstruse* and *metaphysical treatise,* but simply to present a *plain, practical,* and *comprehensive work,* which should meet the wants of our business men, and arithmetical students. We have rejected many old, useless, and obsolete rules, and supplied their place with such modern improvements as were deemed of any importance.

In short, we have endeavored to say everything that was necessary, and no more. The work is designed for *use,* and not merely for *show.* It was prepared by the Author whilst engaged as a teacher, and is the result of arduous labor, and hard study, aided by practical observation and experience in the school room.

It may be proper to remark here, that this is designed as the second book of a series, now in course of preparation by the Author. The first work, entitled *"Arithmetic; Mental and Practical"* for be-

ginners, is in progress and will be published probably some time the ensuing season.

How successfully our work has been executed, time alone will decide. For the present we submit it to teachers and scholars for examination, convinced that it must stand or fall, upon its own merits alone. And should it be found to lessen the labors of teachers, or to promote the intellectual attainments of scholars, its highest aims will be accomplished.

THE AUTHOR.

CONTENTS

Page

Suggestions for Teachers xvi

SECTION I.

Arithmetic defined 1
NOTATION 1
" the *Roman* method 2
" the *Arabic* method 3
NUMERATION 4
Numeration Table 5
Exercises in Numeration 7
Exercises in Notation 8

SECTION II.

ADDITION—Mental Exercises 9
Definitions—Addition Table 10
Proof of Addition 11
Carrying in Addition 12
General Rule—Exercises for the slate 13

SECTION III.

SUBTRACTION—Mental Exercises—Definitions 16
Subtraction Table 17
Proof of Subtraction 18
Borrowing and carrying in Subtraction illustrated 19
General Rule—Exercises for the slate 20

SECTION IV.

MULTIPLICATION—Mental Exercises—Definitions 23
Multiplication Table 24
Proof of Multiplication 26
Method of carrying illustrated—Rule when the multiplier is less than 12 27
General Rule—Exercises for the slate 30

Page.

Contractions in Multiplication—When the multiplier is 10, 100, 1000, &c.—To multiply by a Composite number........ 32
When there are ciphers at the right of either or both factors. 34

SECTION V.

Division—Mental Exercises........ 36
Definitions—Division Table........ 37
Proof of Division—Proof of Multiplication by Division........ 39
Remarks respecting the Remainder........ 40
Method of carrying illlustrated........ 41
Rule for Short Division........ 42
Exercises for the Slate........ 43
Long Division........ 44
Rule for Long Division—Exercise for the Slate........ 46
Contractions in Division—To divide by 10, 100, 1000, &c.. 48
To divide by a composite number........ 49
When there are ciphers at the right of the divisor........ 50

SECTION VI.

Recapitulation—*Remarks and inferences*—Notation........ 51
Numeration—Addition........ 52
Subtraction........ 53
Multiplication........ 54
Proof by casting out the 9's........ 55
Division........ 57
Proof by casting out the 9's........ 59
Principles deduced from Division........ 60–62
Contractions and Abbreviations—To multiply by 25. By 50. By 125........ 62
—When part of the multiplier is a factor of another part.... 62
—By aid of exponents........ 63
To divide by 25........ 64
—By 125. When there is a remainder after dividing by several divisors, to find the true remainder........ 65
To contract operations in Long Division........ 66

SECTION VII.

Cancelation—Definitions and Illustrations........ 67
General Rule........ 70
Exercises for the slate........ 71
Greatest Common Divisor........ 72
Rule—for two numbers........ 73
—For more than two numbers—two rules........ 74
Least Common Multiple........ 76
Rule—Exercises for the slate........ 80

Page.

SECTION VIII.

FRACTIONS—*Mental Exercises* 81
COMMON FRACTIONS—*Definitions* 85
Fundamental Propositions 87
Reduction of Fractions—To reduce fractions to its lowest terms 90
—An improper fraction to a whole or mixed number 91
—A whole or mixed number to an improper fraction 92
—A compound fraction to a simple one 93
—Ditto, by cancelation 94
To reduce fractions to their least common denominator 94
General Rules for the Reduction of Fractions 96
Exercises for the slate 97
Addition and Subtraction of Fractions—Mental Exercises 98
General Rule—Exercises for the slate 100
Multiplication of Fractions—Mental Exercises 102
Exercises for the slate—To multiply a fraction and a whole number together 103
To multiply a whole number and a mixed number together 105
—Fractions together 106
—Mixed numbers together 107
General Rules for the Multiplication of Fractions 108
Exercises for the slate 108
Division of Fractions 109
Mental Exercises—Exercises for the slate—To divide a fraction by a whole number 110
To divide a whole number by a fraction 111
—One fraction by another 113
Method of proceeding when mixed numbers occur 114
Complex fractions 116
To change a fraction to any required denominator 118
General Rules for the Division of Fractions 120
Exercises for the slate 121
Miscellaneous exercises for the slate 123
DECIMAL FRACTIONS—*Definitions, &c.* 124
Decimal Numeration Table 125
Numeration, and Notation of Decimals 127
Federal Money—Table 128
To read sums in Federal Money 130
To write sums in Federal Money 131
Reduction of Federal Money, &c. 131
Exercises for the slate—Reduction of Decimal Fractions 132
To change a Decimal to a Common Fraction 132
To change a Common fraction to a Decimal 133

Addition and Subtraction of Decimals and Federal Money 135
Rule 137
Exercises for the slate 138
Multiplication of Decimals and Federal Money 139
Rule—Exercises for the slate 140
Method of buying and selling articles by the 100 or 1000 141
Division of Decimals and Federal Money 142
Rule 143
Exercises for the slate 144
Miscellaneous exercises for the slate 145
Bills, Accounts, &c. 146

SECTION IX.

COMPOUND NUMBERS—*Definitions* 148
Reduction 149
WEIGHTS—*Troy Weight* 150
Exercises in Troy weight 151
Rules for Reduction, Ascending and Descending 152
Avoirdupois Weight 152
Exercises in Avoirdupois Weight 153
Apothecaries Weight 153
Exercises in Apothecaries Weight 154
MEASURES OF CAPACITY—*Dry Measure* 154
Exercises in Dry Measure 155
Wine Measure 156
Exercises in Wine Measure 156
Ale or Beer Measure 157
Exercises in Ale or Beer Measure 157
MEASURES OF EXTENSION—*Long Measure* 158
Exercises in Long Measure 159
Cloth Measure 159
Exercises in Cloth Measure 160
Land or Square Measure 161
Exercises in Land or Square Measure 162
Solid or Cubic Measure 164
Exercises in Solid or Cubic Measure 165
Time 166
Exercises in Time 167
Circular Measure or Motion 168
Exercises in *Circular Measure or Motion* 169
Miscellaneous Exercises in Reduction 169
Fractions of Compound Numbers 174
To reduce higher to lower fractions—Rule 174
To reduce fractions to integers, and integers to fractions—Rule 175

	Page.
To reduce decimals to fractions and fractions to decimals	177
Addition of Compound Numbers	179
Rule	180
Exercises in Addition of Compound Numbers	181
Subtraction of Compound Numbers	183
Rule	184
Exercises in Subtraction of Compound Numbers	185
Multiplication and Division of Compound Numbers	187
Rules	187
Exercises in Multiplication and Division of Compound Numbers	188
Difference in time of two places—Rule	193

SECTION X.

Exchange—*Definitions, &c.*	195
Table of Foreign coins	196
Rule—To change foreign coins to Federal Money	196
Rule—To change Federal Money to Foreign Coins	197

SECTION XI.

Analysis.—*Definitions, &c.*	198
Mental Exercises in Analysis	199
Exercises for the Slate	201
Practice—Definitions, &c.	207
Exercises in do.	208
Barter, Definition and Exercises.	212
Alligation—Medial	213
Porportional Division	214
Exercises in Analysis	215

SECTION XII.

Ratio and Proportion—Definition of *Ratio*	219
Mental Exercises in do	220
Rules and Exercises in Ratio	220
Simple and Compound Ratio	223
Proportion—Definition, &c.	224
Difference of Ratio and Proportion	225
Extremes and Means in Proportion	225
Rules in Proportion	226
Mental Exercises	227
Simple Proportion, Definitions, &c.	227
Rules for working questions in Simple Proportion	229
General Rule for Simple Proportion	232
Exercises for the slate	233

	Page.
COMPOUND PROPORTION, Definitions, &c.	235
Solution of questions in do.	236
How to prove Compound Proportion	237
General Rule for Compound Proportion	238
Exercises for the slate n do.	239
PARTNERSHIP, Definition, Solutions, &c.	240
Rule for Partnership, Exercises	241
Bankruptcy, Exercises	242
General Average, Exercise	242

SECTION XIII.

PERCENTAGE—Definitions	244
Mental Exercises	245
Rate for Percentage, Exercises	247
COMMISSION AND INSURANCE	249
Exercises in *Commission*	250
Exercises in *Insurance*	251
PROFIT AND LOSS, Definitions	254
Exercises in do.	255
STOCKS, Definitions	260
Exercises in do.	261
BROKAGE, Definitions, Exercises	262
DUTIES, Definitions	262
Specific Duties, Definitions	263
Exercises in do.	264
AD VALORAM DUTIES, Definitions	264
TAXES, Definitions	265
General Rule for Taxes	266
Table for Taxes	267
INTEREST, Definitions and Illustrations	268
Simple Interest, Illustration	269
Rule—Exercises	270
Rule for years and months	271
Rule for days	272
General Rule for Interest	272
Table for Interest	273
Exercises in Interest	274
Interest at ANY *rate*—Rule	276
Second method for "	277
Rule for Interest on Notes and Bonds	279
Vermont Rule " " "	281
Connecticut Rule " " "	282
Exercises in the above	283
Problems in Interest	285

To find the *Interest,* To find the *rate* 285
To find the *time* 286
To find the *Principal* 287
To find the *Principal* and *Interest* 288
DISCOUNT, Definitions 289
Exercises in 290
BANKING, Definitions 291
Exercises in 292
Equation of Payments, illustrations 293
Rule for finding equated Time 294
COMPOUND INTEREST, Illustrations 296
Table for Compound Interest 297
Exercises in " 298

SECTION XIV.

DUODECIMALS, Table, Multiplication of 299
Rule for Multiplication of Decimals 301
Painters' Powers and Plasterers' Work 301
Masons' Work 303

SECTION XV.

POWERS AND ROOTS, Definitions 303
INVOLUTION, Definitions, Exercises 304
Rule for Involution, Exercises 305
Involution of Fractions 306
EVOLUTION, Definitions 308
Mental Exercises, Illustrations 309
Extraction of the Square Root, Illustrations 311
Illustrations, Diagrams, 313
Rule for Extracting the Square Root 315
Exercises in " " 317
Rule to Extract the Square Root of Fractions 318
TRIANGLES, Illustrations 319
Mean Proportionals 321
CIRCLES, Definitions, Illustrations 323

SECTION XVI.

MENSURATION, *Mensuration of Surfaces,* 324
Area of Triangles, Rule 325
Area of Trapezoids, Rule 326
Area of Circles and Polygons 327
Circumscribed and Inscribed Polygons 328
Quadrature of the Circle 329
Ratio of the Diameter to Circumference 330

To find the Circumference, Diameter 330
Rule for finding the Area of the Circle 331
Area of a Square inscribed in a Circle 332
Dimensions of the Circumscribed Circle 336
MENSURATION OF SOLIDS, *Prisms*, Area of a Prism 338
To find the solidity of a Prism 339
Cylinders, Definitions 339
To find the solidity of a Cylinder 340
Boilers, To find the contents of 340
To find the contents of a cistern 341
Parallelopiped, to find the surface and solidity 342
Pyramids, Definitions 342
To find the surface of a Pyramid or Cone 343
To find the surface of the Frustrum of a Pyramid or Cone 344
To find the solidity of a Pyramid or Cone 344
To find the solidity of the Frustrum of a Pyramid or Cone 345
SPHERES OR GLOBES. To find the surface of a Sphere or Globe 346
To find the solidity of a Globe or Sphere 346
TONNAGE OF VESSELS, Definitions 346
Rule for calculating tonnage 347
To find the solidity of Irregular Bodies 348
Guaging. To find the number of bushels a vessel will contain 348
To find the number of wine and beer gallons in a cask 349

SECTION XVII.

PHILOSOPHICAL AND MISCELLANEOUS QUESTIONS 350
THE MECHANICAL POWERS, Definitions 350
The Lever, Definitions and Rules 351
The Wheel and Axle, Definitions and Rules 352
The Pully, Definitions and Rules 352
The Inclined Plane, Definitions and Rules 353
The Wedge, Definitions and Rules 354
The Screw, Definitions aud Rules 354
Machinery, Defininitions and Rules 355
Book Keeping, Definitions and Forms 356
Book Keeping for drovers and farmers 357
FORMS. Form of *Orders*—Of *Receipts* 358
Form of *Due-bills*—Of *Notes* 359
Remarks on Notes 356
Miscellaneous Problems and Rules 360
To find the heighth of an object—its distance 261
To find the time a time a body has been falling—Its velocity 361
To find the space fallen through 362

Problem relating to the right-angled triangle 362
To measure standing timber 362
To find the temperature of any day—De Witt's Rule 363
Various Problems 363
Analytic Questions ....I........................... 365

SECTION XVIII.

Miscellaneous Examples for the Slate 368
Errata 387

INTRODUCTORY REMARKS:

FOR TEACHERS.

There are at least three different departments of science with which every person should be familiar, and they are, *Reading*, *Writing*, and *Arithmetic*. These, in consequence of their universal application to all kinds of business, might with propriety be termed the *Golden Branches of Science*. Other departments may be desirable for ornament, or necessary in some particular profession or occupation, or useful in storing the mind with general information; but there are none that will apply so universally to all classes of individuals, and to every department of business, as the three branches above named.

Such being the case, how necessary is it that they should be taught *correctly*, *thoroughly*, and *completely*: that the learner should be taught *correct theories*, *correct principles*, and the *correct method* of applying these theories and principles to practical use—in short, that he should be taught to work *intellectually*, instead of *mechanically*.

In *Arithmetic*, especially, is it necessary to be particular, as much is often at stake by a single operation; and the greater the *risk*, the more the need of *care*.

In *teaching* Arithmetic, many things are to be considered, the principal of which are—

1st. *The age of the pupil;* and

2nd. *His natural abilities for receiving instructions.*

With young pupils, the subject under consideration should be presented in *clear*, *simple*, and *definite* language—free, on the one hand, from childishness and puerility, and on the other hand from complication and mystery. If possible, *technical terms* should be

avoided; but if it is absolutely necessary to make use of any, they should be *fully* and *thoroughly explained.* As he advances, additional principles may be presented to his view, and new theories explained for his consideration; and thus he may be led on, step by step, gradually and thoroughly, until he has mastered the first elementary rudiments of mathematical science, and is able to grapple successfully with the more abstruse and difficult propositions.

As his mind enlarges, and his intellect expands, and his ability for receiving instruction increases, the explanatory language may be proportionably altered, and some of the technicalities of the science may be introduced; the pupil has now learned the first rudiments and is able to think and reason to some extent for himself, and needs not the particular demonstration of every simple problem, or illustration of every common term.

As he advances still farther, the illustrations may be in *strict scientific language,* and he may be required to investigate theories, examine principles, demonstrate abstruse propositions, unravel difficult questions, or solve knotty problems—to prove why a theory or statement is, or is not correct, and in short, to reason, think, examine, and demonstrate a truth by his own investigating powers, independent of the "say so" of the book.

Practical Applications of principles to business should be introduced throughout the course, thus uniting practice with theory, and science with art, and making the pupil both a scientific scholar, and a judicious man of business.

To accomplish this would of course require years of labor, care, and study; but the pupil thus educated would finally become an honor to himself, a credit to his teacher, and an ornament to society.

Throughout this series we have endeavored to follow the plan here laid down, with what success, remains yet to be decided.

But whilst a work may be prepared with care, and the principles presented clearly and lucidly, yet the instructor must second the aim of the author, and enforce theories and the method of application upon the attention of the scholar, and be sure to

have them correctly understood. To teach with success, however, the teacher must possess—

1st. *A thorough knowledge of his subject;* and
2nd. *The entire confidence and good will of his pupils.*

If in addition to this, he has a good *black-board* and *numerical frame,* he need not fear, but push on, certain of success.

The first book of these series will propably be published the ensuing season. It is recommended that this or some other mental work should be studied before studying written Arithmetic.

A few errors were discovered in this work after the forms were printed, and are mentioned in an *Errata* at the end of the volume. It may be possible some errors have escaped detection. Those discovered will all be corrected in future editions.

Should some portions of this work be thought too difficult at first, they may be omitted, until a review. It is left to the judgment of the teacher, however, what part to omit, and what to study.

In teaching, the scholar should be tanght to depend upon his own resources. Examples should be explained *indirectly,* and the method adopted by some, of working sums for scholars on the slate, without illustration, should be discarded at once. The teacher, however, should not permit the learner to pass by a question or principle until it is perfectly familiar to him.

In conclusion, the writer would say to Teachers—Your success must depend principally upon your own exertions. You have an important charge committed to your keeping, and therefore yours is a heavy responsibility. Let your motto then be "ONWARD AND UPWARD." And whilst you instil the principles of science into the youthful mind, endeavor to train it in such a manner that it shall reflect credit upon its teacher, and honor upon itself. Excite in the minds of your pupils a noble spirit of generous rivalry and emulation; and not only excite this spirit, but keep up the excitement, and your success will astonish even yourself.

To *Students* he would say—*Persevere.* Never give up a question because it is hard, or pass by a demonstration because you cannot

comprehend it at once. Remember, "Where there is a will there is a way," and "Nothing so hard but search will find it out." If you ever expect to gain a knowledge of Arithmetic without *close application*, and *hard study*, you will be sadly disappointed. Therefore meet every difficulty *firmly*, and combat every obstacle *resolutely*, and success will eventually crown your efforts.

COMMON ARITHMETIC.

SECTION I.

NOTATION AND NUMERATION.

A single thing is called a *unit*, or *one;* one and one more are called *two;* two and one more are called *three;* three and one more are called *four;* four and one more are called *five;* five and one more are called *six*, &c.

The expressions *one, two, three, four*, &c., are called NUMBERS.—Hence:

Observation 1. *Number is an expression signifying a unit, or a collection of units.* It always answers to the question, "How many?" Numbers are subject to certain laws and regulations, and are applied to practical business. These laws and relations, when arranged in systematic order, form what is called an ARITHMETIC.—Hence:

Obs. 2. *Arithmetic is the science of numbers.* It is both theoretical and practical.

Obs. 3. The *theory* of Arithmetic consists in the analization of the *laws* and *principles* of numbers.

Obs. 4. The *practice* of Arithmetic consists in the application of its principles to *common business transactions.*

REMARK.—*Science* is knowledge reduced to order.

ARTICLE I. NOTATION.

Obs. 1. *Notation is the method of expressing numbers by characters or signs.*

Obs. 2. There are *two* different methods of notation in use, called the ROMAN and ARABIC *methods,* because it is supposed that these nations first invented them.

What is a single thing called? What does unit mean? What are one and more called? Two and one more? Three and one more? Four and one more? Five and one more? Six and one more? Seven and one more? Eight and one more? Nine and one more? What are the expressions one, two, three, &c., called? What then is number? To what question does it answer? To what are numbers subject? To what are they applied? What do these laws and regulations form when arranged in order? What then is Arithmetic? How is Arithmetic divided? What is the theory of Arithmetic? The practice of Arithmetic? What is Science? What is Notation? How many methods of Notation are there in use? What are they? Why are they so called?

CASE I. *The Roman Method.*—By the *Roman method* we express numbers by the use of the seven following letters, viz: I. V. X. L. C. D. and M.

When standing alone, their values are as follows: I., one; V., five; X., ten; L., fifty; C., one hundred, and M., one thousand; and by various *repetitions* and *combinations*, may express any other number.

Their manner of use is clearly shown in the following

TABLE.

I	denotes	one
II	"	two
III	"	three
IV	"	four
V	"	five
VI	"	six
VII	"	seven
VIII	"	eight
IX	"	nine
X	"	ten
XI	"	eleven
XII	"	twelve
XIII	"	thirteen
XIV	"	fourteen
XV	"	fifteen
XVI	"	sixteen
XVII	"	seventeen
XVIII	"	eighteen
XIX	"	nineteen
XX	"	twenty
XXI	"	twenty-one
XXII	"	twenty-two &c.
XXX	"	thirty
XL	"	forty
L	"	fifty
LX	"	sixty
LXX	"	seventy
LXXX	"	eighty
XC	"	ninety
C	"	one hundred
CC	"	two hundred
CCC	"	three hundred
CCCC	"	four hundred

How do we express numbers by the Roman method? What value have these letters when standing alone? Can any number be expressed by these letters? How?

D	denotes	five hundred
DC	"	six hundred
DCC	"	seven hundred
D C	"	eight hundred
DCCCC	"	nine hundred
M	"	one thousand
MDCCCXLVIII	denotes	one thousand, eight hundred and forty-eight.

By examining this table, it will be perceived that when a letter is repeated, its *value* is repeated; as II *two*, XX *twenty* &c; also, when two letters of different values are joined together, if the *less* be placed *before* the *greater*, the *greater* is *diminished* by a sum *equal* to the value of the *less;* but if the *less* be placed *after* the *greater*, the value of the *greater* is increased by a sum *equal* to the value of the *less*. Thus: X denotes *ten;* but IX denotes *nine*, and XI denotes *eleven*.

Obs. 3. A line (—) placed over a letter increases its value a *thousand times*. Thus: V denotes *five;* but $\overline{\text{V}}$ denotes *five thousand*.

Note.—This method of expressing numbers is but little used, except in denoting Chapters, Sections, &c.

Case 2. *The Arabic Method.*—Numbers are expressed by the common, or *Arabic method*, by the following characters or figures, viz:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	cipher.

Obs. 4. The first *nine* characters are called *significant* figures, because *they always express some number*. They are also termed *digits*, from the Latin word *digitus*, signifying *finger*.

Obs. 5. The last character (0), when standing alone, has *no value;* therefore it is called a *cipher*, which means *nothing;* but when placed at the right of a significant figure, it increases the value of this figure *ten times*.

Numbers larger than nine are expressed by different combinations of the foregoing characters. Thus: ten is expressed by 1 and 0, thus, 10; twenty is expressed by 2 and 0, thus, 20, &c.

What effect does it have upon its value to repeat a letter? What effect does it have to place a letter expressing a smaller number before a letter expressing a larger number? To place the smaller after the larger? How many times does a line (—) placed over a letter increase its value? For what is this method of expressing numbers used? How are numbers expressed by the Common, or Arabic method? What are the names of these characters? What are the first nine characters called? Why? What other name are they termed? From what? What value has the character (0) when standing alone? What is it called? What does cipher mean? What effect does it have to place a cipher at the right of a significan figure? How are numbers larger than nine expressed? How is ten expressed?

TABLE OF NUMBERS FROM TEN TO ONE THOUSAND.

ten	is expressed	10
eleven	"	11
twelve	"	12
thirteen	"	13
fourteen	"	14
fifteen	"	15
sixteen	"	16
seventeen	"	17
eighteen	"	18
nineteen	"	19
twenty	"	20
twenty-one	"	21
twenty-two	"	22
twenty-three	"	23&c
thirty	"	30
thirty-one	"	31&c
forty	"	40
fifty	"	50
sixty	"	60
seventy	"	70
eighty	"	80
ninety	"	90
one hundred	"	100
one hundred and one	"	101
one hundred and ten	"	110
one hundred and fifty	"	150
two hundred	"	200
three hundred	"	300
four hundred	"	400
five hundred	"	500
six hundred	"	600
seven hundred	"	700
eight hundred	"	800
nine hundred	"	900
nine hundred and ninety-nine is expressed		999
one thousand is expressed		1000

Article 2. Numeration.

Obs. 1. *Numeration is the art of expressing numbers by words, or of reading numbers expressed by figures.*

In Numeration, different places are assigned for figures, and a name given to these places. Hence—

Twenty? Thirty? Fifty? One hundred? Five hundred? What is Numeration?

Obs. 2. *The value of a figure depends upon the place it occupies.* Thus:

4, standing alone, expresses simply 4 things, or 4 *units,* and is written thus.................................... 4

But if we annex a cipher to 4, we increase its value *ten times,* and make it 4 *tens,* or *forty,* written thus.............. 40

If we annex two ciphers to 4, we increase its value *one hundred times,* and make it 4 *hundred,* written thus 400

If we annex three ciphers to 4, we increase its value *one thousand times,* and make it 4 *thousand,* written thus......4000

Obs. 3. Hence—*Every removal of a figure towards the left increases its value ten times ;* or, *it always takes* 10 *units of any one order to make* 1 *of the next higher order.*

It matters not whether the figures at the left are *significant figures* or *ciphers,* the increase is the same. Thus: the above expressions may be written 4444 ; the right hand figure being *units,* the second *tens,* the third *hundreds,* and the fourth *thousands ;* and as the figure in each order is 4, the whole expression is read *four thousand, four hundred, four tens* (*forty*) and *four.*

The different places, or orders for figures, and their names, may be exhibited in the following

TABLE.

quadrillions.	hundred of trillions,	tens of trillions,	trillions,	hundreds of billions,	tens of billions,	billions,	hundreds of millions,	tens of millions,	millions,	hundreds of thousands	tens of thousands,	thousands,	hundreds,	tens,	units.
1	4	5	7	9	6	2	0	5	7	9	3	5	4	2	8

Note.—This is according to the French method. The English, after hundreds of millions, hundreds of billions, hundreds of trillions, &c., reckon thousands of millions, tens of thousands of millions, hundreds of thousands of millions, and the same of billions, trillions, &c., assigning *six* places to millions, billions, &c., instead of *three.*

Upon what does the value of a figure depend? What does the number 4, standing alone, express? If we annex a cipher to 4, what does it express? If we annex two ciphers, what does it express? Three ciphers? What effect does every removal of a figure towards the left have upon its value? How many units does it take of one order to make a unit of the next higher order? Does it make any difference whether the figures at the right are significant figures or ciphers? How is the expression 4444 read? Repeat the Table.

Obs. 4. *Every number must occupy one or more places.* Thus:

	thous.	hund.	tens.	units.
The number *six* occupies the units place only: thus	-	-	-	6
The number twenty-three occupies two places, viz: *units* and *tens;* thus	-	-	2	3
The number two hundred and sixty-seven occupies three places, viz: *units, tens and hundreds;* thus	-	2	6	7
The nnmber two thousand, two hundred and twenty-two occupies four places, viz: *units, tens, hundreds and thousands;* thus	2	2	2	2

From the example we perceive

Obs. 5. *That the same figure has different values, according to the place it occupies.* Hence, we infer that figures have two values, viz: *simple* and *local.*

Obs. 6. *The simple value is the value expressed by the figure when standing alone:* as 5 expresses simply five, and 6 expresses simply six.

The local value is the value expressed by the figure when combined with other figures, and depends upon the place it occupies. Thus:

In the number 55, the 5 in the *unit's* place is simply 5; but the 5 in the *ten's* place, if considered with reference to the other 5, becomes 5 *tens,* or *fifty.* The term *local* is derived from the Latin word *locus,* signifying place.

To facilitate the reading of numbers, the orders are *pointed off into periods of three figures each.* The first right hand period is called the *unit's period;* the second, the *thousand's period;* the third, the *million's period,* &c. This may be illustrated as follows:

Trillion's period.	Billion's period.	Million's period.	Thousand's period.	Unit's period.
Hundreds of trillions.	Hundreds of billions.	Hundreds of millions.	Hundreds of thousands.	Hundreds.
Tens of trillions,	Tens of billions,	Tens of millions,	Tens of thousands,	Tens,
Trillions,	Billions,	Millions,	Thousands,	Units,

NOTE.—The ENGLISH have six places in each period.

How many places will the number six occupy? Twenty-three? Two hundred and sixty-seven? Two thousand, two hundred and twenty-two? Name

As we have said before, every number must occupy one or more places. Thus: The number

23 consists of 2 *tens* and 3 *units*, and is called *twenty-three*.

365 consists of 3 *hundreds*, 6 *tens* and 5 *units*, and is called *three hundred and sixty-five*.

1848 consists of 1 *thousand*, 8 *hundreds*, 4 *tens* and 8 *units*, and reads *one thousand, eight hundred and forty-eight.*.

How many units and tens are there in 63?

Ans. 3 *units* and 6 *tens*.

How many units and tens are there in 47? 54? 27? 80? 36?

How many units, tens, and hundreds are there in 126?

Ans. 6 *units*, 2 *tens*, and 1 *hundred*.

How many units, tens, and hundreds are there in 216? 423? 108? 240? 300? 789? 972?

How many units, tens, hundreds, and thousands are there in 1827? 2362? 4786? 6721?

How many units, tens, &c., in the following numbers: 5? 18? 20? 17? 60? 58? 40? 120? 111? 107? 209? 780? 2820? 2006? 7460? 7000? 12067?

Obs. 7. To numerate figures—*Commence at the right hand, and call the orders as they stand in the Table.*

To read figures—*Commence at the right hand, and point them off into periods of three figures each; then commence at the left hand and read each period as if it stood alone, adding the name of the period.*

Numerate and read the number 623. Ans. 3 *units*, 2 *tens*, 6 *hundreds*, which reads *six hundred and twenty-three.*

In like manner let the pupil numerate and read the following numbers:

46	126	1026	16932	61728421
78	150	1428	214975	74500112
96	197	2764	379681	345678907
47	265	4829	127962	763945316
54	390	7008	987943	5438659123
67	411	9060	7237684	29685734120
75	578	10857	4623794	149769004721
87	976	12764	2776547	691239468721623

the places or orders occupied by each of these numbers. Can the same figure have different values? How? How many values have figures? What are they? What is the simple value? The local value? Upon what does the local value depend? In the number 55, what is the value of the 5 in the unit's place? Of the 5 in the ten's place, when considered with reference to the other figure? How do we facilitate the reading of numbers? How many figures does each period embrace? What is the first period called? The second? The third? The fourth? The fifth? How many places does the number 23 occupy? 365? 1848? What are the names of the places each of these figures occupy? How do we numerate figures? Where do we commence to point off figures into periods? Where do we commence to read? How do we read each period?

The number *forty-six* consists of 6 *units* and 4 *tens;* therefore to write forty-six, we place a 6 in the *unit's* place, and a 4 in the *ten's* place, thus.......................... 46

The number *two hundred and six* consists of 2 *hundreds,* 0 *tens,* and 6 *units,* and is written thus.................. 206

The number *one thousand and sixty* consists of 1 *thousand,* 0 *hundred,* 6 *tens,* and 0 *units,* and is written thus.......... 1060

Note.—When *units* only are mentioned, the right hand place is always understood.

Hence, to write numbers, we have this

Rule.—*Write each significant figure in the order which it belongs, and place a cipher in all vacant orders.*

EXERCISES FOR THE SLATE.

Should the learner find any difficulty in reading the following numbers, it might assist him to proceed thus:

Draw a number of lines, and write the names of the orders in their respective spaces; then by distinguishing the periods by double lines, the numbers can be easily written and read.

hundred	and forty	six MILLIONS,	hundred	and	four THOUSAND,	and	twenty	seven.
3	4	6	1	0	4	0	2	7

Thus: to write three hundred and forty six millions, one hundred and four thousand, and twenty-seven, we write each figure in its order, and place a cipher in the orders left vacant, and then it is easily read, as above.

1 Write twenty-five.
2 Forty-eight.
3 Fifty-four.
4 Seventy-six.
5 Eighty-nine.
6 Ninety-seven.
7 One hundred and seven.
8 One hundred and thirty-two.
9 Two hundred and forty.
10 Three hundred and sixty-five.
11 Four hundred and eighty-six.
12 Five hundred and ninety-nine.
13 Seven hundred.
14 Eight hundred and three.
15 Nine hundred and fifty one.
16 One thousand and four.
17 Three thousand, four hundred.
18 Six thousand, four hundred and thirty.
19 Nine thousand, three hundred and twenty-six.
20 Ten thousand, one hundred.

How do we write the number forty-six? Two hundred and six? One thousand and sixty? When units only are mentioned, what place is understood? What is the rule for writing numbers?

21 Twenty-two thousand and seventy-three.
22 Sixty-four thousand, nine hundred and seventy-seven.
23 Eighty thousand, three hundred and one.
24 One hundred and four thousand and eight.
25. Seven hundred thousand, nine hundred.
26. Nine hundred thousand, two hundred and forty-seven.
27. One million, sixty thousand and nine.
28 Three millions, two hundred and seventy-six thousand.
29 Eight hundred and forty millions.
30 Eight billions, ninety-seven millions, six thousand, seven hundred and nine.
31 Six hundred and ninety-seven trillions, two hundred and four billions, seventy millions, four hundred and six thousand and one

NOTE.—The method of reading and writing numbers should be *clearly explained* on the black-board by the teacher, until the scholar *thoroughly understands it*, and can read or write any number *readily*. A little labor and exertion on the part of the teacher will be found to be more beneficial and satisfactory than an elaborate treatise on the subject.

SECTION II.

SIMPLE ADDITION.

ARTICLE 1. MENTAL EXERCISES.

1. Charles bought a book for 8 cents, a slate for 6 cents, and a pencil for 1 cent. How many cents did he pay for the whole?

Ans. 15 cents.

8 cents, and 6 cents, and 1 cent are how many cents?

2. John gave 6 cents for a writing-book, 6 cents for an ink-stand, and 4 cents for some quills. How many cents did he give for all?

6 and 6 and 4 are how many?

3. A poor boy met some ladies, one of whom gave him 3 cents, another 5 cents, another 7 cents, and another 9 cents. How many cents did he get?

4. A man paid 8 dollars for some sheep, 7 dollars for some hogs, and 2 dollars for some fowls. How many dollars did he spend?

5. A lady bought 4 yards of cloth in one piece, 6 in another, 5 in another, 7 in another, and 9 in another. How many yards did she buy?

6. John has 8 marbles, William 6, Thomas 9, Henry 5, Charles 7, James 3, and Robert 4. How many have they all?

7. One boy has 6 apples, another 7 apples, another 4 apples, another 8 apples, and another 5 apples. How many have they all?

8. One man has 6 horses, another 4, another 9, another 3, another 8, and another 7. How many have they all?

9. On one shelf are 9 books, on another 7, on another 8, on another 6, and on another 5. How many are there in all?

10. I bought a book for 4 shillings, a slate for 2 shillings, some paper for 3 shillings, and a map for 9 shillings. How many shillings did I spend?

11. A man sold some wheat for 8 dollars, some corn for 5 dollars, and some oats for 7 dollars. How much money did he get?

12. 4, and 7, and 5, and 3, and 9, and 2, and 1, and 6, and 8, are how many?

Article 2. Definitions.

Obs. 1. The putting together two or more numbers to find *one number*, as in the preceding examples, is called Addition; and the number thus obtained is called the Sum.*

Addition may be of *Simple* or of *Compound* numbers.

Obs. 2. *Simple Addition* is when the numbers all express things of the *same name* or *kind;* as all dollars, all yards, &c.

Obs. 3. Signs.—A cross (+), one line *horizontal* and the other *perpendicular*, is the sign of addition. It is called *plus*, which is a Latin word, meaning *more*. It *signifies*, that the numbers between which it stands are to be added together.

Two *parallel horizontal* lines (=) are the sign of *equality*. It shows that the number *before* it is equal to the number *after* it. Thus: 6+4=10.

To facilitate the addition of numbers, we subjoin the following

ADDITION TABLE.

2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 " 4	2 " 5	2 " 6	2 " 7	2 " 8	2 " 9	2 " 10	2 " 11
3 " 5	3 " 6	3 " 7	3 " 8	3 " 9	3 " 10	3 " 11	3 " 12
4 " 6	4 " 7	4 " 8	4 " 9	4 " 10	4 " 11	4 " 12	4 " 13
5 " 7	5 " 8	5 " 9	5 " 10	5 " 11	5 " 12	5 " 13	5 " 14
6 " 8	6 " 9	6 " 10	6 " 11	6 " 12	6 " 13	6 " 14	6 " 15
7 " 9	7 " 10	7 " 11	7 " 12	7 " 13	7 " 14	7 " 15	7 " 16
8 " 10	8 " 11	8 " 12	8 " 13	8 " 14	8 " 15	8 " 16	8 " 17
9 " 11	9 " 12	9 " 13	9 " 14	9 " 15	9 " 16	9 " 17	9 " 18
10 " 12	10 " 13	10 " 14	10 " 15	10 " 16	10 " 17	10 " 18	10 " 19

The pupil should be frequently exercised in mental addition, until he is *perfectly familiar* with the operation. Care should be taken, however, that the questions are not answered *mechanically*, from a knowledge of the regular increase of numbers. This may be prevented by asking promiscuous questions.

7 + 6 = how many?
9 + 7 = how many?
9 + 4 = how many?
4 + 3 + 5 = how many?

What is Addition? What is the sum? How may Addition be divided? What is Simple Addition? What is the sign of Addition? What does plus mean?

*By many Authors, the result obtained by adding two or more numbers together, is called the *Amount*: but the term *Amount*, we think properly belongs to *Interest*, and should not be applied to Addition.

$9 + 4 + 7 =$ how many?
$6 + 3 + 4 =$ how many?
$6 + 2 + 7 + 8 =$ how many?
$7 + 8 + 5 + 2 =$ how many?
$6 + 8 + 2 + 4 + 1 =$ how many?
$7 + 2 + 1 + 4 + 3 =$ how many?
$3 + 2 + 1 + 9 + 7 =$ how many?
$9 + 7 + 5 + 6 + 2 + 1 + 3 =$ how many?

Example 1. A man has 2 orchards; one has 24 trees, and the other has 32 trees. How many trees in both orchards?

Solution.—In 24 are 2 *tens* and 4 *units;* in 32 are 3 *tens* and 2 *units.* Now units and tens cannot be added together, because it takes *ten* units to make *one* ten; but we can add *units to units,* and *tens to tens;* and 4 units and 2 units are 6 units, and 2 tens and 3 tens are 5 tens, and 5 tens and 6 units are 56. Ans. 56.

Or we may set the numbers down on the slate, and add them, recollecting to place units under units, and tens under tens. Thus:

```
 32
 24
 --
 56
```

Obs. 4. *We place units under units, tens under tens, &c., because it is more convenient to add them when placed in this manner, as we can only add together numbers of the same name.* Hence—

To add numbers:

Obs. 5. *Write units under units, tens under tens, hundreds under hundreds, &c.*

Commence at the right hand. Add each column separately, and place the result beneath it.

Obs. 6. Proof.—*Begin at the top and add downwards; if the two results are alike, the work is correct.*

1. We add the numbers from the top downwards, in order to take them in a different manner, it being hardly probable that the same mistake would occur by both processes, unless it was intentional.

2. The learner must not think when he takes up the slate, that he can dispense with *thinking and reasoning. This is not the case. He should exercise his mental faculties as much when he uses the slate as when he does not.* His slate is merely used for *convenience* in setting down the operation when it is *too long,* or the numbers *too large* to be easily retained in the mind. That is, it is used to *assist,* and not to *take the place* of the mental faculties.

What does it signify? What is the sign of equality? What does it show? How do we set the numbers down to add? Why do we place units under units, tens under tens, &c.? Where do we commence to add? Where do we place the sum of each column? How do we prove the operation? Why do we add the numbers from the top downwards? Should the scholar dispense with thinking and reasoning when he takes up the slate? What is the use of the slate?

2. In one field are 127 sheep, in another 231, and in another 341. How many are there in all the fields? Ans. 699.

3. In one school are 62 scholars, in another 123, and in another 114. How many are there in all? Ans. 299.

4. One man has 324 dollars, another 232 dollars, another 2123 dollars, and another 6210 dollars. How many dollars have they all? Ans. 8889.

5. In one book are 240 pages, in another 320 pages. in another 1112 pages. in another 1023 pages, and in another 1304 pages. How many pages in all? Ans. 3999.

6. $241 + 122 + 1234 + 21201 + 45101 =$ how many? Ans. 67899.

7. $1234 + 20412 + 30211 + 12132 + 24000 =$ how many? Ans. 87989.

Article 3. Carrying in Addition.

Ex 1. A man has 76 bushels of grain in one box, and 58 in another box. How many bushels has he in both?

Solution.—Here a difficulty presents itself, as 8 units + 6 units = 14 units, which cannot be expressed by one figure; but the learner must recollect that 14 units = 1 ten and 4 units; we will therefore write the 4 units under the units, and reserve the 1 ten for the next column. Thus: 8 units + 6 units = 14 units, which is 1 ten and 4 units; 5 tens + 7 tens = 12 tens, + 1 ten, which was reserved = 13 tens, which is 1 hundred, and 3 tens; and the whole result is, 1 hundred, 3 tens, and 4 units, or 134. Ans. 134.

Hence—When the sum of any column exceeds 9—

Obs. 1. *Set down the unit figure only, and carry the tens to the next column; or in other words, carry* 1 *for every* 10.

This principle may be illustrated as follows:

2. Let it be required to add together 446 and 678.

The sum of 6 units + 8 units = 14 units, or 1 ten and 4 units, which we set down accordingly: 4 tens + 7 tens = 11 tens, or 1 hundred and 1 ten, which we write in their respective orders: 4 hundreds and 6 hundreds = 10 hundreds, or 1 thousand, 0 hundreds, which we also place in their proper places; then adding the several sums together, *as they stand,* we obtain 1124 as the total sum or answer. Ans. 1124.

Operation.

```
  678
  446
 ----
   14 = the sum of the units.
  11  =  "   "   "  tens.
 10   =  "   "   "  hundreds.
 ----
 1124 = the total sum.
```

How do we proceed when the sum of any order exceeds 9? Explain the principle of carrying.

3. Add together 2467, 1924, 2761, and 6279, and prove the operation.

	2467
	1924
	2761
	6279
Sum	13431
Proof.	13431

As the two results are alike, the work is supposed to be correct.

From the preceding illustration, we derive the following

GENERAL RULE FOR ADDITION.

I. *Write the numbers to be added so that the figures of the same order may stand directly under each other, and draw a line beneath.* (Art. 2. Obs. 5.)

II. *Begin at the right hand and add each column separately; and if the sum be* 9, *or less, write it under the column added.* (Art. 2. Obs. 5.)

III. *But if the result should exceed* 9, *set down the unit figure of the result, and carry the tens to the next column.* (Art. 3. Obs. 1.)

IV. *Set down the whole sum of the left hand column.*

Proof.—*Begin at the top and add downwards; if the two results are alike, the work is correct.* (Art. 2. Obs. 6.)

EXERCISES FOR THE SLATE.

1. A man bought a horse for 85 dollars, and a wagon for 67 dollars. What did they both cost him? Ans. 152 dollars.

2. A man has 64 sheep in one field, and 148 in another field. How many sheep has he? Ans. 212.

3. A man being asked his age, answered: "I was 27 years of age when I married, and have been married 48 years." How old was he? Ans. 75 years.

4. A man has owing to him—from A. 463 dollars, from B. 798 dollars, from C. 840 dollars, from D. 56 dollars, and from E. 15 dollars. How much has he owing him? Ans. 2172 dollars.

5. A man has 423 bushels of wheat, 291 bushels of oats, 1479 bushels of corn, and 276 bushels of barley. How many bushels of grain has he in all? Ans. 2469.

6. A man received 1243 dollars for horses, 972 dollars for cattle, 672 dollars for grain, 427 dollars for sheep, 197 dollars for hogs, and has 327 dollars at home. How many dollars has he in all? Ans. 3838.

7. A man has 4 children: one is 18 years old, another is 22 years old, another is 16 years old, and the other is 12 years old;

What is the general rule for Addition? The proof?

the father's age is 22 years more than the sum of his children's ages. Required—the age of the father. Ans. 90 years.

8. A. owes one man 642 dollars, another 927 dollars, another 745 dollars, and another 2457 dollars. What is the sum of his debts? Ans. 4771 dollars.

9. A merchant commenced trading with 14768 dollars; the first year he gained 1794 dollars, the second year he gained 2986 dollars, the third year he gained 3789 dollars, and the fourth year he gained 4697 dollars. How much had he then?

Ans. 28034 dollars.

10. A land-holder has in one farm 1217 acres, in another 976 acres, in another 840 acres, and in another 679 acres. How many acres has he? Ans. 3712.

11. A boy walked—one day 8 miles, another day 12 miles, another day 17 miles, and another day 23 miles. How many miles had he walked in all? Ans. 60.

12. A drover has in one field 329 sheep, and 142 lambs; in another field he has 476 sheep, and 216 lambs; and in a third field he has 512 sheep, and 319 lambs. How many sheep has he?—How many lambs? How many sheep and lambs together?

Ans. to the last. 1994.

13. Four men traded in partnership. A. put in 1269 dollars, B. put in 7642 dollars, C. put in 3768 dollars, and D. put in 5429 dollars. How much did they all put in? Ans. 18108 dollars.

14. A gentleman owns a farm worth 8478 dollars, a store worth 12694 dollars, a house and lot worth 8621 dollars, a vessel worth 10216 dollars, and has 14376 dollars worth of other property.—How much is he worth in all? Ans. 54385 dollars.

15. A man left his property to his wife, two sons, and three daughters: to his wife he gave 14612 dollars, to each of his sons he ave 8279 dollars, and to each of his daughters he gave 6297 dollars. What was the value of his property?

Ans. 50061 dollars.

16. A man has paid on a note—at one time 417 dollars, at another time 312 dollars, at another time 512 dollars, and at another time 794 dollars; he now has 238 dollars to pay. How much was the note? Ans. 2273 dollars.

17. The distances on the Ohio canal are as follows: from Portsmouth to Chillicothe 51 miles; from Chillicothe to Circleville 22 miles; from Circleville to Newark 60 miles; from Newark to Roscoe 41 miles; from Roscoe to Dover 42 miles; from Dover to Massilon 28 miles; from Massilon to Akron 27 miles; from Akron to Cleveland 38 miles. Required—the length of the Ohio canal.

Ans. 309 miles.

18. A man bought a span of horses for 312 dollars, a carriage for 497 dollars, and a harness for 75 dollars. What did they all cost? **Ans. 884 dollars.**

19. A merchant bought calico to the amount of 649 dollars. broadcloth to the amount of 837 dollars, and other goods to the amount of 497 dollars. What did they all cost?

Ans. 1983 dollars.

20 The above merchant sold his goods so as to gain 178 dollars on the calico, 279 dollars on the broadcloth, and 125 dollars on the other goods. How much did he gain, and for what sum did he sell the lot?

Ans. { He gained 582 dollars.
He sold the lot for 2565 dollars.

21. There are four numbers: the first is 1248, the second is 2397, the third is as much as the first and second, and the fourth is as much as the second and third. Required—the third and the fourth numbers, and the sum of the four.

Ans. to the last. 13332.

22. After a battle it was found that 7620 men were killed, 9276 were wounded, 792 had deserted, 5874 had been taken prisoners, 1892 were missing, and 32716 were left fit for action. Of how many men did the army consist at first? Ans. 58170.

23. The second Punic war commenced 529 years after the founding of Rome; Carthage was destroyed 78 years later; the Christian Era commenced 146 after this; the death of Nero was 69 years later; 242 years afterwards Constantine ascended the Roman throne, and the dissolution of the Roman Empire occurred about 165 years after. Required—the age of the Roman Empire.

Ans. 1229 years.

24. Find the sum of the following numbers: two hundred and four; seven hundred and thirty-nine; one thousand and seventeen; three thousand, seven hundred and sixty; fifteen thousand, nine hundred and nine; three hundred and six thousand, one hundred and eight. Ans. 337737.

25. Add the following numbers: two thousand and one; seven thousand, four hundred and seventy-nine; eleven thousand, one hundred and eleven; two hundred and twelve thousand and ninety-five; eight hundred thousand and one; nine hundred and ninety-nine thousand, nine hundred and ninety-nine. Ans. 2032686.

26. In one book are two hundred and thirty-two pages, in another are two hundred and sixty-four pages, in another are three hundred and forty-six pages, and in another are two hundred and ninety five pages. How many pages in all? Ans. 1137.

27. A man's farm cost him eighteen thousand, five hundred dollars; a store cost him twenty thousand, two hundred and fifty dollars; a house and lot ten thousand and fifty dollars, and a boat twelve thousand, five hundred dollars. What was the cost of the whole? Ans. 61300.

28. Required—the sum of 12345 + 67890 + 96432 + 7456 + 6217 + 149321 + 819360. Ans. 1159021.

29. Required—the sum of 97452 + 68714 + 127983 + 15791 + 6829467 + 8932164 + 187437621 + 19734653 + 890072 + 6214573 + 9876543210. Ans. 10106891700.

30. In 1840 the New England States contained 2234822 inhabitants, the Middle States contained 4604345, the Southern States contained 5067843, the Western States contained 4984097, the Territories contained 128534, the District of Columbia contained 43712, and on board vessels of war were 6100. Required—the population and Naval service of the United States in 1840.

Ans. 17069453.

SECTION III.

SIMPLE SUBTRACTION.

Article 1. Mental Exercises.

1. John had 8 apples, and gave 3 of them to his sister. How many had he left?

Take 3 from 8, and how many remains? Ans. 5.

2. Mary had 12 pins, and lost 5 of them. How many had she left?

3. A boy having 18 cents, bought a slate for 10 cents. How many cents had he remaining?

4. A boy had 9 marbles, and gave his brother 5. How many had he left?

5. A lady having 16 yards of cloth, cut off 9 yards. How many yards were left?

6. A man having 14 dollars, paid away 7 dollars. How many dollars had he then?

7. James went 15 miles, and John 5. How much farther did James go than John?

8. Henry had 12 cents, and spent 6 of them. How many had he left?

9. William made 17 marks on his slate, and rubbed out 8 of them. How many marks were there left?

10. John's book had 20 pages in it, and he tore out 10 of them. How many pages were left?

11. Fourteen boys were standing together, and 5 of them went away. How many remained?

12. A man having 22 sheep, sold 11 of them. How many had he left?

Article 2. Definitions, &c.

Obs. 1 The finding the *difference* between two numbers, as in the preceding examples, is called Subtraction. It may be defined —the *taking of a less number from a greater.*

Subtraction may be either of *Simple* or of *Compound* numbers.

Obs. 2. *Simple Subtraction* is when the numbers all express things of the *same name,* or *kind*—as all dollars, all cents, all yards, &c.

Obs. 3. *The greater number is called the* MINUEND, (which means *to be diminished.*) *The lesser number is called the* SUBTRAHEND, (which means *to be subtracted.*) *The result, or answer, is called the* DIFFERENCE, or REMAINDER.

Obs. 4. SIGN.—The sign of Subtraction is a *short horizontal line* (—), called MINUS, which means *less,* and signifies that the number *after* it is to be taken from the number *before* it. Thus: 8—5=3 shows that 5 taken from 8 leaves 3, and reads, 8 *minus* 5 is *equal* to 3.

To facilitate the progress of the learner, we subjoin the following

SUBTRACTION TABLE.

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 “ 1	3 “ 1	4 “ 1	5 “ 1	6 “ 1
3 “ 2	4 “ 2	5 “ 2	6 “ 2	7 “ 2
4 “ 3	5 “ 3	6 “ 3	7 “ 3	8 “ 3
5 “ 4	6 “ 4	7 “ 4	8 “ 4	9 “ 4
6 “ 5	7 “ 5	8 “ 5	9 “ 5	10 “ 5
7 “ 6	8 “ 6	9 “ 6	10 “ 6	11 “ 6
8 “ 7	9 “ 7	10 “ 7	11 “ 7	12 “ 7
9 “ 8	10 “ 8	11 “ 8	12 “ 8	13 “ 8
10 “ 9	11 “ 9	12 “ 9	13 “ 9	14 “ 9
6 from	7 from	8 from	9 from	10 from
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 le’ves0
7 “ 1	8 “ 1	9 “ 1	10 “ 1	11 “ 1
8 “ 2	9 “ 2	10 “ 2	11 “ 2	12 “ 2
9 “ 3	10 “ 3	11 “ 3	12 “ 3	13 “ 3
10 “ 4	11 “ 4	12 “ 4	13 “ 4	14 “ 4
11 “ 5	12 “ 5	13 “ 5	14 “ 5	15 “ 5
12 “ 6	13 “ 6	14 “ 6	15 “ 6	16 “ 6
13 “ 7	14 “ 7	15 “ 7	16 “ 7	17 “ 7
14 “ 8	15 “ 8	16 “ 8	17 “ 8	18 “ 8
15 “ 9	16 “ 9	17 “ 9	18 “ 9	19 “ 9

NOTE.—This Table should be *thoroughly* committed to memory before proceeding farther.

What is Subtraction? How may it be defined? How divided? What is Simple Subtraction? What is the greater number called? What does minuend mean? What is the lesser number called? What does subtrahend mean? What is the result, or answer, called? What is the sign of Subtraction? What does minus mean? What does it signify? What does 8—5=3 show? How is it read?

$6 - 4 =$ how many?
$14 - 3 =$ how many?
$17 - 7 =$ how many?
$12 - 5 =$ how many?
$18 - 10 =$ how many?
$17 - 6 =$ how many?
$18 - 7 + 3 =$ how many?
$17 - 8 + 9 =$ how many?
$14 - 5 + 6 =$ how many?
$12 - 3 + 9 =$ how many?
$16 - 4 + 8 =$ how many?
$19 - 10 + 7 =$ how many?

Obs. 5. *A line, or vinculum (—), drawn over two or more numbers, signifies that they are to be taken together as one number. A parenthesis* ( ) is also sometimes used for the same purpose. Thus:— $18 - \overline{4 + 2}$, or $18 - (4 + 2)$ signifies that $4 + 2$, or 6, is to be taken from 18, which leaves 12; but $18 - 4 + 2$ signifies that 4 is to be taken from 18, and 2 added to the remainder, which makes 16.

$18 - \overline{4 + 7}$ how many?
$7 + 6 - \overline{3 + 8} =$ how many?
$7 + 6 + 3 + 6 - (9 - 4 + 7) =$ how many?
$12 - 3 - 2 - \overline{4 + 7 + 6} =$ how many?
$7 + 8 + 4 - (10 + 7 + 6 - 4) =$ how many?
$6 + 8 - \overline{5 + 2} + 7 + 6 - (9 + 8) =$ how many?

Obs. 6. *Subtraction is the reverse of Addition.* Addition is finding the *sum*, and Subtraction is finding the *difference* of numbers. Now if 3 taken from 9 leaves 6, it is evident that 3 added to 6 will equal 9. Hence:

To prove Subtraction—

Obs. 7. *Add the remainder and subtrahend together, and if their sum is equal to the minuend the work is correct.*

Ex. 1. A man having 98 dollars, paid 52 dollars for a horse.—How much had he left? Ans. 46 dollars.

Solution.—98 is composed of 9 tens and 8 units; 52 is composed of 5 tens and and 2 units: now we cannot take units from tens, nor tens from units, because it takes 10 of the one (units), to make 1 of the other (tens); but we can take units from units, and tens from tens. Thus: 2 units from 8 units leave 6 units, and 5 tens from 9 tens leave 4 tens, and 4 tens and 6 units = 46.

Or, we may set the numbers down, the less under the greater,

What does a line drawn over two or more numbers signify? What other mark is used for the same purpose? Of what is Subtraction the reverse?—Show the comparison. How do wo prove Subtraction? Why cannot we take units from tens, or tens from units? How do we set the numbers down to subtract? Why do we place units under units, &c.? (Sect. 11, Art. 2, Obs. 4.) Where do we commence to subtract? Where do we set the remainder?

and subtract one from the other, commencing at the right hand. Thus:

	Proof.	
98	52	In subtracting, we say, 2 from 8 leaves 6, and
52	46	place the 6 under the 2; then 5 from 9 leaves 4, and
—	—	place the 4 under the 5.
46	98	Hence—To subtract numbers:

Obs. 8. *Place the less number under the greater, setting units under units, tens under tens, &c., and draw a line beneath. Commence at the right hand. Take successively, each figure in the lower line from the figure above it, and write the difference below.*

2. A man had 288 sheep, and sold 184 of them. How many had he left? Ans. 104.

3. A man has lived 97 years, and was married when he was 25 years of age. How many years was he married? Ans. 72.

4. A gentleman having 948 dollars, paid away 736 dollars. How many dollars had he remaining? Ans. 212.

5. One book contains 489 pages, and another contains 372 pages. How many more pages in the one than in the other? Ans. 117.

6. A man's property is worth 6894 dollars, and his debts amount to 4080 dollars.. How much will remain after paying his debts? Ans. 2814 dollars.

Article 3. Borrowing and Carrying in Subtraction.

Ex. 1. A man gave 95 dollars for a horse, and 68 dollars for a wagon. How much more did he give for the horse than for the wagon? Ans. 27 dollars.

Solution.—Here a difficulty occurs; for we cannot take 8 units from 5 units. But 95 is composed of 9 tens and 5 units; now if we take 1 ten from the 9 tens, and add it to the 5 units, we shall have 8 tens and 15 units; then 8 units from 15 units leaves 7 units, and 6 tens from 8 tens leaves 2 tens; and 2 tens and 7 units = 27.

2. Required—the difference between 124 and 86.

Operation.

	tens.		units.
124 =	11	+	14
86 =	8	+	6
Ans.	3	+	8 = 38 rem.

In this example we cannot take 6 units from 4 units, therefore we borrow 1 ten from the 2 tens, and add the 4 units, making 14 units, and 14 — 6 = 8 units. Again, we cannot take 8 tens from 1 ten, so we borrow the 1 hundred and add it to the 1 ten, which makes 11 tens; then 11 — 8 = 3 tens, and 3 tens and 8 units = 38.

Explain the method of separating a number into its numerical parts, as in Examples 1st and 2nd. Explain the principle of borrowing ten. When the lower figure is the largest, can we subtract without resolving the numbers into their numerical parts?

Obs. 1. The learner will perceive in both these examples, that *we add ten to the figure of the minuend when it is less than the corresponding figure of the subtrahend.* Indeed, it cannot be otherwise, when we borrow 1 of the next higher order, as 1 *unit of any order is equal to* 10 *units of the next lower order.* (Sect. 1. Art. 2, Obs. 3.) *This is called borrowing ten.*

3. Required—the difference between 526 and 328.

Operation.

```
 526
 328
 ---
 198
```

In the solution of this example, instead of resolving the numbers into their numerical parts, as above, we will set them down according to Obs. 8., Art. 2. To take 8 from 6 is impossible; therefore we will borrow 1 (ten) from the 2 (tens), and add it to the 6, making 16; then $16-8=8$. Now to compensate for the one we borrowed, we will add 1 to the next figure of the *subtrahend,* (which is the same as taking one from the next figure of the *minuend,*) and $2+1=3$; then $2-3$ is impossible; hence we will borrow 1 (hundred), and add it to the 2 (tens), making 12 (tens), and $12-3=9$. Then $3+1=4$, and $5-4=1$. Ans. 198.

Remark 1.—*We add* 1 *to the next figure of the subtrahend, because it is more convenient than to take* 1 *from the minuend.*

2.—The reason of carrying 1 is evident from this fact: *As we borrow* 1 *from the minuend, we must either make the figure of the minuend,* (*which we borrowed from*) 1 *less, or the figure of the subtrahend immediately beneath it* 1 *greater, to pay for that we borrowed.*

From the foregoing remarks and illustrations, we derive the following

GENERAL RULE FOR SUBTRACTION.

I. *Write the less number under the greater, that the figures of the same order may stand under each other.* (Art. 2, Obs. 8.)

II. *Commence at the right hand. Take successively each figure of the lower number from the one above it, and write the remainder below.* (Art. 2, Obs. 8.)

III. *When the figure in the lower number is the largest, add ten to the figure above it, after which subtract as usual, remembering to add, or carry* 1 *to the next figure in the lower number before the next subtraction.* (Art. 3, Obs. 1. Rem.)

Proof.—*Add the remainder and subtrahend together; if their sum is equal to the minuend, the work is correct.* (Art. 2, Obs. 7.)

EXERCISES FOR THE SLATE.

1. A man had 1000 dollars, and paid away 478 dollars. How many dollars had he left? Ans. 522.

2. A man bought a farm for 7864 dollars, and afterwards sold it

---

Explain the process. Why do we add 1 to the next figure of the subtrahend? Explain the reason of carrying. What is the general rule for Subtraction? The proof?

for 975 dollars less than he gave for it. For how much did he sell it? Ans. 6889 dollars.

3. A merchant bought 8473 dollars worth of goods, and sold them for 10312 dollars. How many dollars did he gain? Ans. 1839.

4. A man paid 6767 dollars for land, and 12843 dollars for merchandise. How much more did he pay for the merchandise than for land? Ans. 6076 dollars.

5. A merchant sold goods to the amount of 15784 dollars; he paid 2897 dollars less for them. How much did they cost him? Ans. 12887 dollars.

6. America was discovered in 1492. How many years since, it now being 1849? Ans. 357.

7. The United States declared their Independence in the year 1776. How many years since, the present year being 1849? Ans. 73.

8. A gentleman's income is 4742 dollars a year, and his expenses are 3953 dollars a year. How much does he save per year? Ans. 789 dollars.

9. A farmer raised 1747 bushels of grain one year, and 1699 bushels the next year. How many bushels did he raise the first year more than the second? Ans. 48.

10. At a certain school are 423 students, of which 137 are young ladies. How many gentlemen are there? Ans. 286.

11. From Columbus, (Ohio,) to Cincinnati, by way of Springfield, it is 127 miles; and from Springfield to Cincinnati it is 85 miles.—How far is it from Columbus to Springfield? Ans. 42 miles.

12. In the year 1800, the population of the United States was 5305925, and in 1840 it was 17069453. Required—the increase of population in 40 years. Ans. 11763528.

13. In 1830 the population was 12861192. Required—the increase in ten years. Ans. 4208261.

14. A drover having 1468 sheep, sold 948; he then bought 467, and sold 987. How many had he left? Ans. None.

15. A vessel, the cargo of which was valued at 84000 dollars, in a storm lost part of her cargo valued at 27212 dollars. What was the value of the remaining part? Ans. 56788 dollars.

16. A man bought 58 dollars worth of wheat, 97 dollars worth of pork, 73 dollars worth of cheese, and a fine horse and buggy for 347 dollars. He gave his note for 497 dollars, and paid the rest in money. How much money did he pay? Ans. 78 dollars.

17. A man borrowed at one time 217 dollars, at another time 313 dollars, and at another time 428 dollars; he afterwards paid 869 dollars. How much did he then owe? Ans. 89 dollars.

18. A man at his death left each of his two sons 5732 dollars, and each of his three daughters 4784 dollars, and his widow the

balance of his property. How much did the widow receive, the estate being worth 40000 dollars? Ans. 14184 dollars.

19. A gentleman's property was worth 34768 dollars; but a store worth 4762 dollars, and 14796 dollars worth of goods were destroyed by fire. How much had he left? Ans. 15210 dollars.

20. A man owning 4821 acres of land, gave one son 623 acres to another son 427 acres, and to another son 873 acres. How many acres had he left? Ans. 2898.

21. A man's income is 6000 dollars a year. He spends 372 dollars for clothing, 724 dollars, for house rent, 892 dollars for provisions, 429 dollars for servants, and 527 dollars for traveling.—How much has he left at the end of the year? Ans. 3056 dollars.

22. A man owing 1894 dollars, paid at one time 723 dollars, at another time 674 dollars, and at another time 500 dollars. How did the account then stand? Ans. He overpaid 3 dollars.

23. In 1840 the population of the New England States was 2234822; of the Middle States 4604345; of the Southern States, including Florida, 5122320; and of the Western States, including Wisconsin and Iowa, 5058154. Required—the excess of the population of the Southern and Western States over that of the New England and Middle States? Ans. 3341307 inhabitants.

24. In the last question, required—the excess of the population of the Middle States over that of the New England States.

Ans. 2369523.

25. Also, required—the excess of the population of the Southern States over that of the Western States. Ans. 64166.

26. By the last census, the population of the Northern States* was 9807007, and the population of the Southern States* was 7256346. Required—the excess of the population of the Northern over that of the Southern States. Ans. 2550661.

27. A man bought 27 dollars worth of sugar, 19 dollars worth of spice, 43 dollars worth of coffee, and 10 dollars worth of tea. He gave in payment a one hundred dollar bill. Had he ought to receive any thing back? If so, how much?

Ans. He should receive 1 dollar.

28. A certain store-house contained 5970 bushels of wheat, 3752 bushels of corn, 5978 bushels of rye, and 9847 bushels of oats. It caught fire, and 19768 bushels of grain were saved. How many bushels were destroyed? Ans. 5779.

29. A ship of war sailing with 650 men, lost in one battle 29 men, in another 37, and by sickness 19 more. How many were still living? Ans. 565.

30. Four men bought a lot of land for 978 dollars. The first man paid 386 dollars; the second paid 97 dollars less than the

*In this example, by the Northern are meant the Free States, and by the Southern are meant the Slave States.

first; the third paid 73 dollars less than the second; and the fourth paid the balance. How much did the second, third and fourth men pay?

Ans. The second man paid 289 dollars.
" third " 216 "
" fourth " 87 "

31. From six trillions, eighty-seven millions, twelve thousand and three, take twenty billions, one hundred millions, two hundred and sixteen thousand and nine. Ans. 5979986795994.

SECTION IV.

SIMPLE MULTIPLICATION.

Article 1. Mental Exercises.

1. If one lemon cost 4 cents, how much will 3 lemons cost?

Solution.—If one lemon costs 4 cents, 2 lemons will cost $4+4=8$ cents, and three lemons will cost $8+4$, or $4+4+4=12$ cents. Ans. 12 cents.

2. What costs 4 oranges at 6 cents apiece?
$6+6+6+6=$ how many?
3. At 2 dollars a pair, what will 3 pair of boots cost?
4. In one yard are 3 feet; how many feet in 4 yards?
5. How much will 5 yards of ribon cost at 8 cents a yard?
6. How much will 7 lead pencils cost at 5 cents apiece?
7. What cost 9 pounds of saleratus at 6 cents a pound?
8. At 10 cents a pound, how much will 10 pounds of sugar cost?
9. What cost 8 slates at 6 cents apiece?
10. At 4 dollars a barrel, how much will 7 barrels of flour cost?
11. If one sheep costs 2 dollars, how much will 11 sheep cost?
12. At 12 cents a pound, how much will 6 pounds of butter cost?

Article 2. Definitions, &c.

There is an orchard containing 4 rows of trees, and 6 trees in a row. How many trees are there in the orchard?

Instead of adding the 4 six times, or the 6 four times, as in the preceding examples, we will shorten the operation by multiplying one by the other. Thus:

$$\begin{array}{r} 6 \\ 4 \\ \hline 24 \end{array}$$

To illustrate this, we will suppose these stars to be the orchard. We perceive that there are 4 rows of stars, and 6 stars in a row, and 24 stars in all. Therefore 4 times 6 are 24. Now count the stars the other way. We perceive that there are 6 rows, and 4 stars in a row, and 24 stars

```
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
*  *  *  *  *  *
```

in all, as before. Hence, 6 times 4 are 24. Thus, we perceive it makes no difference whether we multiply 6 by 4, or 4 by 6, the result will still be the same—that is, 24. Ans. 24.

At 5 dollars a yard, what will 5 yards of cloth cost?

5 times 5 are how many?

Obs. 1. This method of repeating a number is called MULTIPLICATION. Multiplication may therefore be defined—

A method of repeating any number a given number of times.

Multiplication may be either of *Simple* or *Compound* numbers.

Obs. 2. *Simple Multiplication is when the number to be multiplied expresses things of but one name or kind*—as dollars, yards, &c.

Obs. 3. *The number which we multiply, or which is to be* REPEATED, *is called the* MULTIPLICAND.

The number which we multiply by, or which shows how many times the other is to be repeated, is called the MULTIPLIER.

The result, or answer, is called the PRODUCT.

The multiplicand and multiplier taken together are called the FACTORS, *or producers* of the product.

Obs. 4. SIGN.—The sign of multiplication is a *cross* (×) *like the letter* X, *and signifies that the numbers between which it stands are to be multiplied together.* Thus: 4 × 6 signifies that 4 and 6 are to be multiplied together, the product of which is 24.

Before proceeding farther, the learner must commit accurately to memory the following

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

What is Multiplication? How is it divided? What is Simple Multiplication? What is the number we multiply called? What is the number we multiply by called? What is the result, or answer, called? What are the multiplicand and multiplier, taken together, called? What is the sign of multiplication? What does it signify?

MULTIPLICATION TABLE—CONTINUED.

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 · 18	2 " 20		2 " 24
3 " 24	3 " 27	3 " 30		3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 5	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 1	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

NOTE.—The pupil should not leave this Table until he has *thoroughly* committed it to memory. Without knowing it *perfectly*, no one can become a good arithmetician.

$7 \times 6 =$ how many?
$9 \times 5 =$ how many?
$7 \times 4 =$ how many?
$8 \times 7 =$ how many?
$11 \times 6 =$ how many?
$12 \times 8 =$ how many?

$6 \times 9 =$ how many?
$7 \times 8 =$ how many?
$9 \times 9 =$ how many?
$9 \times 7 =$ how many?
$11 \times 12 =$ how many?
$11 \times 7 =$ how many?

In each of these questions we took one number as many times as there were units in another. Hence—

Obs. 5. *To multiply one number by another is to take the multiplicand as many times as there are units in the multiplier.*

The product of any two numbers is the *same*, whichever factor is taken as the multiplier.

Obs. 6. As it does not alter the name of a number to repeat it, and the multiplicand is always the number repeated, it is evident that *the product will always be of the same name as the multiplicand.*

REMARK 1.—The multiplier must always be an *abstract* number, as it expresses the *number of times* the multiplicand is taken.

2.—An *abstract* number is one that has no relation to any particular object whatever, but merely expresses the number: as 14, 29, &c.

3—When a number has some relation to a particular object, it is called a *concrete* number: as 5 dollars, 8 miles, &c.

To multiply one number by another, is to take the multiplicand how many times? Does it make any difference in the result which factor is used as the multiplier? Does it alter the name of a number to repeat it? Which factor is repeated? Of what name then is the product? What must the multiplier always be? Why? What is an abstract number? A concrete number? What numbers can be multiplied together? If the multiplicand is an abstract number, what is the product? If the multiplicand is a concrete number, what is the product?

4.—As we cannot multiply by a *concrete* number, it follows that we can only multiply *two abstract* numbers together, in which case the product is *abstract*, or an *abstract* and *concrete* number together, in which case the product is *concrete*.

Ex. 1. If 1 yard of cloth cost 5 dollars, what will 6 yards cost?

Here we have two concrete numbers, viz: 5 dollars and 6 yards. Now the multiplier should be an abstract number, for it would be equally as absurd to multiply 5 dollars by 6 yards, as it would be to multiply 5 houses by 6 gardens. But we can resolve the question in this manner: *if* 1 *yard costs* 5 *dollars,* 6 *yards will cost* 6 *times as much as* 1 *yard, or* 6 *times* 5 *dollars, which is* 30 *dollars.* In this way we make the 6 an abstract number.

Let the learner resolve the following questions in the same manner:

2. A man bought 12 cows, paying 11 dollars apiece for them. How much did they cost him?

3. How much would 9 calves come to, at 4 dollars apiece?

4. How much would 8 pair of boots cost, at 5 dollars a pair?

5. How much would 32 yards of cloth cost, at 3 dollars a yard?

Solution.—To obtain the result in this question, we set one number under the other, and multiply each figure of the multiplicand separately. Thus—3 times 2 units are 6 units, and 3 times 3 tens are 9 tens, and 9 tens and 6 units are 96.

$$\begin{array}{r} 32 \\ 3 \\ \hline \text{Ans. } 96 \text{ dollars.} \end{array}$$

Obs. 7. Proof.—*Take* 1 *from the multiplier; multiply the multiplicand by the remainder, and to the product add the multiplicand; if this result is the same as the first, the work is correct.*

Proof of the last Example.

$$\begin{array}{r} 32 \\ 3-1= \quad 2 \\ \hline 64 \\ 32 \\ \hline 96 \end{array}$$

As this result is the same as the first, the work is supposed to be correct.

The reason of this rule is too evident to need demonstration.—Let the learner see if he cannot demonstrate it himself.

6. Multiply 321 by 4. Ans. 1284.

7. If 1 pound of butter cost 8 cents, what will 16 pounds cost?

$6 \times 8 = 48 = 4$ tens and 8 units; we set down the 8 units; then 8 times 1 ten $=$ 8 tens, and 4 tens make 12 tens.

Operation.

$$\begin{array}{r} 16 \\ 8 \\ \hline \text{Ans. } 128 \text{ cents.} \end{array}$$

This principle may be illustrated as follows:

8. Required—the product of 247 by 9.

1st *Operation.*

```
   247
     9
  ----
    63 = product of 9 into 7 units.
   36  =    "      "    " 4 tens.
  18   =    "      "    " 2 hundreds.
  ----
Ans. 2223 = "      "    " 247.
```

2d *Operation.*

$247 = 200 + 40 + 7.$

$200 \times 9 = 1800.$

$40 \times 9 = 360.$

$7 \times 9 = 63.$

Ans. 2223

By each of these methods it will be perceived that the *ten's figure* of each partial product is added to the next partial product.—Thus, in 63, the 6 is added to the next partial product, (36); and in 36, the 3 is added to the next partial product, (18.) Now as this is always the case, we may shorten the operation by not setting down all the work, but merely carrying the *tens*, as follows:

```
     247
       9
    ----
Ans. 2223
```

Here we say 9 times 7 are 63, and set down the 3, and carry the 6; then, 9 times 4 are 36, and 6 to carry = 42; we set down 2 and carry 4; finally, 9 times 2 are 18, and 4 to carry = 22.

The learner will perceive that each figure of the product occupies the same place, (*numerically,*) as the figure multiplied; that is, if the figure multiplied is *units*, the product is *units*; if *tens*, the product is *tens*, &c. Hence—

Obs. 8. *When the multiplier occupies the unit's place, each figure of the product occupies the same place, (numerically,) as the figure multiplied.*

From the preceding illustrations, we deduce the following

RULE—When the Multiplier is less than 12.

I. *Write the multiplier under the unit figure of the multiplicand.*

II. *Commence at the right hand. Multiply successively each figure of the multiplicand by the multiplier, setting down the unit figure of each product, and carrying the tens to the next product, as in Addition.*

Proof.—*Take 1 from the multiplier; multiply the multiplicand by the remainder, and to the product add the multiplicand; if this result is the same as the first, the work is correct.* (Obs. 7.)

Give the solution of Example 1, and show how the multiplier is made an abstract number. How do we prove multiplication? Explain the principle of carrying, and show why it is correct. Is there any shorter way than to set down the whole product of each figure multiplied, or of separating the multiplicand into its numerical parts? Explain it. What place does each figure of the product occupy, numerically? What inference is deduced from this? What is the Rule when the multiplier is less than 12? The proof?

EXERCISES FOR THE SLATE.

1. How much would 356 pair of shoes cost, at 2 dollars a pair? Ans. 712 dollars.

2. How much would 431 yards of cloth cost at 5 dollars a yard? Ans. 2155 dollars.

3. If a man rides 120 miles in one day, how far can he ride in 6 days? Ans. 720 miles.

4. In one mile are 320 rods. How many rods in 8 miles? Ans. 2560.

5. How much would 9 horses cost at 140 dollars apiece? Ans. 1260 dollars.

6. How much would 398 pounds of salaratus cost, at 6 cents a pound? Ans. 2388 cents.

7. How much would 554 hats cost, at 5 dollars apiece? Ans. 2770 dollars.

8. There are 8 quarts in a peck. How many quarts in 389 pecks? Ans. 3112.

9. How much would 674 yards of cloth cost, at 4 dollars a yard? Ans. 2696 dollars.

10. How much would 6 carriages cost, at 494 dollars apiece? Ans. 2964 dollars.

11. How much would 738 barrels of flour cost, at 6 dollars a barrel? Ans. 4428 dollars.

12. At 8 shillings a day, how many shillings can a man earn in 325 days? Ans. 2600.

13. How many yards will 12 pieces of cloth contain, each piece containing 42 yards? Ans. 504.

14. How much would the above cloth cost, at 9 cents a yard? Ans. 4536 cents.

15. How much would 386 hats cost, at 8 dollars apiece? Ans. 3088 dollars.

16. How much would 273 pair of boots cost, at 5 dollars a pair? Ans. 1365 dollars.

17. How much would 12 stoves cost, at 37 dollars apiece? Ans. 444 dollars.

18. How much would 198 pounds of coffee cost, at 11 cents a pound? Ans. 2178 cents.

19 How much would 177 pounds of sugar cost, at 7 cents a pound? Ans. 1239 cents.

20. How much would 789 acres of land cost, at 12 dollars an acre? Ans. 9468 dollars.

Article 3. When the Multiplier exceeds 12.

Ex. 1. What will 243 acres of land cost, at 24 dollars an acre?

Solution.—We meet with a difficulty in this example, as our mul-

tiplier is greater than 12. But 24 is composed of 2 tens and 4 units.

We will therefore work this question as follows:

In the first place we multiply by the 4 *units*, as usual. We next multiply by the 2 *tens*. Now in reality, we multiply by 20, because 2 tens = 20. Then 20 times 3 are 60; or, if we reject the cipher, 2 times 3 are 6; but as the 2 is 2 *tens*, the 6 is also 6 *tens*, and should therefore occupy the *ten's place*, the same as if the cipher were added. We multiply the remaining figures as usual. Finally, we add the two products together, (because 20 + 4 = 24,) *as they stand*, and their sum is the product required.

Operation.

```
 243
  24
 ---
 972 = product by 4 units.
486  =    "       2 tens.
----
5832 =    "      24
```

REMARK.—In writing our factors, we place figures of the same order under each other.

2. Multiply 5297 by 642.

The 6 in the multiplier, in this example, occupies the *hundred's* place, and to multiply by it is the same as to multiply by 600. Therefore, we place the first figure of the product in the *hundred's place.* Otherwise, we proceed as in the last example.

Operation.

```
      5297
       642
     -----
     10594
    21188
   31782
   -------
Ans. 3400674
```

Obs. 1. From the preceding illustrations we perceive that *the first figure of each partial product occupies the same place, numerically, as the figure by which we multiply.* Hence—When the multiplier exceeds 12:

Obs. 2. *We first multiply by each figure of the multiplier separately, remembering to place the first figure of the product directly under the figure by which we multiply; and then add together the several partial products, as they stand, for the required product.*

3. Multiply 49832 by 148. Ans. 7375136.

4. How much would 1927 tons of iron cost, at 108 dollars a ton?

Explain the solution of Example 1st, and show the several steps of the operation. What place, numerically, does the first figure of each partial product occupy? Give the reason of this. How do we proceed when the multiplier exceeds 12? When there are ciphers between the significant figures of the multiplier how do we proceed? What is the general rule for Multiplication? The proof?

In this example there are no *tens*, but as 0 multiplied into any number produces only 0, we may omit the 0, and multiply by the significant figures, only, observing to place the first figure of each partial product according to Obs. 2. Hence—

Operation.

```
  1927
   108
 -----
 15416
1927
------
208116
```

Ans. 208116 dollars.

When there are ciphers between the significant figures of the multiplier:

Obs. 3. *We multiply by the significant figures only, and in all other respects proceed according to* Obs. 2.

5. Multiply 1776 by 305. Ans. 541680.

From the preceding remarks and illustrations, we derive the following

GENERAL RULE FOR MULTIPLICATION.

I. *Write the multiplier under the multiplicand, placing figures of the same order under each other.*

II. *Commence with the units, and multiply each figure of the multiplicand by each significant figure of the multiplier, placing the first figure of each partial product directly under the figure by which we multiply.* (Obs. 2.)

III. *Add together the several partial products as they stand; their sum will be the product required.* (Obs. 2.)

Proof.—*Take* 1 *from the multiplier; multiply the multiplicand by the remainder, and to the product add the multiplicand; if this result is the same as the first, the work is correct.* (Art. 2, Obs. 7.)

EXERCISES FOR THE SLATE.

1. How much would 596 sheep cost, at 3 dollars apiece? Ans. 1788 dollars.

2. How much would 1487 yards of cloth cost, at 5 dollars a yard? Ans. 7435 dollars.

3. How much would 276 horses cost, 98 dollars apiece? Ans. 27048 dollars.

4. How much would 138 acres of land cost, as 47 dollars an acre? Ans. 6486 dollars.

5. How much would 976 yoke of oxen cost, at 67 dollars a yoke? Ans. 65392 dollars.

6. How much would 235 tons of hay cost, at 16 dollars a ton? Ans. 3760 dollars.

7. How much would 2798 acres of land cost, at 23 dollars an acre? Ans. 64354 dollars.

8. If a man can raise 47 bushels of corn on one acre of land, how many bushels can he raise on 179 acres? Ans. 8413.

9. How much would 329 carriages come to, at 574 dollars apiece? Ans. 188846.

10. How many days has a man lived who is 67 years old, allowing 365 days to the year? Ans. 24455.

11. Allowing he has lived but 59 years, how many days would it be? Ans. 21535.

12. If he has lived 84 years, how many days is it? Ans. 30660.

13. If it takes 872 men 204 days to dig a canal, how long will it take 1 man to dig it? Ans. 177888 days.

14. How much would 279 horses cost, at 207 dollars apiece? Ans. 57753 dollars.

15. How much would the above horses cost, at 196 dollars apiece? Ans. 54684 dollars.

16. How much would they cost, at 175 dollars apiece? Ans. 48825 dollars.

17. How many dollars would 4072 men receive, if each received 408 dollars? Ans. 1661376.

18. If a vessel should sail 129 miles a day, how many miles would she sail in 437 days? Ans. 56373 miles.

19. How much would 19407 pounds of opium cost, at 52 shillings a pound? Ans. 1009164 shillings.

20 How much would 96 thousand feet of boards cost, at 13 dollars a thousand? Ans. 1248 dollars.

21. How much would 284 hogsheads of molasses cost, at 29 dollars a hogshead? Ans. 8236 dollars.

22. If 47 men should form a partnership, each man paying 7684 dollars, what amount of capital would be invested? Ans. 361148 dollars.

23. How much would 923 pieces of broadcloth cost, at 312 dollars a piece? Ans. 287976 dollars.

24. How much would 867 town lots cost, at 489 dollars apiece? Ans. 423963 dollars.

25. Suppose 476 men were to receive 237 dollars apiece for doing a piece of work, how much would they all receive? Ans. 112812 dollars.

26. If a carriage wheel turn over 348 times in going a mile, how many times will it turn over in going 957 miles? Ans. 333036 times.

27. Multiplyone hundred and two thousand, four hundred and seven, by threethousand and seventeen. Ans. 308961919.

28. Multiply four hundred and ninety-six thousand, five hundred and fourteen, by thirty-three thousand, nine hundred and ninety-nine. Ans. 16880979486.

29. Multiply two million, seven hundred and five, by six hundred and seventy-two thousand, two hundred and fifteen. Ans. 1344903911575.

30. Multiply sixty-six trillion, six hundred and sixty-six billion, six hundred and sixty-six, by one hundred and forty-four million, one hundred and forty-four.

Ans. 9599913599999904095904.

Article 4. Contractions in Multiplication.

Case 1.—*When the Multiplier is* 10, 100, 1000, &c.—

Ex. 1. How much must I pay for 47 cows, at 10 dollars apiece?

Ans. 470 dollars.

As every removal of a figure towards the left increases its value ten times, (Sect. 1. Art. 2. Obs. 3.) it follows, that to multiply any number by 10, we need only remove it one place to the left, or simply annex a cipher; to multiply by 100, remove it *two* places, or annex *two* ciphers; to multiply by 1000, remove it *three* places, or annex *three* ciphers, &c. Hence—To multiply by 1 with any number of ciphers annexed:

Obs. 1. *Annex to the multiplicand as many ciphers as there are at the right of the multiplier.*

Note.—*Annex* means to *place after, or at the right hand.*

2. There are 10 cents in one dime; how many cents are there in 327 dimes?

3. There are 100 cents in one dollar; how many cents in 385 dollars?

4 There are 1000 mills in one dollar; how many mills in 476 dollars?

5. There are 10000 mills in an eagle; how many mills in 835 eagles?

6. How many pages are there in 575 books, each book having 1000 pages?

Case 2.—*Multiplication by Composite Numbers.*

Obs. 2. *A Composite number is a number which can be obtained by multiplying two or more factors together, each of which is greater than unity.* Thus, 12 can be obtained by multiplying 3 and 4, or 2 and 6 together; therefore, 12 is a composite number. Also, 24 is composed of 8×3, or 4×6, or 12×2, or $4 \times 2 \times 3$, or $2 \times 2 \times 2 \times 3$, either of which expressions multiplied together will produce 24. Hence, 24 is a Composite number.

Is 48 a composite number? Is 49? 54? 81? 120? 144? 132? Name the factors, or parts, of each of the above numbers.

The learner will find all these, as well as all the composite num-

---

What is it necessary to do to multiply by 10? Why? By 100? Why? By 1000? Why? How then can we multiply by 1 with any number of ciphers annexed? What does annex mean? What is a Composite number? Give examples. Will any number admit of more than one set of factors? Name some such numbers.

bers following, in the *Multiplication Table*, if he examines it attentively, together with one set of factors of which they are composed. Many of them, however, will admit of other factors than those named in the Table. Let the learner see if he cannot ascertain what they are.

Obs. 3. *The numbers, or factors, which are multiplied together to produce any number, are called the component parts of that number.* Thus: 3 and 4 are the component parts of 12, because $3 \times 4 = 12$; and 3, 2 and 5 are the component parts of thirty, because $3 \times 2 \times 5 = 30$.

What are the component parts, or factors, of 36? 42? 56? 72? 84? 90? 96? 100? 121? 132? 144?

REMARK 1.—*A number which cannot be produced by the multiplication of two or more factors, is called a* PRIME *number.* Thus: 2, 3, 5, 7, 11, &c., are prime numbers.

2.—*Every number must be either a prime or a composite number.* Every even number is a composite number. An even number always ends with 0, 2, 4, 6, or 8, and can always be divided by 2—that is, 2 is one factor of all such numbers All numbers ending with 5, or 0, are composite also, because 5 is one factor of all such numbers.

3.—*A prime number is always odd.* An *odd* number always ends with 1, 3, 5, 7, or 9, and cannot be divided by 2—that is, 2 is not a factor of such numbers. A composite number may be either odd or even.

4.—2 is a *prime* number, although it is even. This is the only exception. The learner will understand the nature of odd and even numbers better when he has become acquainted with Division.

Tell which are prime and which are composite numbers in the following: 4, 7, 12, 15, 17, 19, 20, 21, 23, 25, 27, 29, 54, 56, 59, 63, 79, 84, 121, 143, 156, 108, 120.

The following is a list of all the prime numbers, from 1 to 100: 2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

1. How much would 18 cows cost, at 23 dollars apiece?

1st *Operation.*	2nd *Operation.*	
23	23	18 = 6 × 3.
18	6	one factor of 18.
184	138	= cost of 6 cows.
23	3	the other factor of 18.
Ans. 414 dollars.	Ans. 414	= cost of 3 times 6, or 18 cows.

In this case 18 is composed of two factors, 6 and 3. Then if we

What are the component parts of a number? Give examples. What is a prime number? Give examples. What must every number be? What are all even numbers? With what does an even number always end? What are all numbers ending with 5? What is a prime number always? With what does an odd number always end? What may a composite number be? What even number is a prime? Are there any other exceptions?

multiply by 6, we shall find how much 6 cows would cost. Now 18 cows will evidently cost 3 times as much 6 cows, because $6 \times 3 = 18$. Therefore, we multiply the price of 6 cows by 3, which gives the price of 18 cows.

Hence—To multiply by a composite number:

Obs. 4. *Resolve the multiplier into its component parts, or factors, then multiply the multiplicand first by one of these parts, and the product thence arising by the other, and so on. The last product will be the answer required.*

REMARK 1.—It matters not by which factor we multiply first; if no mistake is made, the *final result* will be the same. The reason of this is, because the *product of all the factors of any number, in whatever order taken, is equal to that number.*

2.—The learner will observe that there is a difference between resolving a number into *factors*, and separating it into *parts*. The former has reference to *Multiplication*, and the latter to *Addition*. The *product of the factors* of any number is the same as the *sum of its parts*; and it is an established axiom in mathematics, that *the whole is equal to the sum of all its parts*; or, *the product of all its factors.** Thus: 2 and 8, or 4 and 4, or 2 and 2 and 4, or 2 and 2 and 2 and 2, are the factors of 16; but 14 and 2, 12 and 4, 10 and 6, 3 and 13, &c., are its parts.

2. How much would 26 tons of hay cost, at 14 dollars a ton? [$14 = 7 \times 2$.] Ans. 364 dollars.

3. At 75 dollars apiece, how much would 27 horses cost? [$27 = 3 \times 3 \times 3$.] Ans. 2025 dollars.

4. How much would 147 acres of land cost, at 36 dollars an acre? [$36 = 4 \times 3 \times 3$.] Ans. 5292 dollars.

5. How much would 41 books cost, at 25 shillings apiece? [$25 = 5 \times 5$.] Ans. 1025 shillings.

6. How much would 54 town lots cost, at 378 dollars apiece? [$54 = 9 \times 6$.] Ans. 20412 dollars.

7. How much would 478 sheep cost, at 24 shillings apiece? [$24 = 6 \times 4$.] Ans. 11472 shillings.

8. How much would 123 wagons cost, at 72 dollars apiece? [$72 = 9 \times 8$.] Ans. 8856 dollars.

CASE 3.—*When there are ciphers at the right of either, or both factors.*

Ex. 1. How much would 64 acres of land come to, at 20 dollars per acre?

Operation.

```
     64
     20
   ----
Ans. 1280
```

20 is a composite number, the factors of which are 2 and 10. Then $64 \times 2 = 128$; and annexing a cipher—which is the same as multiplying by ten, (Obs. 1.)—we have 1280 dollars as our answer.

How do we multiply by a composite number? Does it make any difference with the final result which factor we multiply by first? Why not? Is there any difference between resolving a number into factors, and separating it into parts? Explain the difference, and give examples.

* That is, all of one set of parts, or factors.

2. How much would 40 horses cost, at 60 dollars apiece?

Operation. 60 and 40 are both composite numbers. The component parts of the one are 6 and 10, and of the other 4 and 10. Now $6 \times 4 \times 10 \times 10$ is the same as $6 \times 10 \times (4 \times 10.)$ (Obs. 4, Rem. 1.)

40
60
——
Ans. 2400 doll.

Then $6 \times 4 = 24$, and $10 \times 10 = 100$, and $24 \times 100 = 2400$. (Obs. 1.) The learner will perceive that the significant figures only are multiplied, and the ciphers at the right of both factors are merely annexed to the product. He will also perceive that 10 is one factor of all numbers ending with a cipher, and the significant figures another factor. Hence—

When there are ciphers at the right of either or both factors:

Obs. 5. *Multiply the significant figures together, only, and to the product annex as many ciphers as there are at the right of both factors.*

3. There are 50 cents in a half dollar. How many cents in 320 half dollars? Ans. 16000.

4. There are 320 rods in a mile. How many rods are there in 1820 miles? Ans. 582400.

5. How much would 3900 pounds of tea come to, at 80 cents per pound? Ans. 312000 cents.

6. An army of 13000 men having plundered a city, each man's share of the spoil was 480 dollars. What was the amount of plunder taken? Ans. 6240000 dollars.

7. There are 160 square rods in an acre; how many square rods in 3900 acres? Ans. 624000.

8. How many pages would there be in 270 books, if there were 360 pages in each? Ans. 97200.

9. Sound moves at the rate of 1130 feet per second; how far will it move in 4716 seconds? Ans. 5329080 feet.

10. In a certain building there are 24 rooms; 12 of these rooms have one window each, and 12 have 2 windows each; each window has 24 lights. How many lights in the building? Ans. 864.

11. It has been computed that an elm produces annually, at an average, three hundred and twenty-nine thousand grains, or seeds, each of which would produce a tree. How many trees might be produced from 178 elms, in 97 years? Ans. 5680514000.

12. Multiply 956802700 by 324007020.
Ans. 310010791554954000.

13. Multiply 1234567890 by 9876543210.
Ans. 12193263111263526900.

Of what numbers is 10 always a factor? What is another factor? When there are ciphers at the right of either, or both factors, how do we proceed? Explain this.

14. Multiply two million, three hundred and fifty-seven thousand, by two hundred and fourteen thousand, three hundred.

Ans. 505105100000.

15. Multiply seven hundred and seventy-seven thousand, seven hundred and seventy, by fifty-five thousand, five hundred and fifty.

Ans. 43205123500.

16. Multiply eight hundred million, nine hundred and forty thousand, three hundred, by twenty million, one hundred and forty thousand, six hundred.

Ans. 16131418206180000.

SECTION V.

SIMPLE DIVISION.

Article 1. Mental Exercises.

1. At 4 cents apiece, how many lemons can you buy for 12 cents?

Solution.—The first lemon costs 4 cents, and you have 12 — 4 = 8 cents left. The second lemon costs 4 cents, and you have 8 — 4 = 4 cents left, with which you can buy but 1 lemon. Therefore, you can buy 3 lemons for 12 cents.

2. How many oranges, at 5 cents apiece, can you buy for 15 cents?

5 is one factor of 15; what is the other factor? (See Multiplication Table.) Ans. 3.

3. How many hats, at 5 dollars apiece, can be bought for 20 dollars?

4. How many yards of cloth can be bought for 24 dollars, at 4 dollars a yard?

5. At 3 shillings a bushel, how many bushels of potatoes may be bought for 18 shillings?

6. At 6 shillings a pair, how many pair of gloves may be bought for 36 shillings?

7. At 3 dollars apiece, how many sheep can be bought for 33 dollars?

8. At 7 shillings apiece, how many books may be bought for 49 shillings?

9. If a pound of tea costs 7 shillings, how many pounds may be bought for 63 shillings?

10. If I pay 6 cents a mile for riding on the stage, how many miles can I ride for 54 cents?

11. In one peck are 8 quarts. How many pecks are there in 72 quarts?

12. There are 7 days in a week; how many weeks are there in 42 days?

Article 2. Definitions, &c.

The separating a number into equal parts, as in the preceding examples, is called Division. Hence—

Obs. 1. Division may be defined—*the separating a number into any given number of equal parts*; or, *finding how often one number is contained in another.*

Division may be either of *Simple* or *Compound* numbers.

Obs. 2. *Simple Division* is when the number to be divided expresses things of but one *name*, or *kind*—as dollars, bushels &c.

Obs. 3. *The number which is to be divided is called the* Dividend.

The number we divide by is called the Divisor.

The result, or answer is called the Quotient, which is derived from the Latin word *quoties*, signifying *how many.*

Obs. 4. Sign.—The sign of Division is *a short horizontal line between two dots.* (÷). It shows that the number *before it*, is to be divided by the number *after it.* Thus: $24 \div 3$ shows that 24 is to be divided by 3.

a. Division is also sometimes expressed *by writing the divisor under the dividend, with a line between them*; thus: $\frac{24}{3}$ shows that 24 is to be divided by 3, as before.

Before proceeding farther, the learner must commit accurately to memory the following

DIVISION TABLE.

2 in		3 in		4 in		5 in		6 in		7 in	
2	once	3	once	4	once	5	once	6	once	7	once
4	2	6	2	8	2	10	2	12	2	14	2
6	3	9	3	12	3	15	3	18	3	21	3
8	4	12	4	16	4	20	4	24	4	28	4
10	5	15	5	20	5	25	5	30	5	35	5
12	6	18	6	24	6	30	6	36	6	42	6
14	7	21	7	28	7	35	7	42	7	49	7
16	8	24	8	32	8	40	8	48	8	56	8
18	9	27	9	36	9	45	9	54	9	63	9

8 in		9 in		10 in		11 in		12 in	
8	once	9	once	10	once	11	once	12	once
16	2	18	2	20	2	22	2	24	2
24	3	27	3	30	3	33	3	36	3
32	4	36	4	40	4	44	4	48	4
40	5	45	5	50	5	55	5	60	5
48	6	54	6	60	6	66	6	72	6
56	7	63	7	70	7	77	7	84	7
64	8	72	8	80	8	88	8	96	8
72	9	81	9	90	9	99	9	108	9

How is Division defined? How divided? What is Simple Division?

$21 \div 7 =$ how many?	$63 \div 7 =$ how many?
$48 \div 6 =$ how many?	$64 \div 8 =$ how many?
$54 \div 9 =$ how many?	$32 \div 4 =$ how many?
$27 \div 9 =$ how many?	$49 \div 7 =$ how many?
$35 \div 5 =$ how many?	$24 \div 3 =$ how many?
$54 \div 6 =$ how many?	$72 \div 9 =$ how many?

Ex. 1. A person paid 369 dollars for 3 horses; how much was that apiece?

Solution.—$369 = 300 + 60 + 9$. Then 3 in 300 goes 100 times; 3 in 60 goes 20 times; and 3 in 9 goes 3 times; and $100 + 20 + 3 = 123$. Ans. 123 dollars.

We divide thus by 3, because it is evident that *if* 3 *horses cost* 369 *dollars, one horse will cost one-third as much as* 3 *horses, or one-third of* 369 *dollars.*

But there is a method of dividing numbers without separating them into their numerical parts, which we will illustrate as follows:

Operation.

Divisor Dividend
3) 369
———
123 Quotient.

By this method we place the divisor at the left of the dividend, with a line between them. We next draw a line below the divdend to separate it from the quotient, which is to be placed below.

Then we say 3 in 3 (hundreds) goes 1 (hundred) times, and write the 1 below the 3; then 3 in 6 (tens) goes 2 (tens) times, and we write the 2 under the 6; then 3 in 9 (units) goes 3 (units) times, and we write the 3 under the 9, and the work is done.

Obs. 5. The pupil will observe that the *form* used is of comparatively little importance, it being used *merely for the sake of convenience.* In short, there is *no form* by which any operation in Arithmetic is performed, that is *absolutely necessary, if the pupil will only keep the principles of numbers firmly fixed in his mind,* and without this he cannot succeed if he has a book full of forms. They are used only for *looks* and *convenience* in illustrating our operations.

It will be noticed that *each figure of the quotient occupies the same place, numerically, as the figure of the dividend that was divided to produce it. This is always the case.*

What is the number to be divided called? The number we divide by? The result or answer? From what? What does Quoties signify? What is the sign of Division? What does it show? How is division otherwise expressed? Give the solution of Ex. 1st. Is there any other method of working such questions? How do we write the numbers by this method? How proceed next? Is this form of any particular importance? Why then is it used? Is any form absolutely necessary in arithmetic? What must a scholar do if he dispenses with forms? Can he succeed well without this? Why not? What then is the use of forms? What place does each figure occupy numerically? Is this always the case?

2. A man spent 488 cents for ribon, at 4 cents a yard; how many yards did he get? Ans. 122.

3. Four men received 845 dollars; how much was that apiece? Ans. $211\frac{1}{4}$ dollars.

In this example, after dividing by 4, there was 1 dollar left.—Now this 1 dollar is to be divided between 4 men; but as 1 will not contain 4, we will merely express the division; thus: $\frac{1}{4}$. This is called one-fourth, and annexing it to the right of our quotient, we find that each man received $211\frac{1}{4}$ dollars.

The 1 which was left after the division was performed, is called a REMAINDER. Hence—

Obs. 6. *When there is a remainder, it is written over the divisor, and annexed to the quotient.*

Obs. 7. It will be perceived that *Division is exactly the reverse of Multiplication.* In the *latter* we have *two factors* given to find their *product:* in the *former* we have the *product and one factor* given to find the *other factor,* the *quotient* and *divisor* answering to to the two factors, and the dividend answering to to the *product,* in Multiplication. Hence—

Obs. 8. *If we multiply the divisor and quotient together, we shall obtain the dividend.*

But if, after a division has been performed, there is a remainder, it is evident that the product of the quotient and divisor will want just this remainder to make the dividend. Hence—

To prove Division:

Obs. 9. *Multiply the divisor and quotient together, and to the product add the remainder, (if any); if the result is equal to the dividend, the work is correct.*

From the preceding remarks, we derive the following method of proving Multiplication by Division:

Obs. 10. *Divide the product by one factor, and if the result s the other factor, the work is correct.*

NOTE.—In this case there is never any remainder.*

4. At 3 dollars a yard, how many yards of cloth can be bought for 697 dollars? Ans. $232\frac{1}{3}$.

Let the learner prove this and the following examples:

5. At 5 dollars an acre, how many acres of land can be bought for 558 dollars? Ans. $111\frac{3}{5}$.

If there is a number left after dividing, what is it called? What is done with it? Of what is Division the reverse? Show the comparison. To what do the different terms of Division answer in Multiplication? What inference is deduced from this? But suppose after the division has been performed, there is a remainder; what then is the conclusion? How then do we prove Division? How can we prove Multiplication by Division? Is there any remainder in this case? Why not?

*That is, in integral numbers.

6. I wish to set out 126 trees in 6 rows; how many must I set in a row? Ans. 21.

In this example the first figure of the dividend did not contain the divisor, therefore we took *two* figures; if *two* figures had not contained it, we should have taken *three* figures, &c. Hence—

Obs. 11. *If the left hand figure of the dividend will not contain the divisor, take two figures; if two figures will not contain it, take three figures; and universally take as many figures at the left of the dividend, as will contain the divisor once or more, to obtain the first quotient figure, which we write under the right hand figure of the part taken.*

7. Six men had 366 dollars to divide between them. How much was that apiece? Ans. 61 dollars.

8. A man had 976 acres of land, which he wished to divide between his 4 sons. How many acres would each receive?

Operation.

4)976
——
Ans. 244 acres.

976 = 9 (hundreds) + 7 (tens) + 6. 9 (hundreds) = 8 (hundreds) + 1 (hundred). Now 4 will go in 8(hundreds) just 2(hundreds) times; consequently it will go in 9 (hundreds) 2 (hundreds) times, and have 1 (hundreds) remainder. Adding the 1 (hundreds) remainder, (which is 10 tens,) to the 7 (tens), we have 17 (tens,) which divided by 4 gives 4 (tens) for a quotient, and 1 (ten) remainder. This 1 (ten) is equal to 10 units, which added to 6 (units) gives 16 (units) This divided by 4 gives 4 (units), and our work is done. Hence, onr quotient is 2 (hundreds), 4 (tens), and 4 (units), or 244.

Obs. 12. By attentively examining the above solution, the learner will perceive the following considerations, viz:

a. 1st. *The remainder each time is of the same name as the dividend. This is always the case,* whether the remainder occurs after each quotient figure is obtained, or only at the end of the operation; because *the remainder is always a part of the dividend.*

b. 2nd. *The remainder each time should be less than the divisor;* because, if it were *equal to,* or *greater* than the divisor, the divisor could be contained another time in the dividend.

c. 3d. *The remainder each time has ten times the value of the next lower order.* This is evident, according to Sect. 1, Art. 2, Obs. 3. Hence, we must multiply it by 10 before adding it to the

---

If the first figure of the dividend will not contain the divisor, how do we proceed? How many figures should we take, in all cases from the dividend, to obtain our first quotient figure? Explain the solution of Ex. 8. What is the first consideration deduced from this? Is this always the case? Why? What is the second consideration? Why? What is the third consideration? Why? What then must be done before it can be added to the next lower figure?

next figure, (Sect. II., Art, 2, Obs. 4.) It has the same effect, however, to prefix it to the next figure, as the cipher always occupies the place filled by this figure.

NOTE.—*Prefix* means *to place before*, or *at the left hand.*

This may be illustrated as follows :

9. Divide 1074 by 3.

Operation.

```
3)10  7  4
     10 20
     -- --
     17 24
   -------
   3  5  8
```

In the first case we have 1 remainder, which, multiplied by 10, equals 10, and 7 equals 17. We next have 2 remainder, which, multiplied by 10, equals 20, and 4 makes 24, which contains 3 without a remainder. Hence—

Obs. 13. *When there is a remainder after dividing any figure of the dividend, prefix it (mentally) to the next figure of the dividend, and proceed as before.*

10. At 8 dollars an acre, how many acres of land can be bought for 8984 dollars? Ans. 1123.

11. At 5 dollars a pair, how many pair of boots may be bought for 685 dollars? Ans. 137.

12. At 4 shillings a bushel, how many bushels of potatoes can be obtained for 1628 shillings.

Operation.

```
4)1628
  ----
   407
```

In this case the divisor is not contained in the ten's figure, therefore we write a cipher in the quotient, and prefix the 2 to the next figure, as if it were a remainder, which it really is. Then 4 in 28 goes 7 times. Hence—

Obs. 14. *When any partial dividend will not contain the divisor, write a cipher in the quotient, call the partial dividend a remainder, and proceed as before.*

REMARK.—The *partial dividend* is that part taken to find each quotient figure.

Obs. 15. Also—*After the first quotient figure is obtained, either a cipher or a significant figure must be put in the quotient, for each figure or cipher in the dividend.*

13. A man traveled 1624 miles in 8 weeks; how many miles was that a week? Ans. 203.

Why? By what other method can we effect the same result? Why is this correct? What does prefix mean? Give the solution of Ex. 9, and show why it has the same effect to prefix the remainder to the next lower figure as to multiply it by 10, and then add it. What inference is deduced from this?—When the divisor cannot be contained in any partial dividend, how do we proceed? What is the partial dividend? How many figures, or ciphers, must be put in the quotient after the first quotient figure is obtained? When the divisor is small, and the operation is carried on chiefly in the mind, what is it called?

When the divisor is small, as in the preceding examples, the operation is performed chiefly in the mind; it is then called Short Division.

From the remarks and illustrations already given, we derive the following

RULE FOR SHORT DIVISION.

I. *Write the divisor at the left of the dividend, with a line between them; also, draw a line below the dividend.*

II. *Take from the left of the dividend as many figures as will contain the divisor once or more; divide them, write the quotient under the right hand figure of the part taken,* (Obs. 11); *prefix the remainder (if any,) to the next figure of the dividend, and divide as before, until all the figures of the dividend have been divided.* (Obs. 13.)

III. *If the divisor is not contained in any partial dividend, write a cipher in the quotient, prefix this partial dividend to the next figure, as if it were a remainder, and proceed as usual.* (Obs. 14.)

IV. *If a remainder occurs after dividing the last figure of the dividend, write it over the divisor, and annex it to the quotient.*—(Obs. 6.)

Proof.—*Multiply the divisor and quotient together, and to the product add the remainder (if any); if the sum is equal to the dividend, the work is correct.* (Obs. 9.)

1. We commence at the left hand to perform division, because the remainders which occur must be reduced to lower orders before the division can be continued. (Obs. 12, 3d.) Besides, it is evident that if we divide any order of figures by any number, and there is a remainder, that *this remainder cannot be reduced to a higher order, and still contain the same divisor.* Hence, it must be reduced *lower;* and as numbers decrease from the left hand towards the right, we commence at the left to divide. The learner will better understand the reasons for carrying in the Fundamental Rules, when he has become acquainted with Compound numbers.

2. The *divisor, dividend* and *quotient* may any of them be either abstract or concrete. The divisor and quotient, however, *cannot both be concrete in the same question,* and when *either* of them is concrete, the *dividend* is also *concrete,* and of the *same name.* Nei-

---

What is the rule for Short Division? Why do we commence at the left hand to divide? Why must the remainders be reduced to lower orders before we can continue the operation? What is said respecting the divisor, dividend, and quotient being abstract or concrete? Can the divisor and quotient both be concrete in the same question? When one of them is concrete, what must the dividend be? Can the dividend be abstract, without the divisor and quotient both being abstract? What name, then, must the dividend always be? Give examples.

ther can the dividend be abstract, without the *divisor* and *quotient both being abstract* also. In short, the dividend must *always* be of the same name as the divisor, or quotient, or both. Thus: in the 6th question, we reason—If 126 trees are put in 6 rows, each row will contain one-sixth of 126 trees, which is 21 *trees*, and not 21 *times*, or merely 21, as is often asserted. Here, the divisor (6) is *abstract*, whilst the dividend and quotient are both *concrete*, and of the same kind. In the 11th example we divide 685 dollars by 5 dollars; but the learner must not infer that 685 dollars divided by 5 dollars gives 137 boots for an answer. In this case, as the boots cost 5 dollars a pair, we can buy as many pair as 5 dollars is contained in 685 dollars, which is 137 *times*. Here our *divisor* and dividend are *concrete*, and of the same name, and the quotient is *abstract*. It would be as absurd to make the quotient concrete in this case, as it would be to say, (in the 8th Ex.) that 976 *acres* contain 4 *sons* 244 *times*, or that one fourth of 976 *acres* is any thing else than 244 *acres*.

EXERCISES FOR THE SLATE.

1. At 6 cents a ball, how many balls of tape may be bought for 186 cents? Ans. 31.

2. How many sheep can I buy for 786 dollars, at 3 dollars apiece? Ans. 262.

3. How many pair of gloves can I buy for 1728 shillings, at 9 shillings a pair? Ans. 192.

4. There are 3 feet in a yard; how many yards in 479 feet? Ans. $159\frac{2}{3}$.

5. A man bought 9 horses for 1382 dollars. How much was that apiece? Ans. $153\frac{5}{9}$ dollars.

6. A man sold 6 yoke of oxen for 492 dollars. How much was that a yoke? Ans. 82 dollars.

7. If 7 pieces of cloth cost 438 dollars, how much is that apiece? Ans. $62\frac{4}{7}$ dollars.

8. If 8 wagons cost 548 dollars what is that apiece? Ans. $68\frac{4}{8}$ dollars.

9. At 4 shillings apiece, how many books can be bought for 6488 shillings? Ans. 1622.

10. At 11 miles a day, how many days would it take to travel 483 miles? Ans. $43\frac{10}{11}$.

11. There are 8 quarts in a peck; how many pecks in 9952 quarts? Ans. 1244.

12. At 5 cents a mile, how far can I ride for 8730 cents? Ans. 1746 miles.

13. At 12 dollars a month, how many months would it take to earn 4623 dollars? Ans. $385\frac{3}{12}$.

14. How many calves, at 7 dollars apiece, can I buy for 368 dollars? Ans. 52, and have 4 dollars left.

15. A man has 2832 pair of shoes in 8 boxes. How many are there in each box? Ans. 354 pair.

16. At 5 dollars a barrel, how many barrels of flour can I buy for 2540 dollars? Ans. 508.

17. If 8 laborers should perform a piece of work for 1628 dollars, how much would that be apiece? Ans. $203\frac{4}{8}$ dollars.

18. 16 times 489271 is how many times 2? Ans. 3914168.

19. 15 times 14962 is how many times 5? Ans. 44886.

20. 17 times 6217 is how many times 11? Ans. $9608\frac{1}{11}$.

Article 3. Long Division.

Obs. 1. In *Short Division* the operation is performed mentally to a considerable extent; but in *Long Division* the operation is all set down upon the slate. *The principles of both, however, are the same.* The *former* is generally used when the divisor is less than 12, and the *latter* when the divisor exceeds 12.

1. A man paid 4096 dollars for some land, at 16 dollars an acre. How many acres did he buy?

Operation.

```
16)4096(256 Ans.
   32..
   --
    89
    80
    --
     96
     96
     --
     00
```

We write our numbers as before, except the quotient, which we place at the *right hand.* We do this *because it is more convenient than to place it below.* We next say, 16 in 40 (hundreds) goes 2 (hundreds) times, and write our 2 in the quotient. We then multiply our divisor (16) by our quotient figure (2), and place the product (32) under the part taken of the dividend.—*This is done to ascertain whether there is any remainder after dividing.* Next we subtract this product (32) from the dividend, (40,) which leaves 8(hundreds) as our remainder. We then bring down the 9 (tens) at the right of the 8 (hundreds,) (which is the same as prefixing the remainder to the 9 (tens), which makes 89 (tens). Then 16 in 89 (tens) goes 5 (tens) times, which we write in the quotient. *We place the 5 at the right of the 2 because the 2 is hundreds, and the 5 is tens, and tens occupy the place at the right of hundreds.* Then 5 times 16 is 80, which we place under the 89, and subtracting, we have 9 (tens) as our second remainder. To this we bring down our next figure in the dividend (6), making 96 (units), which divided by 16, gives 6 (units)

What is the difference between the operation of Long and Short Division? What is the difference between the principles of the two? When is Short Division generally used? Long Division? How do we write the numbers in Long Division? Why do we place the quotient on the right? What is the first step of the operation? The second? Third? Fourth? Why do we multiply the divisor by the quotient figure? What is meant by bringing down? Why do we place our second quotient figure at the right of the first?

for our next quotient figure. Finally, multiplying and subtracting as before, we find there is no remainder; and as all the figures of the dividend have been brought down and divided, our work is done.

NOTE.—The dots are placed under figures of the dividend when they are brought down, *to prevent their being brought down again.* They are not necessary if the pupil is observing.

Obs. 2. By examining this solution attentively, the learner will observe the two following considerations, viz:

1st. *Each figure in the quotient occupies the same place, numerically, as the right hand figure of that part of the dividend taken to produce it.* (Art. 2, Obs. 5.)

2nd. *The operations of Long Division consist of four steps, viz:* 1st, *to divide;* 2nd, *to multiply;* 3rd, *to subtract;* and 4th, *to bring down. This is the whole process.*

2. A man bought 15 acres of land for 375 dollars. How much was that per acre? Ans. 25 dollars.

3. If a man travel 625 miles in 25 days, how many miles is that per day? Ans. 25.

4. A man paid 32 dollars an acre for some land, and bought enough to come to 34576 dollars. How many acres did he buy?

Operation.

```
32)34576(1080 16/32.
   32
   —
    257
    256
    ——
     16
```

32 in 34 goes 1 time, and 2 remainder. Bring down the 5, we find that 25 will not contain 32; therefore we write a cipher in the quotient, (Art. 2, Obs. 14), and bring down the next figure, 7. 32 in 257 goes 7 times, and 1 remainder. Bringing down the 6, we find that 16 will not contain the 32, and therefore write another cipher in the quotient; then as there are no more figures in the dividend to bring down, 16 is our remainder, which we write over our divisor, and annex to the quotient, (Art. 2, Obs. 6), and thus obtain for our Ans. $1080\frac{16}{32}$.

5. How many times will 6144 contain 24?

1st *Operation.*

```
24)6144(257
   48
   —
   134
   120
   ——
    144
    168
    ——
```

Here, we supposed our last quotient figure to be 7; but by multiplying, we find that our product is greater than our dividend; therefore the quotient figure is too large.

2d *Operation.*

```
24)6144(256
   48
   —
   134
   120
   ——
    144
    144
    ———
    000
```

Here, we supposed it to be 6, which we find to be correct.

Hence—

Obs. 3. *When the product of the divisor by any quotient figure is greater than the dividend, the quotient figure must be made less.* The remarks respecting the remainder, (Art. 2, Obs. 12,) are equally as applicable to *Long* as to *Short Division.*

From the preceding remarks and illustrations, we derive the following

RULE FOR LONG DIVISION.

I. *Write the divisor and dividend as before directed, and draw a line at the right of the dividend to separate it from the quotient.*

II. *Take as many figures at the left of the dividend as will contain the divisor once or more, and place the number of times they contain it at the right of the first quotient figure.*

III. *Multiply the divisor by this quotient figure, and subtract the product from that part of the dividend taken.*

IV. *To the right of the remainder bring down the next figure of the dividend, and proceed as before. So continue to do till all the figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not contain the divisor, write a cipher in the quotient, bring down the next figure of the dividend, and proceed as before.* (Art. 2, Obs. 14.)

Proof.—*Multiply together the divisor and quotient, and to the product add the remainder (if any); if the result is equal to the dividend, the work is correct.* (Art. 2, Obs. 9.)

EXERCISES FOR THE SLATE.

1. If a man walk 4 miles an hour, how many hours will it take him to walk 128 miles? Ans. 32.

For what are the dots used? Are they really necessary? What place does each quotient figure occupy numerically? Of how many steps does the operation in Long Division consist? Name them. When the dividend will not contain the divisor, how do we proceed? What is done with the final remainder? When the product of the divisor by any quotient figure is greater than the dividend, what is to be done? Are the remarks respecting the remainder, (Art. 2, Obs. 8.) applicable to Long as well as Short Division? What were these remarks? What is the rule for Long Division?

2. There are 12 months in a year; how many years in 20176 months? Ans. $1681\frac{4}{12}$.

3. There are 8 quarts in a peck. How many pecks in 32024 quarts? Ans. 4003.

.4 There are 6 working days in a week. How many weeks in 1256 working days? Ans. $209\frac{2}{6}$.

5. A half eagle is worth 5 dollars. How many half eagles would it take to be worth 394820 dollars? Ans. 78964.

6. In one square yard are 9 square feet. How many square yards in 42786 square feet? Ans. 4754.

7. How many calves, at 14 dollars apiece, can be bought for 476 dollars? Ans. 34.

8. At 17 dollars a month, how many months would it take to earn 969 dollars? Ans. 57.

9. The steamer Great Western was 15 days in crossing the Atlantic Ocean, a distance of 3000 miles. How far did she sail per day? Ans. 200 miles.

10. Suppose 21528 dollars were equally divided between 104 men; how much would each receive? Ans. 207 dollars.

11. Suppose a vessel has sailed 15340 miles, at the rate of 126 miles per day; how many days has she sailed? Ans. $121\frac{94}{126}$.

12. There are 63 gallons in a hogshead. How many gallons in 7560 hogsheads? Ans. 120.

13. At 16 dollars a month, how long would it take to earn 624 dollars? Ans. 39 months.

14. How m ny days would it take to walk 576 miles, walking 24 miles per day? Ans. 24 days.

15. In every day are 24 hours. How many days in 4096 hours? Ans. $170\frac{16}{24}$.

16. At 18 dollars a ton, how many tons of hay could be bought for 450 dollars? Ans. 25.

17. How many casks of wine could be bought for 4950 dollars, at 25 dollars a cask? Ans. 198.

18. If a man has traveled 6255 miles in 15 weeks, how far is that per week? Ans. 417 miles.

19. If 75 wagons cost 5625 dollars, how much is that apiece? Ans. 75 dollars.

20. If 53 horses cost 3286 dollars, how much is that for each? Ans. 62 dollars.

21. If 32 pounds of raisins cost 800 cents, how much is that per pound? Ans. 25 cents.

22. A man has a note to pay, amounting to 1204 dollars. If he pays 86 dollars at a time, how many payments will he have to make to pay up his note? Ans. 14.

23. There are 52 weeks in a year. How many years in 1248 weeks? Ans. 24.

24. Suppose a firm to consist of 37 partners, and their capital to amount to 1178450 dollars. How much is that for each?
Ans. 31850 dollars.

25. At 144 shillings a month, how long would it take a man to earn 7488 shillings? Ans. 52 months.

26. 25 times 8469 is how many times 9? Ans. 23525.

27. 144 times 7324 is how many times 48? Ans. 21972.

28. 1728 times 20736 is how many times 288? Ans. 124416.

29. Two numbers multiplied together produce one billion, thirty million, six hundred and three thousand, six hundred and fifteen, and one of the numbers is three thousand, two hundred and fifteen. What is the other number? Ans. 320561.

30. Divide thirty-seven billion, four hundred and twenty-three million, eight hundred and thirty-four thousand, five hundred and sixty, by one hundred and twenty-three thousand, four hundred and fifty-six. Ans. 303135.

Article 4. Contractions in Division.

Case 1.—*To divide by* 10, 100, 1000, &c.

1. At 10 dollars a month, how many months would it take to earn 534 dollars?

Operation.
1|0 | 53|4
——
Ans. $53\frac{4}{10}$.

As a cipher annexed to any figure increases its value 10 times, (Sect. 1, Art. 1, Obs. 5,) and two ciphers annexed increases its value 100 times, &c., so to cut off *one* figure from the right of any number, diminishes the value of that number 10 times, to cut off *two* figures diminishes its value 100 times, to cut off *three* figures diminishes its value 1000 times, &c.

Hence—To divide by 1 with any number of ciphers annexed:

Obs. 1. *Cut off from the right of the dividend as many figures as there are ciphers at the right of the divisor; the figures at the left will be the quotient, and those at the right the remainder.*

2. Suppose you wished to divide 4267 dollars equally between 100 men, how much would you give to each man?
Ans. $42\frac{67}{100}$ dollars.

3. There are 10 mills in a cent. How many cents in 150 mills?

4. There are 100 cents in a dollar. How many dollars in 4875 cents?

5. There are 10 dollars in an eagle. How many eagles in 3854 dollars?

6. There are 1000 mills in a dollar. How many dollars in 65832 mills?

7. There are 10000 mills in an eagle. How many eagles in 489621 mills?

How do we divide by 10, 100, 1000, &c.? Explain why this rule is correct.

CASE 2. *To divide by a Composite Number.*

If we divide 12 by 6, we obtain 2. Again: if we divide 12 by 3, we obtain 4; and if we divide 4 by 2, we also obtain 2. Now 3 and 2, (our latter divisors,) are the factors of 6, and to divide 12 by them produces the same result as to divide 12 by 6. Hence—

Obs. 2. *To divide the dividend by the factors of the divisor, will produce the same result as to divide it by the divisor itself.*

a. Therefore, conversely, *Any dividend that will contain a divisor will also contain the factors of that divisor.*

1. If I pay 240 dollars for 48 yards of cloth, what is that per yard? Ans. 5 dollars.

1st *Operation.*	2d *Operation.*
48)240(5 dollars. Ans.	8)240
240	——
——	6)30
000	—
	Ans. 5 dollars.

48 is composed of two factors, 6 and 8, ($8 \times 6 = 48$,) and according to the above remark, we may divide 240 first by one factor, and the quotient thence arising by the other, and it will produce the same result as to divide it by 48, that is, 5 dollars a yard.

Hence—When the divisor is a composite number:

Obs. 3. *Divide the dividend first by one factor, and the quotient thence arising by another factor, and so on; the last quotient will be the answer.*

NOTE.—It makes no difference in any case which factor we divide by first; if no mistake is made, the *final result* will be the same.

2. If a man travel 32 miles per day, how many days will it take him to travel 1472 miles? ($32 = 4 \times 8$.) Ans. 46.

3. If a man pay 2835 dollars for horses, at the rate of 63 dollars apiece, how many can he buy? ($63 = 9 \times 7$.) Ans. 45.

4. How many acres of land, at 25 dollars an acre, can be bought for 875 dollars? ($25 = 5 \times 5$.) Ans. 35.

5. How many yoke of oxen can I buy for 1904 dollars, at 56 dollars a yoke? ($56 = 8 \times 7$.) Ans. 34.

6. How many watches can be bought for 1875 dollars, at 75 dollars apiece? ($75 = 5 \times 5 \times 3$.) Ans. 25.

What is the difference in the result of dividing 12 by 6, or by 3 and 2, the factors of 6? What is the first inference deduced from this? The second? Give the solution of example 1st, and show why it is correct. How do we proceed when the divisor is a composite number? Does it make any difference in the final result which factor we divide by first? Give the solution of Ex. 1, Case 3, and show why it is correct. What is done with the remainder, 3, in Ex. 2? How do we proceed when there are ciphers at the right of the divisor?

7. A school teacher wishes to divide 256 apples between 32 pupils. How many must he give to each? ($32 = 4 \times 8$.) Ans. 8.

8. If 175 watches cost 4375 dollars, how much is that apiece? ($175 = 5 \times 5 \times 7$.) Ans. 25 dollars.

CASE 3. *When there are ciphers at the right of the divisor.*

1. There are 40 rods in a furlong. How many furlongs in 689 rods?

1st *Operation.*

$$\begin{array}{l} 40)689(17\tfrac{9}{40}.\text{ Ans.} \\ \underline{40} \\ 289 \\ \underline{280} \\ \quad 9 \end{array}$$

2d *Operation.*

$$\begin{array}{c} 4|0)68|9 \\ \hline \text{Ans. } 17\tfrac{9}{40}. \end{array}$$

40 is composed of two factors, 4 and 10, ($10 \times 4 = 40$); therefore, we may first divide by ten, (which is only to cut off one figure at the right—Obs. 1,) and then divide the quotient thence arising by 4, (Obs. 3;) this is done by cutting off the cipher at the right of the divisor, and a figure at the right of the dividend, and then dividing the remaining figures of the dividend (68,) by the remaining figure of the divisor (4); the figure that was cut off from the dividend is the remainder.

2. There are 60 minutes in an hour. How many hours in 754 minutes?

Operation.

$$\begin{array}{c} 6|0)75|4 \\ \hline \text{Ans. } 12\tfrac{34}{60} \end{array}$$

In this example, after dividing by 6, there was 3 remainder, to which we annexed the 4 that was cut off, making 34 as our true remainder.

Hence—When there are ciphers at the right of the divisor:

Obs. 4. *Cut off these ciphers; also, cut off as many places at the right of the dividend; then divide the remaining figures of the dividend by the remaining figures of the divisor, and to the last remainder annex the figures, or ciphers, that were cut off from the dividend, for the true remainder.*

3. How many horses can be bought for 6400 dollars, at 80 dollars apiece? Ans. 80.

4. At 30 dollars an acre, how much land can be bought for 1568 dollars? Ans. $52\frac{8}{30}$ acres.

5. How many barrels will 9700 pounds of pork pack, allowing 200 pounds to the barrel? Ans. $48\frac{100}{200}$.

6. How many bins would it take to contain 43200 bushels of wheat, allowing each bin to contain 320 bushels? Ans. 135

7. Allowing each bin to contain 360 bushels, how many would it take? Ans. 120.

8. At 60 dollars a yoke, how many yoke of oxen could be bought for 480 dollars? Ans. 8.

9. At 80 dollars a yoke, how many yoke could be bought? Ans. 6.

10. An army of 13500 men having plundered a city, took 5130000 dollars. What was each man's share? Ans. 380 dollars.

SECTION VI.

RECAPITULATION.

ARTICLE 1. REMARKS AND INFERENCES DEDUCED FROM THE PRECEDING RULES.

Obs. 1. The preceding Rules, viz.—*Notation* and *Numeration Addition, Subtraction, Multiplication* and *Division*—are called the FUNDAMENTAL RULES OF ARITHMETIC, because they are the *basis,* or *foundation* of a l the other Rules, and without the aid of one or more of these, no operation in numbers can be performed.

In this Section it is intended to review these Rules, and make a few remarks, and present a few ideas and suggestions in addition to what has already been said. The rules, as a general thing are so evident, that they need no demonstration. If the pupil cannot demonstrate them himself, he should turn back and study over the preceding Sections again. It is *absolutely necessary* that he should understand the preceding Rules *thoroughly* before he can ever become an arithmetician.

Obs. 2. NOTATION.—*All numbers are expressed* (by the common, or Arabic method,) *by nine digits and a cipher*. As the cipher, however, is only used to give value to significant figures, (it having no value itself,) it might with propriety be called *an auxiliary digit.*

Obs. 3. Our system of Notation is also called the *decimal system,* because the *numbers increase in a ten-fold ratio.* The term *decimal* is derived from the Latin word *decem,* which signifies *ten.*

Obs. 4. The *names,* as well as the *characters* which represent

---

What are the preceding rules called? Why? Is it necessary for the learner to understand these rules, before he can become an arithmetician? Why so? How are numbers expressed by the common method of notation? For what is the cipher used? What, then, might it properly be called? By what other name is the common system of notation known? Why is it thus called? From what is the term decimal derived? What is said respecting the names and characters that we use to represent numbers? Would any other names or characters have answered as well had they been adopted?

numbers, are entirely arbitrary. Any other names or characters would have served the same purpose had they been adopted. It is *use* only that renders them familiar.

Obs. 5. The names of numbers below ten, are either *primitive* words, or derived from the Latin. *Eleven* and *twelve* are regarded by some as *derivative* words, formed from ten and one, ten and two; thus: eleven means *ten leave one*—that is, if you take ten from it you leave one; twelve means, *ten leave two*—that is, if you take ten from it you leave two. This is, however, rather a forced signification, and they are generally regarded as primitive words.

Names of numbers higher than twelve are but repetitions of the preceding names. Thus: thirteen, fourteen, &c., signify *three and ten, four and ten,* &c., and twenty, thirty, &c., signify *two tens, three tens,* &c., and so on. The different powers of ten, as *one hundred, one thousand,* &c., are *primitive* words.

Obs. 6. Numeration.—It may be asked by some, *why do we commence at the left hand to read numbers, and at the right hand to numerate them?* The reason is obvious. In speaking of numbers we always name the highest orders first, as it is more convenient; and therefore, as the higher orders are at the left hand, by our system of notation, we commence at the left hand to read.

Again—In numerating, we call the lower orders first; therefore, as numbers increase from the right hand towards the left, the lower orders must be at the right; consequently, we commence there to numerate. Had our system of notation been such, that numbers increased from the *left hand towards the right,* we should have commenced at the *right hand to read,* and at the *left hand to numerate.*

Obs. 7. Addition.—*We cannot add numbers of different names together;* because if I have 5 pear trees and 8 cherry trees in an orchard, it cannot be said that their sum is 13 *pear trees,* or 13 *cherry trees.* We can, however, say that their sum is 13 *trees.* Again: if I saw 5 hay stacks, 7 wheat stacks, 12 trees, 2 barns, 12 horses, and 15 cattle in a lot, their sum would not be all stacks, trees, barns, horses, or cattle; but if asked how many *objects* I saw in the

---

Are the names of numbers below ten primitive or derivative words? What are eleven and twelve regarded by some? Formed from what? How? Is this a natural signification? How are they generally regarded? How are numbers higher than twelve formed? Give examples. What is said respecting the different powers of ten? Why do we commence at the left hand to read numbers? Why do we commence at the right to numerate? Had our system of notation been such that numbers increased from the left hand towards the right, where would we have commenced to read, and where to numerate? Explain why we cannot add numbers of different names together. What is necessary in adding several numbers together? When we have given several numbers, how do we find their sum?

field, their *sum* would express the number. Also, the sum of 2 *units* and 6 *tens*, is neither 8 *units* nor 8 *tens;* but if we reduce the 6 *tens* to *units,* their sum would be 68 *units.* Thus, we perceive that *we cannot add several numbers together, unless they can be classified under one order in which one name is common to them all.*

Obs. 8. When we have given several numbers, to find their sum—*Add them together.*

Ex. 1. There are two numbers; one is 144, and the other is 62. What is their sum? Ans. 206.

2. A. has 1200 dollars, and B. has 480 dollars. How many dollars have both? Ans. 1680 dollars.

3. The distance A. has traveled is 180 miles; B. has traveled 114 miles. How many miles have both traveled? Ans. 294.

Obs. 9. It may be asked by some—Why, in Addition, we carry 1 for every *ten,* instead of one for every *eight, nine, twelve,* or some other number? The answer is—*because our system of notation increases in a ten-fold ratio; consequently, as often as we have ten of one order, we must have* 1 *of the next higher order.* Had the base of our system of notation been *eight, nine, twelve,* or any other number, instead of *ten,* we should have carried 1 for every *eight, nine, twelve,* or whatever other number was the base of our scale of notation. The same remark applies to Subtraction, Multiplication, and Division.

Obs. 10. Subtraction.—*The numbers must all be of the same name, or kind in Subtraction as well as in Addition.* Thus: if I have in a field 13 cherry trees, and 5 pear trees, my taking the *pear* trees away does not diminish the number of *cherry* trees, although it lessens the number of trees in the lot.

Again: we cannot take 2 *units* from 8 *tens,* unless we first reduce the tens to units. Thus, we perceive that *we cannot subtract two numbers, unless they have a name common to both.*

It is not absolutely necessary, in Subtraction, that we place the greater number above the other. This is done only for *convenience.*

Obs. 11. When we have given the sum of two numbers, and either number, to find the other—*From the sum subtract the given number.*

Demonstrate this rule. Why, in Addition, do we carry 1 for every ten, instead of 1 for every eight, twelve, or some other number? Had the base of our system of notation been eight, twelve, or some other number instead of ten, how often would we have carried 1? Explain why, in Subtraction, the numbers must all be of the same name, or kind. What is necessary in order to subtract two numbers? Is it necessary in Subtraction that we place the greater number above the other? Why then is it done? When we have given the sum of two numbers, and either number, how do we find the other number? Demonstrate this rule.

4. The sum of two numbers is 40, and one of them is 18. What is the other number? Ans. 22.

5. Two men have 1000 dollars; A. has 572 dollars. How many dollars has B.? Ans. 428.

Obs. 12. When the *difference* and *greater* of two numbers are given to find the *lesser* number—*From the greater number subtract the difference.*

6. The greater of two numbers is 53, and their difference is 14. Required—the lesser number. Ans. 39.

7. A. has 1000 dollars, which is 346 dollars more than B. has. How many dollars has B.? Ans. 654.

8. A. has traveled 300 miles, which is 163 miles farther than B. has traveled. How far has B. traveled? Ans. 137 miles.

Obs. 13. Given, the *difference* and *lesser* of two numbers, to find the *greater*—*Add the difference and lesser number together.*

NOTE.—This is the same, in reality, as the proof of Subtraction.

9. The difference of two numbers is 7, and the lesser number is 13. What is the greater number? Ans. 20.

10. A. has 560 dollars, and B. has 40 dollars more. How much has B.? Ans. 600 dollars.

11. A. went 140 miles, and B. went 60 miles farther. How far did B. go? Ans. 200 miles.

Obs. 14. Given the sum and difference of two numbers, to find those numbers—*From their sum subtract their difference, and half the remainder will be the lesser number.*

12. The sum of two numbers is 16, and their difference is 4. Required—the numbers. Ans. 6 and 10.

13. The sum of the ages of two men is 66 years, and one of them is 20 years older than the other. Required—their ages.
Ans. 43 and 23 years.

14. Two men together have 500 dollars, and one of them has 240 dollars more than the other. How many dollars have both?
Ans. One has 370 dollars, and the other 130 dollars.

Obs. 15. MULTIPLICATION.—As Multiplication is the repeated addition of the same number to itself, it may be thought by some that the multiplier can sometimes be a concrete number, (as we cannot add numbers of different names together). But the learner must

---

When the difference and greater of two numbers are given, how do we find the lesser number? Demonstrate this rule. When we have given the difference and lesser of two numbers, how do we find the greater? Demonstrate this rule When we have given the sum and difference of two numbers, how do we find the numbers? Demonstrate this rule. Explain why the multiplier is always an abstract number. When we have given the product of two factors, and

recollect that the *multiplicand* is the number to be added, and that the multiplier only expresses *how many times* it is to be added, or repeated. Hence, *the multiplier is always an abstract number.* To multiply *acres* by *dollars*, or *cents* by *yards*, &c., is sheer nonsense. For the method of analyzing concrete questions, see Sect. IV, Art. 2, Ex. 1.

Obs. 16. Given the product of two factors, and either factor, to find the other factor—*Divide the product by the given factor.* (Sect. V., Art. 2, Obs. 10.)

15. The product of two factors is 48, and one of them is 6. What is the other factor? Ans. 8.

16. The product of two factors is 576, and one of them is 48. What is the other factor? Ans. 12.

To prove Multiplication by casting out the 9's—

Obs. 17. *Cast the 9's out the multiplier and multiplicand; multiply their excesses together, and cast the 9's out of their product; then, if the excess of 9's in this, is the same as the excess of 9's in the total product, or answer to the question, the work is correct.*

This method of proof depends upon a property of the number 9, viz:

Any number divided by 9 will leave the same remainder as the sum of its digits, or figures, divided by 9.

Demonstration.—Take any number, as 876. This separated into its numerical parts, equals $800 + 70 + 6$. But $800 = 8 \times 100 = 8 \times (99 + 1) = 8 \times 99 + 8$. Also, $70 = 7 \times 9 + 7$. Hence, $876 = 8 \times 99 + 8 + (7 \times 9 + 7) + 6 = \overline{8 \times 99 + (7 \times 9)} + 8 + 7 + 6$, and $876 \div 9 = (\overline{8 \times 99 + 7 \times 9} + 8 + 7 + 6) \div 9$. But $8 \times 99 + (7 \times 9)$ is evidently divided by 9 without a remainder, (because 9 is one factor of this expression); therefore, $876 \div 9$ will leave the same remainder as $(8 + 7 + 6) \div 9$. The same method of reasoning will apply to any other number.

Now, from this demonstration, the reason of the rule is evident. Because, if we reject the 9's from any number, and also reject the 9's from the *several parts* of the same number, add the latter excesses together, and reject the 9's from their sum, this latter excess ought to be equal to the excess of 9's in the number itself; *the whole being equal to the sum of all its parts.*

Now, in Multiplication, the *product* is a number, of which the *multiplicand* is a part, taken as many times as there are units in the

---

either factor, how do we find the other factor? Demonstrate this rule. How do we prove multiplication by casting out the 9's? Upon what does this method of proof depend? What is this property? Demonstrate it. Show from the demonstration why the rule is evident. How is this applied to Multiplication?

multiplier. Hence, if we multiply the multiplier by the excess of 9's in the multiplicand, the excess of 9's in the product ought to be equal to the excess of 9's in the total product, or answer to the question. But it will produce the same result to multiply the excess of 9's in the multiplicand by the excess of 9's in the multiplier, and reject the 9's from this product. Hence, the rule is correct.

To cast out the 9's in any number, we add together the digits of that number, and as often as we obtain 9, reject it, take the remainder and proceed as before. Thus, to cast the 9's out of 1657324, we say 4 and 2 are 6, and 3 are 9; (rejecting the 9,) 7 and 5 are 12; (rejecting the 9,) 3 and 6 are 9; (rejecting the 9,) we find the excess in the whole number to be 1.

NOTE.—This property of the number 9, belongs to no other digit except 3.

17. Multiply 278 by 745, and prove the operation.

Operation.

278	Excess of 9's in the multiplicand is 8.
745	" " " multiplier is 7.
	" " " product of 8×7 is 2.
1390	" " " whole product is 2.
1112	Hence, the operation is correct.
1946	
207110	

It is customary to write the excesses in the four spaces of a cross; the excesses of the two factors being placed above and below, and the other excesses at the right and left, thus: Then if the excesses at the right and left are alike, the work is correct.

$$\begin{array}{ccc} & 8 & \\ 2 & \times & 2 \\ & 7 & \end{array}$$

18. What is the product of 4682 by 378? Ans. 1769796.
19. What is the product of 4781 by 6213? Ans. 29704353.
20. What is the product of 37682 by 26571?
Ans. 1001248422.

Obs. 18. When we have given the cost of 1, to find the cost of a given quantity, either more or less—*Multiply the cost and quantity together.*

REMARK.—The term *quantity* applies to any thing capable of *increase* or *diminution*—as numbers, lines, cloth, &c.

NOTE.—The learner must not infer from the above that we multiply two concrete numbers together, because we say, multiply together the cost and

---

How do we cast the 9's out of a number? Does this property of the number 9 belong to any other digit? When we have given the cost of 1, how do we find the cost of a given quantity, either more or less? To what does the term quantity apply?

quantity. We express the rule in this way for the sake of brevity, but the multiplier should always be abstract. (Obs. 15, and Sect. IV, Art. 2, Obs. 6, Rem. 1.)

21. If 1 bushel of apples cost 42 cents, how much will 25 bushels cost? Ans. 1050 cents.

22. If 1 stove costs 25 dollars, how much will 19 stoves cost? Ans. 475 dollars.

23. If 1 book costs 84 cents, how much will 216 books cost? Ans. 18144 cents.

Obs. 19. If we add a *unit* to the multiplier, it will produce the same result as to add the multiplicand to the product; if we add *two units*, the result will be the same as to add the multiplicand *twice* to the product; and universally, *to add any number to the multiplier, increases the product as many times the multiplicand as there are units in the number added.*

Again, if we subtract a *unit* from the multiplier, it will produce the same result as to subtract the multiplicand from the product; to subtract *two units*, the result will be the same as to subtract *twice* the multiplicand from the product; and universally, *to subtract any number from the multiplier, diminishes the product as many times the multiplicand, as there are units in the number subtracted.*

It is for these reasons that the proof in Multiplication is correct. (Sect. IV., Art. 2, Obs. 7.)

Remark.—The learner will bear in mind that in all cases where the multiplier is *unity*, or 1, the product is equal to the multiplicand; when the multiplier is *greater* than 1, the product is greater than the multiplicand; and when the multiplier is *less* than 1, the product is less than the multiplicand.

Obs. 20. Division.—Division has been defined by many authors of Arithmetic as *a short method of performing many subtractions, when the numbers to be subtracted are all equal;* and as a consequence it is urged that the divisor is always of the same name as the dividend, (because we cannot subtract numbers of different names from each other,) and that the quotient is always an abstract number, (because it expresses *the number of times* the subtractions have been performed.)

This, however, is not always the case, because we very often have two concrete numbers given—one as a dividend, and the other

---

If we add a unit to the multiplier, what effect does it have on the product? If we add two units what effect does it have? What inference is deduced from this? What effect does it have on the product to subtract a unit from the multiplier? Two units? What inference is deduced from this? If the multiplier is 1, to what is the product equal? If the multiplier is greater than 1, to what is the product equal? If the multiplier is less than 1, to what is the product equal? How is division defined by many authors? What inference is deduced from this? Is this always the case?

as a divisor, which are of entirely different names. For instance, suppose it were required to divide 100 dollars equally between 4 men, and tell how many dollars each would receive. Now, how are we to take 4 *men* from 100 *dollars,* and how many dollars will remain after the operation has been performed? Why, the very idea of such a thing is absurd and preposterous in the extreme.

The learner will bear in mind that the dividend is equal to the product of the divisor and quotient, plus the remainder, if any.—(Sect. V., Art. 2, Obs. 9.) Hence, the divisor or quotient must one of them always be of the same name as the dividend, since the product must always be of the same name as the multiplicand. (Sect IV., Art 2, Obs. 6.)

Now in the above example the dividend is *dollars,* and the answer required is *dollars;* and we reason—*if* 4 *men receive* 100 *dollars, one man will receive one-fourth of* 100 *dollars.* But one-fourth of one hundred dollars is 25 dollars, and it cannot be any thing else. Here, our divisor is *abstract,* whilst our dividend and quotient are both *concrete.* This example could not be performed by *Subtraction;* hence, the above definition of Division cannot be true in all cases. The divisor is *abstract,* and the dividend and quotient are *concrete;* hence, the inferences deduced from this definition are incorrect.

Obs. 21. Given the divisor and quotient to find the dividend—*Multiply the divisor and quotient together, and to the product add the remainder, if any.*

24. The divisor of a certain number is 15, and the quotient is 6. Required—the number. Ans. 90.

25. If the divisor is 24, and the quotient 36, what is the dividend? Ans. 864.

26. If the divisor is 48, the quotient 15, and the remainder 37, what is the dividend? Ans. 757.

Obs. 22. Given the dividend and quotient to find the divisor—*Subtract the remainder (if any,) from the dividend, and divide the result by the quotient.*

27. If the dividend is 456, and the quotient is 8, what is the divisor? Ans. 57.

28. If the dividend is 108, and the quotient 9, what is the divisor? Ans. 12.

29. If the dividend is 1736, the quotient 144, and the remainder 8, what is the divisor? Ans. 12.

Why not? Give an example. To what is the dividend equal? What inference is deduced from this? Why? In the example given what is the dividend? What answer is required? How do we reason? Of what name is our divisor in this case? Our dividend and quotient? Could this example have been performed by Subtraction? What do we conclude from this? **When we have given the divisor and quotient, how do we find the dividend?**

30. If the dividend is 4829, the quotient 37, and the remainder 19, what is the divisor? Ans. 130.

Obs. 23. To prove division by casting out the 9's—*Cast the 9's out of the divisor and quotient; multiply their excesses together, and cast the 9's out of their product; cast the 9's out of the remainder, add this excess to the last, and cast the 9's out of their sum; and if this latter excess is equal to the excess of 9's in the dividend, the work is correct.*

This Rule is demonstrated in the same manner as the rule for proving Multiplication by casting out the 9's, the dividend being the number separated, and the parts being the *divisor*, (or *quotient*,) taken as many times as there are units in the *quotient*, (or *divisor*,) and *remainder*.

It may be proper to remark that Addition and Subtraction can also be proved by casting out the 9's, but as the method of proof given in Sects. II. and III. is full as short, and easier to be understood, we shall let them suffice at present. An ingenious scholar, however, can study out the proof from the demonstration and remarks under Obs. 17, if he thorougly understands the subject thus far.

31. Divide 207748 by 745, and prove the operation.

Operation.

745)207748(278	The excess of 9's in the divisor is	7
1490	" " " " quotient is	8
——		—
5874	" " " " product of 8×7 is	2
5215	" " " " remainder is	8
——		—
6598	" " " " sum of 8+2 is	1
5960	" " " " dividend is	1
——		—
638	Hence, the work is correct.	

32. Divide 456789 by 365. Ans. 1251, and 174 rem.
33. Divide 764218 by 213. Ans. 3587, and187 rem.
34. Divide 932684 by 416. Ans.2242,and12 rem.
35. Divide 897653 by 321. Ans. 2796, and 137 rem.

Obs. 24. When we have given a quantity either more or less, to find the cost of unity, or 1—*Divide the cost by the quantity.*

36. If 8 pounds of coffee cost 100 cents, how much is that per pound? Ans. $12\frac{1}{2}$ cents.

Demonstrate this rule. When we have given the dividend and quotient, how do we find the divisor? Demonstrate this rule. How do we prove division by casting out the 9's? How is this rule demonstrated? Which is the number separated? Which are the parts? Can Addition and Subtraction be proved by casting out the 9's? Can you study out the rule? When we have given a quantity either more or less, and its cost, how do we find the cost of 1?

37. If 24 stoves cost 576 dollars, how much is that apiece?
Ans. 24 dollars.

38. If 124 horses cost 8308 dollars, how much is that apiece?
Ans. 67 dollars.

a. When we have given the cost of a quantity, and the cost of unity, or 1, to find the quantity—*Divide the cost of the quantity by the cost of unity, or* 1.

39. How many acres of land can I buy for 3834 dollars, at 27 dollars per acre? Ans. 142.

40. How many stoves can I buy for 336 dollars, at 28 dollars apiece? Ans. 12.

Obs. 25. Although Subtraction and Division both separate numbers into *parts*, the learner will observe that there is a great difference between them. In *Division* the parts are *always equal*, (being factors of the dividend;) but in Subtraction, the parts may be either *equal* or *unequal*. Thus 24 may be separated into 4 parts by Division, each of which is 6; and likewise it may be separated into 4 parts by Subtraction, the parts being 7, 3, 9, 5, or 8, 6, 4, 6, or 9, 8, 5, 3, &c. But the former parts (factors,) we *multiply* together to produce 24, ($6 \times 4 = 24$,) whilst we *add* the latter to obtain the same result. ($7 + 3 + 9 + 5 = 24$, &c.) (Sect. IV., Art. 4, Obs. 4, Rem. 2.)

Obs. 26. It is plain from the nature of Division, that the *value of the quotient* depends both on the divisor and dividend. Because, it is self-evident, *that with the same dividend, the greater the divisor, the less will be the quotient, and the smaller the divisor, the greater will be the quotient.* Hence—

a. If we increase our divisor we diminish our quotient; and conversely, if we diminish our divisor, we increase our quotient, if the dividend remains unaltered. Because—

b. If our divisor is unity, or 1, *the quotient is equal to the dividend; if our divisor is greater than* 1, *the quotient is less than the dividend; and if our divisor is less than* 1, *the quotient is greater thna the dividend.*

c. It is also self-evident, that *if we increase the dividend, we increase the quotient; and if we diminish the dividend, we diminish the quotient, if the divisor remains unaltered.* Because—

When we have given the cost of a quantity, and the cost of unity, how do we find the quantity? Explain the difference between Subtraction and Division. Give an example. Upon what does the value of the quotient depend? Why? What inference is deduced from this fact? If our divisor is unity, or 1, to what is the quotient equal? If our divisor is greater than unity, what is the value of the quotient? If our divisor is less than unity, what is the value of the quotient? What effect does it have upon the quotient to increase or decrease our dividend, without altering the divisor?

d. When the dividend and divisor are equal, the quotient is unity, or 1 *; when the dividend is greater than the divisor, the quotient is greater than* 1 *; and when the dividend is less than the divisor, the quotient is less than* 1.

Obs. 27. From these remarks, it is evident that it produces the same effect on the quotient, to *multiply the divisor* by any number, as to *divide the dividend* by the same number ; and also, it produces the same effect on the quotient to *divide the divisor* by any number, as to *multiply the dividend* by the same number. From these facts, we deduce the following considerations :

a. 1st. *To divide the divisor by any number, or to multiply the dividend by the same number, is in effect multiplying the quotient by this number.* Thus:

4	is contained in		12	3 times.
$4 \div 2$	"	"	12	6 times, or 3×2 times.
4	"	"	12×2	6 times, or 3×2 times.

b. 2d. *To multiply the divisor by any number, or to divide the dividend by the same number, is in effect dividing the quotient by this number.* Thus:

3	is contained in		12	4 times.
3×2	"	"	12	2 times, or $4 \div 2$ times.
3	"	"	$12 \div 2$	2 times, or $4 \div 2$ times.

Obs. 28. *To multiply or divide both the divisor and dividend by the same number, does not alter the quotient.* Thus:

8	is contained in		16	2 times.
8×2	"	"	16×2	2 times.
$8 \div 2$	"	"	$16 \div 2$	2 times.

Obs. 29. *If we add the same number to both the divisor and dividend, we diminish the quotient ; and if we subtract the same number from both the divisor and dividend, we increase the quotient.* Thus :

$6 \div 2 = 3.$ And $12 \div 4 = 3.$

$6 + 2 \div 2 + 2 = 2.$ $12 - 2 \div (4 - 2) = 5.$

Remark.—In each of these cases the divisor is supposed to be *less* than the dividend. When the divisor and dividend are *equal* it does not alter the quo-

---

Why is this correct? What fact is evident from these remarks? What is the first consideration we notice from this fact? Give an example. What is the second consideration? Give an example. What effect does it have upon the quotient to multiply or divide both the divisor and dividend by the same number? Give an example. If we add the same number to both the divisor and dividend, what effect does it have upon the quotient? If we subtract the same number from both the divisor and dividend, what effect does it have on the quotient? Give examples. In the last two cases, what is the value of the divisor, compared with that of the dividend? If the divisor and dividend are equal, what effect does it have upon the quotient to add the same number to, or subtract the same number from both?

tient to add the same number to, or subtract it from both. Also—When the divisor is *greater* than the dividend, the converse of the above propositions is true.

Obs. 30. *If a given number be both multiplied and divided by the same number, the final result will be the original number.* Thus—

$$8 \times 7 = 56. \qquad 56 \div 7 = 8.$$

Article 2. Contractions and Abbreviations.

Note.—In addition to the rules for contracting the operations in Multiplication and Division, given in Sects. IV. and V. the following may also be of advantage in some particular cases.

Obs. 1. As to multiply by 10, 100, 1000, &c., we have merely to annex ciphers, to the multiplicand, [Sect. IV., Art. 4, Obs. 1.] it follows, that if a multiplier is an exact part of 100, 1000, &c., we can annex two, three, or more ciphers, if necessary, to the multiplicand, and then divide it by such a number as the multiplier is a part of, 100, 1000, &c. Thus:

To multiply by 25—*Annex two ciphers, and divide by* 4: because 25 is one-fourth of 100.

To multiply by 50—*Annex two ciphers, and divide by* 2: because 50 is one-half of 100.

To multiply by 125—*Annex three ciphers, and divide by* 8: because 125 is one-eighth of 1000.

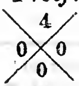
Ex. 1. Multiply 48 by 25.	Ans. 1200.
2. Multiply 72 by 25.	Ans. 1800.
3. Multiply 649 by 25.	Ans. 16225.
4. Multiply 288 by 50.	Ans. 14400.
5. Multiply 897 by 50.	Ans. 44850.
6. Multiply 462 by 125.	Ans. 57750.
7. Multiply 3426 by 125.	Ans. 428250.
8. Multiply 7894 by 125.	Ans. 986750.

9. Multiply 478264 by 12348.

Operation.

```
       478264
        12348
      -------
      1434792
     5739168
     22956672
     --------
Ans. 5905603872.
```

Proof.



If the divisor is greater than the dividend, what is the effect? If a number be multiplied, and the product divided by the same number, what effect does it have upon the final result? Give an example. How do we multiply by 10, 100, 1000, &c.? What inference is deduced from this? How can we multiply by 25? Why is this correct? By 50? Why is this correct? By 125?

We first multiply by 3, and then multiply this product by 4, because $3 \times 4 = 12$, and consequently 4 times 3 times any number is evidently twelve times that number. Next, we multiply this latter product by 4, because $12 \times 4 = 48$, and place the first figure of the product under the 8, because, in reality, we multiply the multiplicand by 48. The first figure of our second partial product we place under the 2, because, in reality, we multiply the multiplicand by 12. The several results are added together, as usual. The operation is proved by casting out the 9's.

10. Multiply 14246 by 819. ($81 = 9 \times 9$.) Ans. 11667474.

11. Multiply 327436 by 126721. ($21 = 7 \times 3$; $126 = 21 \times 6$.) Ans. 41493017356.

12. Multiply 9476245 by 648963. ($63 = 9 \times 7$; $64 = 8 \times 8$.) Ans. 6149732383935.

Obs. 2. The learner will perceive from these examples, that *it makes no difference which figure of the multiplier we multiply by first, provided we place the first figure of the product directly under the figure by which we multiply.*

13. Multiply 326 by 241.

Operation.

$$\begin{array}{r} 3^3 2^2 6^1 \\ 2^2 4^1 1^0 \\ \hline \text{Ans. } 7^5 8^4 5^3 6^2 6^1. \end{array}$$

By this method we number the figures of each factor, commencing (at the right hand,) with 1 in the multiplicand, and 0 in the multiplier. *This is done to assist the memory.* These small figures we call EXPONENTS.

We number each figure of the product, (our first exponent being 1,) *to show which figures we multiply together.* Thus, we wish in the first place to find a figure in the product, the exponent of which is 1. Now we multiply those figures together, the sum of whose exponents is equal to the exponent required in the product: the exponent of 1 is 0, and the exponent of 6 is 1; $0 + 1 = 1$; then we multiply 6 by 1, and $6 \times 1 = 6$, the first figure of the product. Next we wish to find a figure of the product whose exponent is 2. The exponent of 1 is 0, and that of 2 is 2; $0 + 2 = 2$: also the exponent of 4 is 1, and that of 6 is 1; $1 + 1 = 2$; then $2 \times 1 = 2$, and $6 \times 4 = 24$; $24 + 2 = 26$. We set down the 6 and carry the 2 as usual. Now we wish a figure in the product whose exponent is 3: $0 + 1 = 3$; $1 + 2 = 3$; and $2 + 1 = 3$; then $3 \times 1 + (2 \times 4) + (6 \times 2) = 23$, and 2 to carry makes 25. Then 5 is our third figure

Why is this correct? Explain the operation of Ex. 9. What conclusion is drawn from this and the three following examples? What is our first step in the operation of Ex. 13? Why is this done? What are these small figures called? What is our first exponent in the product? Why do we number the product? What do we wish to obtain first? Which figures do we multiply together?

in the product, and 2 to carry. The next figure in the product must have 4 for its exponent: $1+3=4$, and $2+2=4$; then $3\times 4+(2\times 2)=16$, and 2 to carry makes 18; we set down 8 and carry 1. Our next exponent in the product is 5; $2+3=5$; then $3\times 2=6$, and 1 to carry makes 7 as our next figure in the product, and as the sum of no two exponents exceeds 5, we conclude that our work is done.

Obs. 3. By examining this operation attentively we notice the following considerations:

1st. *We add but two exponents together at a time, one of which belongs to each factor.*

2d. *We multiply those figures together, the sum of whose exponents is equal to the exponent sought, add their several products together mentally, and set down and carry as usual.*

Obs. 4. The only difference between this and the usual method of multiplying is, that by this method the operation is performed *mentally*, whilst by the other it is all written upon the slate; *the mental exercise and discipline of mind*, (no contemptible considerations,) being the chief advantages gained by this manner of operating.

14. Multiply 379 by 647. Ans. 245213.
15. Multiply 1846 by 1234. Ans. 2277964.
16. Multiply 27345 by 97216. Ans. 2658371520.
17. Multiply 894832 by 687219. Ans. 614945552208.

REMARK.—We have already learned how to multiply by 25, 50 and 125, and it is evident that we can divide by these numbers, by merely reversing the operation, as division is the opposite of multiplication.

Obs. 5. Hence, to divide by 25—*Multiply the dividend by 4; cut off two figures from the right, and take one-fourth of these for the true remainder.*

18. Divide 176 by 25. Ans. 7, and 1 rem.

Operation.—$176\times 4=7|04$; $4\div 4=1$.

19. Divide 275 by 25. Ans. 11.
20. Divide 1284 by 25. Ans. 51, and 9 rem.
21. Divide 4294 by 25. Ans. 171, and 19 rem
22. Divide 36812 by 25. Ans. 1472, and 12 rem.

REMARK.—The reason why we divide the remainder by 4, is because the remainder is *one-hundredths*, when it should be *twenty-fifths*. We reduce it to twenty-fifths by dividing it by 4. The learner will understand this better when he has studied Fractions.

Give the remaining solution of this question. How do we know when our operation is completed? In examining this operation what is the first consideration we notice? The second? What is the difference between this and the common method of operating? What are the chief advantrges derived from this method? How may we divide by 25? Why do we take one-fourth of the figures cut off for the true remainder?

Obs. 6. To divide by 125: *Multiply the dividend by 8; cut off three figures from the right, and take one-eighth of these for the true remainder.*

REMARK.—The figures cut off are *thousandths*, whereas they should be 125*ths*. We reduce them to 125*ths* by dividing by 8. (See remark above.)

23. Divide 1462 by 125. Ans. 11, and 87 rem.

Operation.—$1462 \times 8 = 11|696$; $696 \div 8 = 87$.

24. Divide 3216 by 125. Ans. 25, and 91 rem.
25. Divide 4781 by 125. Ans. 38, and 31 rem.
26. Divide 4821 by 125. Ans. 38, and 71 rem.
27. Divide 3976 by 125. Ans. 31, and 101 rem.

Obs. 7. When there is a remainder after dividing by several numbers, to find the true remainder—*Multiply each remainder by all the preceding divisors, and to the sum of their product add the first remainder.*

28. Suppose a teacher had 158 apples to divide between 6 classes, and each class contained 4 scholars. How many should he give to each scholar?

Ans. 6, and he would have 14 apples left, or $6\frac{14}{24}$ apples each.

Operation.

$$\begin{array}{r|l} 6 & 158 \\ \hline 4 & 26 \text{— 2 rem.} \quad 2 \times 6 = 12;\ 12 + 2 = 14 \text{ true rem.} \\ \hline & 6 \text{— 2 rem.} \end{array}$$

He first divides the apples into 6 piles, one for each class, and finds that each pile will contain 26 apples, and there will be 2 apples left. He then divides each pile into 4 parts, one for each scholar, and finds that there are 2 apples left in each pile. Now as there are 6 piles, there must be $2 \times 6 = 12$ apples left in all the piles, and the two that were left in the first place make 14. The same method of reasoning will apply to any number of divisors; hence, the above rule is correct.

How do we divide by 125? Why do we take one-eighth of the figures cut off for the true remainder? When there is a remainder after dividing by several divisors, how do we find the true remainder? Explain the operation of Ex. 28, and show why it is correct. From what divisor is each remainder exempt from being multiplied? How may we often contract operations in Long Division? Are the general rules, Sects. IV. and V., sufficient for all calculations in Multiplication and Division? Why then do we use contractions and abbreviations?

29. Divide 2373 by 2, 3, 4, 2 and 3. Ans. 16, and 69 rem.

Operation.

```
2|2373
 ------          Last rem.  1 × 2 × 4 × 3 × 2 = 48
3|1186 — 1 rem.  Third rem,     3 × 3 × 2 = 18
 ------          Second rem, 1 × 2        =  2
4|395 — 1 rem.   First rem.,                 1
 ----                                       --
2|98 — 3 rem.                      Sum 69 true rem.
 --
3|49
 --
 16 — 1 rem.
```

REMARK.—The learner will perceive that no remainder is multiplied by th divisor from which this remainder accrued.

30. Divide 1706 by 2, 3, 7, 3, and 4. Ans. 3, and 194 rem.
31. Divide 2903 by 3, 4, 2, and 3, Ans. 40, and 23 rem.
32. Divide 3721 by 2, 4, 2, 3, and 2. Ans. 38, and 73 rem.
33. Divide 4973 by 4, 2, 4, 2, and 5. Ans. 15, and 173 rem.
34. Divide 79641 by 4, 7, 6, 8, 9. and 5.
Ans. 1, and 19161 rem.

Obs. 8. We may often contract operations in Long Division, by *rejecting factors; that is, by dividing both the dividend and divisor by any number that will divide the divisor without a remainder.*

35. Divide 625 by 25. Ans. 25.

Operation.

```
5)25|625
  --|---     We first divide both the divisor and dividend by
   5|125     5, and then proceed as in Simple Division.
    |---
Ans.  25
```

36. Divide 34921 by 1728. Ans. 20, and 361 rem.

Operation.

```
8)1728|34921              Last rem.  5 × 8 × 8 = 320
  ----|-----              Second "   5 × 8 =      40
 8)216| 4365 — 1 rem.     First  "                 1
   ---|-----                                     ---
  9)27|  545 — 5  "                 Sum 361 true rem.
    --|-----
     3|   60 — 5  "
      |  ---
          20
```

37. Divide 4276 by 288. ($288 = 9 \times 8 \times 4$.)
Ans. 14, and 244 rem.

38. Divide 6953 by 256. ($256 = 8 \times 8 \times 4$.)
Ans. 27, and 41 rem.

39, Divide 7491 by 625. ($625 = 5 \times 5 \times 5 \times 5$.)
Ans. 11, and 616 rem.

40. Divide 17426 by 1296. ($1296 = 9 \times 6 \times 8 \times 3$.)
Ans. 13, and 578 rem.

Remark.—The ingenious pupil, when he becomes acquainted with Fractions, can easily study out more contractions, both in Multiplication and Division, (and can derive both pleasure and profit from his labor); but he must recollect that the General Rules, in Sects. IV. and V., will apply to any case of either that may occur. The chief advantage derived from abbreviations is, that by the aid of these we can often solve questions *mentally*, which would otherwise require the aid of the slate. More methods might be given, but is thought unnecessary, as these are sufficient for any calculations in common business.

SECTION VII.

CANCELATION.

Article 1. Definitions and Illustrations.

Obs. 1. Cancelation *is a short method of performing many operations of numbers, when Multiplication and Division are both concerned.* To *Cancel* means to *erase*, or *reject*.

Obs. 2. It very often happens in arithmetical calculations that we have, in the same question, several numbers that are *multipliers*, and several that are *divisors*. Now, if we should so arrange these numbers that all the divisors would be by themselves, and all the other numbers by themselves, it is highly probable that the same number might occur both as a *multiplier* and *divisor;* or, if not, there might be *factors* common to both multipliers and divisors, which might be rejected, and thus very much shorten the operation.—*This is the object of Cancelation.*

Obs. 3. The first thing we do towards arranging these numbers, so as to cancel them, is to *draw a perpendicular line, and place all our divisors at the left, and all our other numbers at the right hand side of this line. This is done because it is most convenient.*

Obs. 4. Now from the very manner in which our numbers are placed, it follows, *that the numbers at the right are all factors of some number which is a dividend, and the numbers on the left are all factors*

What is Cancelation? What does cancel mean? What often happens in arithmetical calculations? How may we often shorten the operation in such cases? What is the first thing we do in order to cancel numbers?

of some number which is a divisor, and the answer to the question is the quotient of the one divided by the other.

Therefore, in the following examples, we have used the term *Dividend* to represent the numbers at the right, when taken *collectively*, and the term *Divisor* to represent the numbers at the left, when taken in the same manner, and the term *Factor* to represent the numbers *on either side of the line* when taken *separately*.

Ex. 1. Multiply 24 by 6 and 8, and divide the result by 16 and 6.

Operation.

$$\begin{array}{r|l} & \cancel{24}-12 \\ \cancel{2}-\cancel{16} & \cancel{6} \\ \cancel{6} & \cancel{8} \\ \hline & \text{Ans. } 12 \end{array}$$

We first find that the factor 6 is common to both our divisor and dividend, and therefore cancel it. (Sect. VI., Art. 1, Obs. 30.) We next divide both our divisor and dividend by 8; this is done by canceling the 8 in our dividend, into the 16 in our divisor, and setting the other factor of 16 (2) on the side of our divisor (Sect. VI., Art. 1, Obs. 28.) Finally, we cancel the 2 into 24, which gives 12 as our answer.

2. Multiply 4667 by 24, 50 and 63, and divide the result by 36, 28, 26 and 40.

Operation.

$$\begin{array}{r|l} \cancel{2}-\cancel{26} & \cancel{4667}-359 \\ \cancel{3}-\cancel{36} & \cancel{24}-\cancel{2} \\ 4-\cancel{28} & 5\cancel{0} \\ 4\cancel{0} & \cancel{63}-\cancel{9}-3 \\ \hline 16 & 5385(336\tfrac{9}{16}\ \text{Ans} \\ & 48 \\ & \overline{\ 58} \\ & \ \ 48 \\ & \overline{\ 105} \\ & \ \ \ 96 \\ & \overline{\ \ \ \ 9} \end{array}$$

We first divide both the divisor and dividend by 10; this is done by canceling the cipher in 40 and 50. (Sect. V., Art. 4, Obs. 1.) Then we divide both divisor and dividend by 12; this is done by rejecting 12 in the factors 24 and 36, canceling these factors, and placing each quotient (from dividing them by 12,) on the side of the number divided to produce it. In the same manner we cancel 28 and 63, dividing both by 7. We also cancel 26 and 4667, by dividing both by 13. The value of the result as yet remains unaltered. (Sect. VI., Art. 1, Obs. 28.) We now have a 2 on each side of the line, which we cancel, and also cancel the 3 into the 9. Now we have 359, 5, and 3 left on the right, and two 4's on the left, and as these will not cancel, we multiply those on the left together for a divisor, and those on the right for a dividend, and proceed as in Simple Division.

Why are they placed in this manner? What conclusion is deduced from the numbers being placed in this way? In the examples given, what do the terms divisor, dividend and factor represent? Explain the operation of Ex. 1. Explain the operation of Ex. 2.

3. Multiply 132 by 9, 7, 16 and 12, and divide the result by 11, 144, 63, 8 and 2.

Operation.

$$\begin{array}{r|r}
\cancel{11} & \cancel{132} \\
\cancel{12} - \cancel{144} & \cancel{9} \\
\cancel{63} & \cancel{7} \\
\cancel{8} & \cancel{16} \\
\cancel{2} & \cancel{12} \\
 & \overline{} \\
\text{Ans. } 1. &
\end{array}$$

9 times 7×63; therefore, we cancel these two into 63. 8 times $2 = 16$; we cancel these into 16. We cancel 12 in 144, which leaves 12 as the other factor (of 144). 12 times $11 = 132$; therefore we cancel these into 132. Thus every number cencels, and consequently the divisor is equal to the dividend, and the answer is 1. (Sect. VI., Art. 1, Obs. 26, *d.*)

4. Multiply 72 by 32, and 16, and divide the product by 144, 48, and 24.

Operation.

$$\begin{array}{r|l}
2 - \cancel{144} & \cancel{72} \\
3 - \cancel{48} & \cancel{32} - \cancel{4} - 2 \\
3 - \cancel{24} & \cancel{16} \\
\hline
9 & 2 = \text{Ans. } \frac{2}{9}.
\end{array}$$

72 cancels into 144; 16 into 48; 8 divides 24 and 32, and 2 cancels into 4. Then we multiply the numbers on the left together, which gives 9 as our divisor, whilst our dividend is 2. In this case our result is fraction, or less than unity, (Sect. VI., Art. 1, Obs. 26, *d.*,) and we can only express the divisions by writing the dividend over the divisor.

5. Multiply 48 by 24, and divide the product by 96 and 72.

Operation..

$$\begin{array}{r|l}
2 - \cancel{96} & \cancel{48} \\
3 - \cancel{72} & \cancel{24} \\
\hline
6 & \frac{1}{6}. \text{ Ans.}
\end{array}$$

In this case all the numbers cancel on the right of the line; but when all the numbers cancel on either side of the line, the remaining factor on that side is unity, or 1. Thus, 48 in 48 goes 1 time; 48 in 96 goes 2 times; 24 in 24 goes 1 time; 24 in 72 goes 3 times; $1 \times 1 = 1$; $2 \times 3 = 6$; hence, we must divide 1 by 6, and $1 \div 6 = \frac{1}{6}$. (Sect. 5. Art. 2. Obs. 4. *a.*)

Obs. 5. From attentively examining these operations, we notice the following considerations:

a. 1st. *In all questions in Cancelation we have an operating number or leading term, which is always placed at the right.*

b. 2d. *It does not alter the result to divide numbers on both sides of the line by the same number, and canceling one number into another is in reality dividing both sides by that number.*

c. 3d. *When we divide two factors, (one of the divisor and the*

Explain the operation of Ex. 3. Explain the operation of Ex. 4. When all the numbers cancel on both sides, what is the result? When all the numbers cancel on either side of the line what is the remaining factor on that side? Illustrate this. In examining these operations, what is the first consideration we notice? The second?

other of the dividend,) by any number, we place each result on the side of the number divided to produce it.

d. 4th. *When we have canceled all we can, we multiply together the remaining numbers at the left for a divisor, and those at the right for a dividend, and proceed as in Division of Simple numbers.*

e. 5th. *If all the numbers on both sides of the line cancel, the answer is* 1.

f. 6th. *If, after canceling, the divisor is greater than the dividend, the answer is less than unity, and the division can only be expressed.*

g. 7th. *Unity is a factor on either side of a line, when all the numbers cancel on that side.*

Obs. 6. The learner must bear in mind, in all his operations, that *the principle of cancelation consists, simply, in rejecting the factors common to both the divisor and dividend.* This being the case, all our canceling is performed by *Division.* Some, however, attempt to explain it on the principles of *Subtraction,* saying, that *if equals are taken from equals, the remainder will be equal.* The principle is correct; but as our divisor and dividend are not always *equal,* unfortunately, this principle cannot apply to Cancelation; because the subtraction of the same number from *unequal* quantities, where one is a divisor and the other a dividend, will materially alter the value of the quotient. (Sect. VI., Art. 1, Obs. 29.)

From the preceding remarks and illustrations, we derive the following

GENERAL RULE FOR CANCELATION.

I. *Draw a perpendicular line, and place the operating number, or leading term, at the right, together with all the multipliers, and all the other numbers at the left.* (Obs. 3 and 5, *a.*)

II. *When the same factor occurs on both sides of the line, cancel it in both places.*

III. *After canceling all common factors, multiply together all the remaining numbers at the right for a dividend, and those at the left for a divisor, and then proceed as in Division of Simple numbers.* (Obs. 5, *d.*)

Note.—Read carefully the considerations under Obs. 5.

Third? Fourth? Fifth? Sixth? Seventh? In what does the principle of Cancelation consist? How is it performed? How do some explain it? Why do they explain it thus? Is this principle correct? Why, then, will it not apply to Cancelation? Will it produce the same final result to subtract the same number from unequal quantities, when one is a divisor and the other a dividend? Why not? In the General Rule, what is the first thing we do? What is the second? The third?

EXERCISES FOR THE SLATE.

1. If 6 bushels of potatoes cost 18 shillings, how much would 9 bushels cost?

Operation.

$$\begin{array}{l|l} \cancel{6} & \cancel{18} - 3 \\ & 9 \\ \hline \end{array}$$

Ans. 27 shillings.

If 6 bushels cost 18 shillings, 1 bushel will cost $18 \div 6 = 3$ shilllings, and 9 bushels would cost 9 times as much as 1 bushel.— Hence, in this example we divide by 6 and multiply by 9.

2. If 16 yards of cloth cost 64 dollars, how much would 19 yards cost? Ans. 76 dollars.

3. Paid 50 dollars for 16 sheep. how much should I pay for 40 sheep at the same rate? Ans. 125 dollars.

4. How much must I pay for 36 horses, if I pay 450 dollars for 11 horses? Ans. $1472\frac{8}{11}$ dollars.

5. How much must I pay for 32 yards of broadcloth, if 8 yards cost 48 dollars? Ans. 192 dollars.

6. How much must I pay for 18 cows, if 6 cows cost 90 dollars? Ans. 270 dollars.

7. There are 365 days in a year, and 24 hours in a day, and the earth moves around the sun at the rate of 68000 miles an hour. Now how many years would it take a man to travel the distance the earth moves in a year, allowing a man to travel 40 miles a day? Ans. 40800.

8. How long would it take him, if he traveled 60 miles a day? Ans. 272000 years.

9. How long, if he traveled 80 miles a day? Ans. 20400 years.

10. If a man walk 200 miles in 6 days, how many miles can he walk in 28 days? Ans. $933\frac{1}{3}$.

11. If I pay 24 dollars for 12 books, how many dollars must I pay for 19 books? Ans. 38.

12. How much would 40 acres of land cost, if 30 acres cost 150 dollars? Ans. 200 dollars.

13. How much would 60 bushels of wheat cost, if 25 bushels cost 30 dollars? Ans. 72 dollars.

14. How much would 76 bushels of oats cost, if 15 bushels cost 4 dollars? Ans. $20\frac{4}{15}$ dollar.

15. If I pay 150 dollars for 6 stoves, how much must I pay for 14 stoves? Ans. 350 dollars.

16. 18 times 432 are how many times 12. Ans. 648 times.

17. Multiply 847 by 19, 28, and 54, and divide the result by 57, 36, and 98. Ans. 121.

18. Multiply 8700 by 91, 46, and 144, and divide the result by 390, 132, and 1740. Ans. $58\frac{6}{11}$.

19. Multiply 50 by 128, 75, 35, and 85, and divide the result by 1200, 1360, and 875. Ans. 1.

20. Multiply 380 by 57, 115, 186, and 323, and divide the result by 760, 171, 92, 589, and 765. Ans. $\frac{1}{36}$.

Article 2. Greatest Common Divisor.

Suppose it were required to find some number that would divide 8 and 12 without a remainder. We find that 2 will divide both, because $8 \div 2 = 4$, and $12 \div 2 = 6$. Then 2 is a common divisor of 8 and 12. Hence—

Obs. 1. *A common divisor of two or more numbers, is a number that will divide them without a remainder.*

But suppose we wished to find the *greatest* number that would divide 8 and 12. This we find to be 4, as $8 \div 4 = 2$, and $12 \div 4 = 3$, and 2 and 3 are both prime numbers, and cannot be divided again. Then 4 is the greatest common divisor of 8 and 12.—Hence—

Obs. 3. The greatest Common Divisor *of two or more numbers is the greatest number that will divide them without a remainder.*

Remark 1.—One number is said to be a *measure* of another number, when the *former* is contained in the *latter* without a remainder. Therefore, a common divisor is often called a Common Measure; and the greatest common divisor of two or more numbers, is often called the Greatest Common Measure of those numbers.

2. It must be observed, however, that whilst a *measure* can exist with reference to *two* numbers, the *common measure*, and the *greatest common measure* must both always have reference to at least *three*, one of which must *divide* the other two. Thus, 4 is a measure of 16, a common measure of 20 and 24, and the greatest common measure of 8 and 12.

Ex. 1. What is the greatest common divisor of 9 and 126?

Solution.—The greatest common divisor cannot be greater than 9, because it must divide 9. 9 will divide itself; let us see if it will divide 126. We find by trial, that 126 contains 9 exactly 14 times; then 9 must be the greatest common divisor of 9 and 126.

2. What is the greatest common divisor of 15 and 80?

Operation.

```
15)80(8
   75
   —
    5)15(3
      15
      —
      00
```

We first try if 15 is the greatest common divisor. After dividing 80 by 15, we have a remainder of 5. Now if 5 will divide 15, it will also divide 80; because if 15 contains 5, 75 will also contain 5, as 75 is divisible by 15; then if 75 contains 5, 80 will also contain 5; be-

---

What is a common divisor of 2 or more numbers? The greatest common divisor? When is one number said to be a measure of another? What is a common divisor often called? The greatest common divisor? What is the difference between a measure, a common measure, and the greatest common measure of numbers? Give an example. Explain the solution of Ex. 1.

cause $80 = 75 + 5$; that is, 80 will contain 5, one more time than 75. We find that 15 will contain 5, therefore 5 is the greatest common divisor of 15 and 80.

From this example, we perceive that the greatest common divisor of two numbers, *must be a common divisor of the least number and their remainder after division*; therefore, it cannot be *larger* than this remainder, because the remainder must be divided by it.—Hence—

To find the greatest common divisor of two numbers, we have the following

RULE.—*Divide the greater number by the less, and that divisor by the remainder, and so on, continuing to divide the last divisor by the last remainder, till nothing remains. The last divisor will be the greatest common divisor.*

NOTE.—1. The learner will perceive that a common divisor of two or more numbers, is simply a *common factor* of those numbers; and the greatest common divisor is their *greatest common factor.*

2. We learn, (Sect. IV. Art. 4, Obs. 3, Rem. 1.) that a prime number cannot consist of two factors, both greater than unity, or 1. Therefore, *a prime number is divisible only by itself and unity.*

3. One number is said to be *prime to another,* when only a unit will divide both of them. Hence—

Obs. 3. *Two or more prime numbers cannot have a common divisor, because they have no common factors.*

EXERCISES FOR THE SLATE.

1. What is the greatest common divisor of 18 and 78?
Ans. 6.

2. What is the greatest common divisor of 23 and 120?
Ans. 1.

3. What is the greatest common divisor of 32 and 122?
Ans. 2.

4. What is the greatest common divisor of 69 and 291?
Ans. 3.

5. What is the greatest common divisor of 231 and 517?
Ans. 11.

6. What is the greatest common divisor of 5191 and 8497?
Ans. 29.

To find the greatest common divisor of more than two numbers:

Obs. 4. *Find the greatest common divisor of two of them; then*

Explain the operation of Ex. 2. What consideration do we notice in the solution of this example? How do we find the greatest common divisor of two numbers? What is a common divisor? The greatest common divisor? Of what cannot a prime number consist? What inference do we deduce from this? When is a number said to be prime to another? Can two or more prime numbers have a common divisor? Why not? What is the first rule for finding the greatest common divisor of more than two numbers?

of this greatest common divisor, and another given number, and so on, through all the given numbers. The last common divisor found will be the one required. Or,

Obs. 5. *Write the numbers in a line. Divide them by any number that will divide them all without a remainder, and write the quotients below, as in Division.*

Divide these quotients in the same manner, and so continue to do, till no number greater than 1 will divide them all without a remainder.

Finally, multiply together all the divisors, and their product will be the greatest common divisor.

Demonstration.—We learn, Sect. IV, Art. 4, Obs. 2, that *every composite number is composed of factors;* and if we resolve two or more numbers into their several factors, and the *same factor* occurs in *all* the numbers, this factor is evidently a *common divisor* of these numbers, and the *product* of all these common factors is their *greatest common divisor.* Now these common factors are the numbers by which we *divide;* hence, the above rule is correct.

7. What is the greatest common divisor of 64 and 96?

Operation.

```
8)64__96
  ------
 4)8__12
   -----
   2__ 3
8×4=32 Ans.
```

We first divide by 8 and then by 4. 2 and 3 are not divisible by any number greater than 1, hence, $8 \times 4 = 32$ the greatest common divisor required.

8. What is the greatest common divisor of 2457, 3213, and 1197?

By the first Rule.

```
2457)3213(1
     2457
     ----
      756)2457(3
          2268
          ----
           189)756(4
               756
               ---
               000

189)1197(6
    1134
    ----
Ans. 63)189(3
        189
        ---
        000
```

By the second Rule.

```
9|2457_3213_1197
 |--------------
7| 273__357__133
 |--------------
    39 __ 51 __ 19

9 × 7 = 63. Ans.
```

Obs. 6. The following rules may assist the learner in finding common divisors:

a. Any even number may be divided by 2, because as the remain-

The second rule? Demonstrate this rule. What numbers can be divided by 2? By 3? By 9?

der must always be less than the divisor, (Sect. V, Art. 2, Obs. 12, *b*.) the last partial dividend must be 0, 2, 4, 6, 8, 10, 12, 16, or 18, either of which will contain 2.

b. Any number can be divided by 3 *when the sum of its digits can be divided by* 3.

c. Also, *Any number can be divided by* 9 *when the sum of its digits can be divided by* 9.

The last two rules are both demonstrated in the same manner, (Sect VI, Art. 1, Obs. 17, demonstration.)

d. Any number can be divided by 4, *when its two right hand digits can be divided by* 4. Because, if the number can be divided by 4, it can be divided by 2 × 2; (Sect. V., Art. 4, Obs. 2, *a*.) Therefore, the half of such a number must be an even number; and if half of any number will contain 2, the number itself will contain 4.

e. Any number can be divided by 5, *when its right hand figure is* 5 *or* 0. Because, as the remainder is always less than the divisor, (Sect. V., Art. 2, Obs 12, *b*.,) the last partial dividend must be 5, 10, 15, 20, 25, 30, 35, 40, or 45, all of which will contain 5.

f. Any number ending with 0, 00, 000, &c., *can be divided by* 10, 100, 1000, &c. (Sect V., Art. 4, Case 1.)

g. No even number will divide an odd number, and no number will contain a greater number than its half. Because—

1st. If an odd number will contain an even number, it will also contain 2, which is a factor of all even numbers; (Sect. V., Art. 4, Obs. 2, *a*.) But this cannot be according to Sect. IV., Art. 4, Obs. 3, Rem. 3. The reason why an odd number will not contain 2, is because the last partial dividend is always 1, 3, 5, 7, 9, 11, 13, 15, 17, or 19, none of which will contain 2 without a remainder.

2nd. Any number will contain its half just twice. Hence, if the divisor is greater than the half of the number, it must be equal to the number itself, or there will be a remainder after the division is performed; else it must be greater than the number, when the result is a fraction. (Sect. VI., Art. 1, Obs. 26, *d*.)

Find the greatest common divisor of the following numbers:

9. 48 and 60. Ans. 12.
10. 24, 36, and 84. Ans. 12.
11. 36, 96, and 144. Ans. 12.
12. 25, 45, 85, and 115. Ans. 5.
13. 22, 44, 143, 209, and 297. Ans. 11.
14. 18, 63, 99, 117, and 171. Ans. 9.

By 4? By 5? By 10, 100, 1000, &c.? Will an even number divide an odd number? Why not? What is the greatest divisor any number will contain? Demonstrate these rules.

15. 48, 54, 75, and 111. Ans. 3.
16. 249, 332, and 415. Ans. 83.
17. 94, 188, 282, and 423. Ans. 47.
18. 78, 117, 143, and 169. Ans. 13.
19. 12, 20, 36, 44, and 48. Ans. 4.
20. 21, 49, 63, 91, and 133. Ans. 7.

ARTICLE 3. LEAST COMMON MULTIPLE.

Obs. 1. *One number is said to be a* MULTIPLE *of another number, when the former contains the latter without a remainder.* Thus, 4 or 6 is a multiple of 2, because either of them will contain 2.

Obs. 2. A COMMON MULTIPLE *is any number that will contain two or more numbers without a remainder.* Thus, 12 is a common multiple of 3 and 4, because it will contain both 3 and 4 without a remainder.

REMARK.—A *multiple,* whether common of *one, two,* or more numbers, *is always a composite number,* and the numbers contained in it are its *factors.* Thus, 2 is a factor of 4, and 3 and 4 are factors of 12.

Hence—To find a common multiple of two or more numbers :

Obs. 3. *Multiply them together.*

Ex. 1. What is a common multiple of 3, 4, and 6. Ans. 72.
2. Of 4, 8, and 10. Ans. 320.
3. Of 2, 4, 6, 8, 10, and 12. Ans. 46080.

REMARK 1.—It is sometimes desirable to find the *least* number that will contain two or more numbers without a remainder. This is called their LEAST COMMON MULTIPLE. Thus, 72 is a *common* multiple of 3, 4, and 6 ; but 12 is their *least* common multiple.

2. A common multiple of two or more numbers must contain *all the different factors* of these numbers. Thus, 48, a common multiple of 6 and 8, contains 3 and 2, the factors of 6, and also 4 and 2, or 2, 2, and 2, the factors of 8 ; but 32 will not contain the factors of 6 and 8, therefore it is not a common multiple of these numbers.

4. What is the least common multiple of 3, 5, and 7?

These numbers are all *prime,* and have no *factors ;* therefore, no number less than their continued product will contain them ; that is $3 \times 5 \times 7 = 105$. Ans. Hence—

To find the least common multiple of prime numbers—

Obs. 4. *Multiply them together.*

When is one number said to be a multiple of another? Give an example. What is a common multiple? Give an example. What kind of a number is a multiple? What are its factors? Give examples. How do we find a common multiple, or two or more numbers? What is the least common multiple of two or more numbers? Give an example. What must all common multiples of two or more numbers contain? Give an example. What factor of 6 or 8 will not 32 contain?

5. What is the least common multiple of 2, 3, and 5?
Ans. 30.

6. What is the least common multiple of 7, 11, and 13?
Ans. 1001.

7. What is the least common multiple of 17, 19, 23, and 29?
Ans. 215441.

To find the least common multiple of composite numbers:

Obs. 5. As we have said before, (Sect. V., Art. 4, Obs. 2, *a*,) *if any number will contain a composite number, it will also contain its factors.* Thus, 24 will contain 12, and it will also conta'n 2 and 6, or 3 and 4, the factors of 12.

8. What is the least common multiple of 6 and 8?

Solution.—The factors of 6 are 2×3; of 8, $2 \times 2 \times 2$. Now we wish our multiple to contain all these factors, but it is evident that if the same factor occurs in both numbers it will occur *twice* in their product, or common multiple, where it need only occur once. Then as the factor 2 occurs in both 6 and 8, we will reject it in one of them, and the product of the other factors $3 \times 2 \times 2 \times 2 = 24$, the least common multiple required.

9. What is the least common multiple of 2, 4, 6, 8, 12, and 16?

Operation.

$\cancel{2}$--$\cancel{4}$--$\cancel{6}$--$\cancel{8}$--$\cancel{12}$--16
3
$16 \times 3 = 48$ Ans.

Any number that we can divide by 16, we can divide by 2, 4, and 8, be-cause these numbers are factors of 16; also, any number that we can divide by 12, we can divide by 6, as 6 is a factor of 12. (Obs. 5.) Now we have remaining 12 and 16, but these numbers have a common factor, (4.) This we reject in the 12, and multiplying 16 by the other factor of 12, (3,) we obtain 48 as our answer.

Remark.—We write our numbers, placing the *largest* at the right, because it is *more convenient* to place them in this manner.

10. What is the least common multiple of 3, 9, 8, 15, 6, 5, 14, 10 and 12?

Operation.

1st line, $\cancel{3}$--$\cancel{5}$--$\cancel{6}$--$\cancel{8}$--$\cancel{9}$--$\cancel{10}$--$\cancel{12}$--14--15
2nd " 3-- $\cancel{2}$-- $\cancel{4}$
3rd " $\cancel{4}$ -- 2
4th " 2

$2 \times 2 \times 3 \times 14 \times 15 = 2520$ Ans.

When any number will contain a composite number, what must it also contain? Give an example. Explain the solution of Ex. 8. Why do we reject the factor 2 from one of the numbers in this example. Explain the operation of Ex. 9. How do we write our numbers? Why? Explain the operation of Ex. 10.

We first cancel 3, 5, and 6, because they are factors of 9, 10, and 12. We next find that the factor 3 is common to 9 and 15; we cancel it in the 9; 5 is a common factor of 10 and 15; we cancel it in the 10; 3 is a common factor of 12 and 15; we cancel it in the 12; the remaining factors of these numbers (9, 10, and 12,) we write below, and these (the factors 3, 2, and 4), form the second line. Next, we find that the factor 2 is common to 8 and 14; we cancel it in the 8; 2 (in the 2nd line,) will divide 14; we cancel it, (2;) 2 is a factor common to 4, (in the 2nd line,) and 14; we cancel it in the 4; we write the remaining factors (of 8 and 4) below, as our third line. Next, we find the factor 2 common to 4 and 2, (in the third line;) rejecting it in the 4, we write the remaining factor (of 4) below as our fourth line. Finally, we multiply together the numbers remaining (in all the lines,) and their product is the least common multiple required.

Obs. 6. From examining this operation, we notice the following considerations:

1st. *We work, in all cases, from the left hand towards the rtght,* because it is more convenient, from the manner in which the numbers are written. Thus:

a. We cancel a number at the left, when it will divide one at the right, because the number at the left is a *factor* of the number at the right.

b. When we cancel a common factor, we cancel it in the nvmber at the left. This is done, because the number at the left being *smaller* than the number at the right. the factor that is *not* canceled (at the left) is smaller than the corresponding factor of the number at the right, and by this means we obtain the smallest numbers as our final multipliers.

2d. *When we reject common factors from two numbers, we reject the* GREATEST FACTOR *common to them.*

This is done to prevent the *same factor* from occurring more times than is necessary in the multiple. This would frequently be the case if we simply canceled the smaller factors. Besides, by canceling the *greater* factors, our final multipliers are smaller.

3d. *We cannot reject factors twice from the same numbers.* Because the first time, we cancel the *greatest* factor common, and if we should again cancel factors from the *same* numbers, our multiple might not contain both these numbers. Thus, (in the last example) after canceling the factor 3 from 9, (because 3 is a common fac-

---

What is the first consideration we notice from examining this operation? Why? Show how we do this in the first place. Why? How in the second place? Why do we cancel the factor in the number at the left? What is the second consideration we notice? Why do we reject the greatest factor? What is the third consideration? Why not?

tor of 9 and 15,) some might say that 3 (the other factor of 9) was a factor of 15, and cancel it accordingly. This would, in reality, be canceling 9 into 15, or making 9 a factor of 15, which is not the case, and the consequence would be that the final result would not contain 9.

4th. *We first cancel factors with reference to the right hand number; next, with reference to the second number from the right, and then with reference to the third number from the right, and so on.* We do this, because the numbers at the right being the *largest*, it is highly probable that many of the smaller numbers will cancel *out entirely* in these, and thus we would have fewer numbers to multiply together to obtain our final result.

5th. *After canceling all we can, we multiply the remaining numbers together.* We do this, because, after all the common factors are canceled, the least common multiple must be the *product* of the remaining numbers.

Some may think that we can reject factors indiscriminately and obtain the same result. The following example will show their error:

11. What is the least common multiple of 3, 9, 12, 16, 18, and 24?

First Operation.	*Second Operation.*	*Third Operation.*
$\cancel{3}$-$\cancel{9}$-$\cancel{12}$-$\cancel{16}$-$\cancel{18}$-24	$\cancel{16}$-$\cancel{9}$-$\cancel{18}$-$\cancel{3}$-24-$\cancel{12}$	$\cancel{3}$-$\cancel{9}$-$\cancel{12}$-$\cancel{16}$-$\cancel{18}$-24
2- 3	$\cancel{8}$ 3	$\cancel{4}$-6
		2
$24\times2\times3=144$	$24\times3=72$ an incorrect result.	$24\times6\times2=288.$ an incorrect result

In the second operation we canceled 2 into the 16, (it being common to 16 and 18;) and 6 into the 18, (it being common to 18 and 24;) and 8 into 24. Then $24 \times 3 = 72$, a result too small.

In the third operation we did not cancel the *greatest* common factor each time, and the result was too large.

To show that the first result is correct, we will resolve 144 into its several factors, and see if all the different factors of the given numbers occur in it.

The factors of 144 are $3 \times 3 \times 2 \times 2 \times 2 \times 2$.

The factors of the given numbers are $3\times \overbrace{3\times3} \times \overbrace{2\times2\times3} \times \overbrace{2\times2\times2\times2} \times \overbrace{2\times3\times3} \times \overbrace{2\times2\times2\times3}$.

Give an example illustrating this, and show the consequence upon the result by departing from this rule? What is the fourth consideration? Why do we do this? Why will it not do to reject factors indiscriminately? (Explain a few examples on the black-board illustrating this point.) How do we show that the result of the first operation, Ex. 11, was correct?

Canceling superfluous factors —— $\overbrace{\cancel{3}\times\cancel{3}}\times\cancel{3}\times\overbrace{\cancel{2}\times\cancel{2}\times\cancel{3}}\times$
$\overbrace{2\times2\times2\times2}\times\overbrace{\cancel{2}\times3\times3}\times\overbrace{\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{3}}$

We have $2\times2\times2\times2\times3\times3=144$. Therefore, it is correct.

The braces include the factors of each number.

Hence—To find the least common multiple of two or more numbers, we have the following

RULE.—I. *Set the numbers down in a line, placing the greatest at the right.* (Rem. 3.)

II. *When the numbers at the left have factors common to the right hand number, cancel the factors at the left only; so proceed till all the common factors at the left are canceled.*

III. *Multiply together the remaining factors, and their product will be the least common multiple.* (Obs. 6, 5th.)

NOTE.—The learner will perceive that when the second and third numbers from the right cancel in the first part of the operation, we take their *factors below*, and proceed as with the numbers themselves. No numbers are used after being canceled, as all their factors are found in the other numbers.

EXERCISES FOR THE SLATE.

What is the least common multiple of the following numbers:

1. 6, 8,	Ans. 24.
2. 7, 12,	Ans. 84.
3. 9, 12, 18,	Ans. 36.
4. 8, 15, 24,	Ans. 120.
5. 6, 9, 15, 20,	Ans. 180.
6. 5, 7, 10, 25,	Ans. 350.
7. 10, 12, 16, 18, 24,	Ans. 720.
8. 10, 16, 20, 24, 32,	Ans. 480.
9. 6, 8, 10, 16, 18, 20,	Ans. 720.
10. 3, 5, 2, 9, 12, 15, 18,	Ans. 180.
11. 3, 4, 8, 12, 16, 20, 24, 28,	Ans. 1680.
12. 4, 2, 6, 8, 9, 12, 14, 15, 16,	Ans. 5040.
13. 5, 2, 36, 10, 12, 18, 20, 24,	Ans. 360.
14. 6, 4, 8, 12, 16, 24, 32,	Ans. 96.
15. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,	Ans. 2520.
16. 8, 12, 14, 18, 24, 32, 36, 40,	Ans. 10080.
17. 9, 12, 16, 32, 36, 48, 96,	Ans. 288.
18. 10, 3, 8, 12, 32, 16, 27, 18,	Ans. 4320.
19. 7, 11, 22, 49, 55, 70, 84,	Ans. 32340.
20. 12, 8, 14, 18, 24, 15, 36, 28,	Ans. 2520.

What is the first step in finding the least common multiple of numbers?—The second? Third? When the second and third numbers from the right cancel how do we proceed? Do we use a number after it is canceled? Why not?

SECTION VIII.

FRACTIONS.

Article 1. Mental Exercises, Definitions, &c.

Obs. 1. A Fraction *is a part of any thing.*

Thus if I speak of an apple, I evidently mean a whole apple; but if I cut the apple into *two* equal parts, each part is called *one-half;* if I cut it into *three* equal parts, each part is called *one-third;* if I cut it into *four* equal parts, each part is called *one-fourth,* and two of these last named parts are called *two-fourths;* three parts are called *three-fourths,* &c., and the expressions, one-half, one-third, one-fourth, two-fourths, three-fourths, &c., are called Fractions.

We can divide numbers into parts as well as apples; thus we can take one-half of 2, one-third of 3, one-fourth of 8, or any other part of any number.

To take one-half of any number, we divide it by 2; to take one-third of any number, we divide it by 3; to take one-fourth of any number, we divide it by 4, &c.

Ex. 1. What is one-half of the following numbers? 4, 6, 8, 10, 12, 14, 18, 16, 24, 22, 20, 36.

2. What is one-third of the following numbers? 6, 9, 12, 15, 18, 24, 21, 36, 33, 30, 27.

3. What is one-fourth of the following numbers? 4, 8, 12, 16, 24, 20, 36, 32, 28, 40, 44.

4. What is one-fifth of the following numbers? 5, 10, 15, 25, 20, 45, 30, 40, 35, 60, 50, 45, 55.

5. What is one-sixth of the following numbers? 12, 18, 48, 24, 36, 30, 42, 60, 66.

6. What is one-seventh of the following numbers? 14, 35, 42, 21, 28, 49, 56, 63.

Remark.—In each of these examples, the learner will perceive that he divides a number into *equal parts,* the same as in division. (Sect. V, Art 2, Obs. 1.) Hence—*Fractions partake of the nature of Division.*

If a unit is divided into *two* parts, each part is called *one-half;* if into *three* parts, each part is called *one-third,* &c. Hence—

What is a Fraction? If I speak of an apple, what do I evidently mean? If I cut the apple into two equal parts, what is one part called? If I cut it into three equal parts, what is one part called? If I cut it into four equal parts, what is one part called? What are two parts of the latter division called? Three parts? What are the expressions one-half, one-third, one-fourth, &c., called? How do we take one-half of a number? One-third of a number? One-fourth of a number? In each of the given examples what does the learner do? To which of the Fundamental Rules are fractions similar? If a unit is divided into two parts, what is each part called? If it is divided into three parts, what is each part called? From what then does the fraction take its name?

Obs. 2. *The fraction takes its name from the number of parts into which the unit is divided.* Thus, if a unit is divided into two parts, each part is called *one-half*; if into three parts, each part is called *one-third*, &c. Therefore, in a unit, or whole number, there are two halves, three thirds, four fourths, nine ninths, twenty twentieths, &c.

7. How many fifths in a unit or whole number? How many sixths? Tenths? Fifteenths? Twenty-ninths? Fifty-fifths.

Remark. — Every fraction must have some *value*, and this value also *depends upon the number of parts into which the unit is divided.* Thus if a unit is divided into 3 parts, or thirds, the parts will evidently be less than if divided into 2 parts, or halves; and if divided into 4 parts, will be less than if divided into only 3. Hence:

Obs. 3. *The greater the number of parts into which a unit is divided the less will be the value of each part.*

8. Which is of the greatest value, one-half or one-third? one-fourth or one-fifth? 2 fourths or 2 fifths? 3 fifths or 3 sixths?

9. Which is of the greatest value, one-sixth or one-seventh? one-tenth or one-twelfth? 3 eighths or 3 ninths? 2 sevenths or 2 eighths?

MENTAL EXERCISES.

1. What is one-half of the following numbers? 10, 18, 26, 40, 84, 100, 150.

2. What part of 2 is 1? Ans. one-half.

3. What part of 3 is 1? of 4? 8? 5? 7? 9? 6? 10? 12? 20? 25?

4. What part of 3 is 2? Ans. two-thirds.

Solution.—One is one-third of 3, and 2 is twice 1; therefore 2 is twice one-third of 3, and twice one-third of 3 is two-thirds of 3.

5. What part of 5 is 2? of 6? 7? 8? 12? 9? 10? 14? 18? 25?

6. What part of 12 is 3? is 4? 7? 8? 6? 5? 9? 11? 10? 15? 12?

7. If half a bushel of corn cost 2 shillings what would 1 bushel cost? Ans. 4 shillings.

Solution.—There are two halves in a whole number; therefore, a bushel would cost twice as much as half a bushel, and twice 2 are 4.

8. If one-third of a pound of raisins costs 8 cents, what cost two-thirds of a pound? What cost a whole pound?

9. If one-fourth of a pound of spice cost 4 cents; what cost two-fourths of a pound? 3 fourths? A whole pound?

10. If 1 fifth of a yard of cloth cost 5 shillings what cost 2 fifths of a yard? 3 fifths? 4 fifths? a whole yard?

What must every fraction have? Upon what does this value depend? If one apple were divided into two parts, or halves, and another apple were divided into three parts or thirds, in which would the parts be the least? If one were divided into three parts, and the other into four parts, in which would the value of the parts be the least? What do we conclude from this?

11. If 1 sixth of a barrel of cider cost 3 shillings, what cost 2 sixths? 3 sixths? 4 sixths? 5 sixths? a whole barrel.

12. If 1 eighth of a bolt of cloth cost one dollar, what cost 2 eighths? 4 eighths? 6 eighths? 3 eighths? 7 eighths? 5 eighths? a whole bolt? 2 bolts? 3 bolts? 4 bolts? 6 bolts? 9 bolts?

13. If a pound of coffee cost 12 cents, what would a half a pound cost?

Solution.—A half a pound would evidently cost half as much as a pound, and one-half of 12 cents is 6 cents.

14. If 1 yard of cloth cost 24 cents, what will a third of a yard cost? 2 thirds? 3 yards?

15. If 1 acre of land cost 20 dollars, what cost 1 fourth of an acre? 3 fourths? 2 fourths? 5 acres?

16. What cost 1 sixth of a ton of hay at 12 dollars a ton? 3 sixths? 5 sixths? 2 sixths? 4 sixths?

17. What costs 3 sevenths of an acre of land at 14 dollars an acre.

Direction.—First find what 1 seventh of an acre would cost and then multiply this by 3.

18. What cost 5 ninths of a pound of tea at 72 cents a pound? 3 ninths? 7 ninths? 1 ninth? 6 ninths? 2 ninths? 5 ninths? 8 ninths?

19. If 1 book cost half a dollar, what will 2 books cost?

Solution.—2 books would cost twice as much as one book; twice 1 half are 2 halves, and two halves are a unit or whole number.

Ans. 1 dollar.

20. If 1 man eat half a pie, how many pies will 2 men eat? 3 men? 4 men? 7 men? 12 men? 20 men?

21. If a boy plants 1 third of an acre of corn in a day, how much will he plant in 2 days? 3 days? 4 days? 5 days? 6 days? 7 days?

22. If a horse eats 1 fourth of a bushel of oats in a day how much will he eat in 3 days? 2 days? 4 days? 7 days? 5 days? 8 days? 6 days.

23. What cost 8 marbles at 1 eighth of a cent a piece? What cost 10? 12? 16? 20? 24? 32? 40? 56? 72? 88? 96?

24. If 6 apples were divided equally between 3 boys, what part of them would each boy receive? How many apples would each receive?

Solution.—1 is one-third of 3; therefore, each boy receives one-third of the apples. One-third of 6 is 2, hence each boy receives 2 apples.

25. Four men are to receive 40 dollars, each receiving an equal share, what part of the money must 1 man receive? 2 men? 3 men? How many dollars must 1 man receive? 2 men? 3 men?

26. If 6 bushels of wheat cost 12 dollars, what part of that does 1 bushel cost? 5 bushels? 3 bushels? 2 bushels? 4 bushels? How many dollars does 1 bushel cost? 2 bushels,? 5 bushels 3 bushels? 4 bushels?

27. 2 is one-third of what number?

Solution.—There are 3 thirds in a whole number, hence, if 2 is one-third, the number must be 3 times 2, or 6. Ans. 6.

28. 3 is 1 third of what number? 1 fourth? 1 fifth? 1 sixth? 1 seventh?

29. 4 is 1 third of what number? 1 sixth? 1 tenth? 1 eighth? 1 twelfth?

30. 5 is 1 fifth of what number? 1 seventh? 1 tenth? 1 eleventh?

31. 4 is 2 thirds of what number?

Solution.—If 4 is 2 thirds, one-half of 4, or 2, is 1 third of the number. Then $2 \times 3 = 6$. Ans. 6.

32. Paid 3 dollars for 3 fourths of a yard of cloth; what was that a yard? 3 is 3 fourths of what number? Ans. 4.

33. If 7 eighths of a bushel of wheat cost 84 cents, what is that a bushel? 84 is 7 eighths of what number?

34. If 5 sixths of an acre of land costs 30 dollars, how much is that per acre? 30 is 5 sixths of what number?

35. 9 tenths of a dollar is 90 cents; how many cents are there in a dollar? 90 is 9 tenths of what number?

36. 7 ninths of a hogshead (wine measure) is 49 gallons; how many gallons in a hogshead? 49 is 7 ninths of what number?

37. 6 is 2 thirds of what number?

38. 9 is 3 fourths of what number?

39. 10 is 2 fifths of what number?

40. 28 is 7 elevenths of what number?

41. What is 3 fourths of 12?

Solution.—1 fourth of 12 is 3; then 3 fourths is $3 \times 3 = 9$. Ans. 9.

42. What is 7 eighths of 24?

43. What is 4 sixths of 18?

44. What is 4 tenths of 40?

45. What is 9 twelfths of 24?

46. What is 8 ninths of 27?

47. What is 8 sixths of 18? Ans. 24.

48. What is 12 sevenths of 35?

49. What is 15 twelfths of 24?

50. What is 16 elevenths of 33?

Obs. 4. When a number is divided into *equal parts*, as in the preceding examples, these parts are called FRACTIONS.

When a number is divided into equal parts what are these parts called?

Fractions are of three kinds; COMMON, DECIMAL, and DUODECIMAL.

1. COMMON FRACTIONS.

Obs. 5. COMMON FRACTIONS *are expressed by two numbers, one written above the other, with a line between them.* Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{16}{9}$, &c.

Obs. 6. *The number below the line is called the* DENOMINATOR, *because it shows into how many parts a unit or thing is divided.*

Obs. 7. *The number above the line is called the* NUMERATOR, *because it shows the number of parts taken, or expressed by the fraction.*

Thus, in the fraction $\frac{3}{4}$, the denominator 4 shows that the unit is divided into 4 parts, and the numerator shows that 3 of these parts are expressed by the fraction.

Obs. 8. *The numerator and denominator taken together are called the* TERMS OF THE FRACTION.

REMARK. 1. The number below the line is called the denominator, because *it gives a name to the parts.*

2. The number above the line is called the numerator, because *it numbers the parts used.*

3. The word *Fraction* is derived from the Latin, and signifies broken; hence, fractions are often called BROKEN NUMBERS. Also, a whole number is often called an INTEGER.

NOTE.—The learner will perceive that the fraction is the *part,* and not the *number itself* when broken. Thus if I break an apple, it is evidently a broken apple, but only *the piece broken off* is the *fraction* of the apple.

Obs. 9. It will be perceived that Fractions arise from *Division,* and they may be regarded as expressions of *unexecuted division;* the numerator answering to the *dividend,* and the denominator to the *divisor;* the value of the fraction being the *quotient* of the numerator divided by denominator.

Thus, in the fraction $\frac{4}{2}$, 4 is the dividend, 2 the divisor, and $4 \div 2 = 2$, the quotient; and in the fraction $\frac{1}{3}$, 1 is the dividend, 3 the divisor, and $1 \div 3$, or $\frac{1}{3}$ of 1 the quotient. (Sect. V. Art. 2, Obs. 4, *a.*)

Of how many kinds are fractions? What are they? How are common fractions expressed? What is the number below the line called? Why? What is the number above the line called? Why? In the fraction $\frac{3}{4}$ which is the denominator? Which is the numerator? What are the numerator and denominator taken together called? Give another reason why the number below the line is called the denominator? Give another reason why the number above the line is called the numerator? From what is the term fraction derived? What does it signify? What then are fractions often called? What is a whole number likewise often called? From what do fractions arise? What may they be regarded? To what do the numerator and denominator answer in division? What is the value of the fraction? In the fraction $\frac{4}{2}$ which is the divisor, which the dividend, and what the quotient? In the fraction $\frac{1}{3}$ which is the divisor, which the dividend and what the quotient?

Obs. 10. Common Fractions are either *Proper, Improper, Simple, Compound,* or *Complex.*

Obs. 11. A Proper Fraction *is one in which the numerator is less than the denominator;* as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{6}$, $\frac{3}{7}$, &c.

Obs. 12. An Improper Fraction *is one in which the numerator is equal to, or greater than the denominator;* as $\frac{3}{3}$, $\frac{5}{4}$, $\frac{15}{7}$, &c.

Remark.—When a whole number and a fraction are expressed together, it is called a mixed number; as $2\frac{1}{3}$, $3\frac{1}{2}$, $5\frac{2}{4}$, $8\frac{9}{13}$, &c.

Obs. 13. A Simple Fraction *is a single expression;* as $\frac{1}{2}$, $\frac{4}{3}$, &c. *It may be either proper or improper.*

Obs. 14. A Compound Fraction *is a fraction of a fraction; it consists of several fractions connected together by the word* of; as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{5}{12}$, &c.

Obs. 15. A Complex Fraction *is one which has a fraction in its numerator, or denominator, or in both;* as $\dfrac{4}{5\frac{1}{6}}$, $\dfrac{2\frac{1}{2}}{3}$, $\dfrac{3\frac{2}{3}}{6\frac{1}{4}}$, &c.

Note.—Besides these we have continued fractions, but as they are not used in common business, they are not treated of in this work.

From the preceding remarks and definitions, the demonstration of the following rules is evident:

Obs. 16. To read fractions—*First read the number of parts used as shown by the numerator, and then the size of the parts as shown by the denominator.*

1. Read the following expressions:—$\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$, $\frac{5}{6}$; $\frac{2}{6}$; $1\frac{2}{7}$; $\frac{4}{7}$, $\frac{8}{9}$; $4\frac{7}{8}$; $\frac{6}{7}$; $\frac{6}{10}$; $\frac{5}{9}$; $\frac{3}{10}$; $3\frac{7}{9}$; $\frac{7}{11}$; $\frac{17}{12}$, $\frac{11}{13}$; $\frac{25}{16}$; $\frac{19}{20}$; $\frac{8}{14}$; $\frac{21}{17}$; $\frac{22}{31}$; $4\frac{36}{43}$; $\frac{59}{84}$ of $\frac{87}{100}$ of $\frac{146}{2040}$; $16\frac{4892}{5643}$; $40\frac{8413}{10267}$; $99\frac{4551}{10001}$.

2. Read the following expressions: $\dfrac{2}{1\frac{1}{2}}$; $\dfrac{1\frac{1}{4}}{3}$; $\dfrac{3\frac{2}{3}}{4\frac{1}{4}}$; $\dfrac{4\frac{1}{5}}{5\frac{7}{8}}$; $\dfrac{7\frac{8}{9}}{10\frac{11}{12}}$.

3. A teacher wishing to divide some fruit among his scholars, separated it into a certain number of equal parts, and gave some a greater, and some a less number of these parts, according to their deportment in school. What John received could be expressed by the fraction $\frac{3}{24}$. Now can you tell me into how many parts the teacher divided the fruit, and how many of these parts John obtained?

4. What Charles received could be expressed by the fraction $\frac{5}{24}$; how many parts did he get?

How are common fractions divided? What is a proper fraction? Give examples. What is an improper fraction? Give examples. What is a mixed number? Give examples. What is a simple fraction? Give examples. May it be proper or improper? What is a compound fraction? Give examples. What is a complex fraction? Give examples. How do we read fractions?

5. What Henry received could be expressed by the fraction $\frac{2}{24}$; how many parts did he get?

Obs. 17. To write fractions—*First write the number of parts used, as the numerator, and then the size of the parts as the denominator.*

6. Write two-thirds. One-fifth. Two-fifths. Three-fifths.—Four-fifths. Two-sixths. Four-sixths. One-sixth. Three-sixths. Five-sixths. Three-sevenths. Five-eighths. Six-ninths. Three-tenths. Eight-tenths. Four-elevenths. Seven-elevenths. Nine-twelfths.

7. Write eleven-twelfths. Seven-twelfths. Eleven-sixteenths. Eight-twentieths. Sixteen-twentieths. Nineteen-twenty-fourths.—Forty-eight-seventieths. Fifty units, and thirty-six-eighty-ninths. One hundred units, and four hundred-one-thousandths. Eighty units, and three-ten-thousandths.

Art. 2. Fundamental Propositions.

In the following remarks and observations, the learner will remember that the numerator answers to the *dividend,* the denominator to the *divisor,* and the value of the fraction to the quotient of the numerator divided by the denominator. (Art. 1, Obs. 9.) Thus the value of $\frac{2}{2}$ is 1; of $\frac{6}{3}$ is 2; of $\frac{1}{4}$ is one-fourth of 1, &c. Therefore,

Obs. 1. *If the denominator remains the same, to multiply the numerator by any number multiplies the value of the fraction by that number.*

Take the fraction $\frac{4}{2} = 2$; multiplying the numerator by 2, w obtain $\frac{8}{2} = 4$, which is the same as 2×2.

Remark.—By multiplying the dividend, we multiply the quotient. (Sect. VI, Art. 1, Obs. 27. *a.*)

Obs. 2. *If the denominator remains the same, to divide the numerator by any number divides the value of the fraction by that number.*

Take $\frac{4}{2} = 2$; dividing the numerator by 2, we obtain $\frac{2}{2} = 1$, which is the same as $2 \div 2 = 1$.

How do we write fractions? If the denominator remains the same, what effect does it have upon the value of the fraction, to multiply the numerator by any number? Give an example. How do we know this proposition to be true? If the denominator remains the same what effect does it have on the value of a fraction to divide the numerator by any number? Give examples. Why is this proposition correct? What inference is deduced from this? Give examples.

REMARK.—By dividing the dividend, we divide the quotient. (Sect. VI, Art. 1, Obs. 27, *b*.) Therefore

Obs. 3. *With a given denominator, the greater the numerator, the greater the value of the fraction.* For $\frac{2}{2}$ is greater than $\frac{1}{2}$, $\frac{3}{4}$ than $\frac{1}{4}$, &c.

REMARK.—Because, the greater the dividend, the greater the quotient.—(Sect. VI, Art. 1, Obs. 26, *c*.)

Obs. 4. *If the numerator remains the same, to multiply the denominator by any number divides the value of the fraction by that number.*

Take $\frac{4}{2} = 2$; multiplying the denominator by 2, makes it $\frac{4}{4} = 1$, which is the same as $2 \div 2$.

REMARK.—By multiplying the divisor we divide the quotient. (Sect. VI, Art. 1, Obs. 27.)

Obs. 5. *If the numerator remains the same, to divide the denominator by any number multiplies the value of the fraction by that number.*

Take $\frac{4}{4} = 1$; dividing the denominator by by 2, we obtain $\frac{4}{2} = 2$, which is the same as 1×2.

REMARK. By dividing the divisor we multiply the quotient. (Sect. VI, Art. 1, Obs. 27, *a*.) Therefore

Obs. 6. *With a given numerator the greater the denominator, the less will be the value of the fraction.* For $\frac{1}{4}$ is less than $\frac{1}{2}$, $\frac{1}{5}$ is less than $\frac{1}{3}$, &c.

REMARK.—Because, the greater the divisor, the less the quotient. (Sect. VI, Art. 1, Obs. 26.)

Obs. 7. From these observations we notice,

a. 1st. *It has the same effect upon the value of the fraction, to multiply the numerator by any number, or to divide the denominator by the same number.*

Take $\frac{1}{2}$; multiplying the numerator by 2, we obtain $\frac{2}{2} = 1$; dividing the denominator by 2, we obtain $\frac{1}{1} = 1$, as before.

REMARK.—Because, to multiply the dividend, or to divide the divisor, has the same effect upon the quotient. (Sect. VI, Art. 1, Obs. 27, *a*.)

b. 2d. *It has the same effect upon the value of the fraction, to divide the numerator by any number, as to multiply the denominator by the same number.*

Why is this correct? If the numerator remains the same, what effect does it have upon the value of the fraction, to multiply the denominator by any number? Give an example. How do we know this to be correct? If the numerator remains the same, what effect does it have upon the value of the fraction, to divide the denominator by any number? Give an example. How do we know this to be correct? What inference is deduced from this? Give examples. Why is this correct? What is the first consideration we notice from these propositions? Give an example. Why is this correct? What is the second consideration we notice? Give an example?

Take $\frac{6}{3}$; dividing the numerator by 2, we obtain $\frac{3}{3} = 1$: multiplying the denominator by 2, we obtain $\frac{6}{6} = 1$; as before.

REMARK.—Because, to divide the dividend, or to multiply the divisor, has the same effect upon the quotient. (Sect. VI, Art. 1, Obs. 27, *b*.)

Obs. 8. *Multiplying or dividing both the numerator and denominator by any number, changes the form of the fraction without altering its value.*

Take $\frac{4}{4} = 1$; multiplying both terms of the fraction by 2, we obtain $\frac{8}{8} = 1$; dividing both terms of the fraction by 2, we obtain $\frac{2}{2} = 1$.

REMARK.—Because to multiply both the divisor and dividend by the same number does not alter the quotient. (Sect. VI, Art. 1, Obs. 28.)

Obs. 9. *If the same number be added to both terms of a proper fraction, the resulting fraction will be greater than the former fraction.*

Take $\frac{2}{3}$; adding 3 to both terms of the fraction, we obtain $\frac{5}{6}$, which is greater than $\frac{2}{3}$.

a. If the same number be subtracted from both terms of a proper fraction, the resulting fraction will be less than the former fraction.

Take $\frac{4}{5}$, subtracting 3 from both terms of the fraction, we obtain $\frac{1}{2}$, which is less than $\frac{4}{5}$.

REMARK.—These two propositions are the converse of the propositions given in Sect. VI, Art. 1, Obs. 29. When the terms of a fraction are *equal*, however, it does not affect the value of the fraction to add or subtract the same number from both terms of it, because *if equals are added to, or subtracted from, equals, their sums, or differences, will be equal.* (Sect. VI, Art. 1, Obs. 29, Rem.)

Obs. 10. *If the numerator is less than the denominator the value of the fraction is less than* 1. Thus $\frac{1}{2}$ = one-half of 1, $\frac{3}{4}$ = three-fourths of 1, &c.

REMARK.—Because when the numerator is less than the denominator, the dividend is less than the divisor.

a. If the numerator and denominator are equal, the value of the fraction is 1. Thus, $\frac{2}{2} = 1$; $\frac{3}{3} = 1$; $\frac{5}{5} = 1$, &c.

REMARK.—Because when both terms of the fraction are equal, the dividend and divisor are equal.

Why is this correct? What effect does it have upon the value of the fraction to multiply or divide both the numerator and denomiator by the same number? Give an example. Why is this correct? If the same number is added to both terms of a proper fraction, what is the value of the resulting fraction compared with that of the former fraction? Give an example? If the same number be subtracted from both terms of a proper fraction, what is the value of the resulting fraction, compared with that of the former fraction? Give an example. Of what are these two propositions the converse? When the terms of the fraction are equal, what effect does it have upon its value, to add or subtract the same number from both of them? Why? If the numerator is less than the denominator, what is the value of the fraction? Why? If the numerator is greater than the denominator, what is the value of the fraction? Why? If the numerator and denominator are equal, what is the value of the fraction?

b. If the numerator is greater than the denominator, the value of the fraction is greater than 1. Thus $\frac{3}{2} = 1\frac{1}{2}$; $\frac{8}{4} = 2$; $\frac{15}{5} = 3$; &c.

REMARK.—Because, when the numerator is greater than the denominator the dividend is greater than the divisor.

Obs. 11. Fractions may be *added, subtracted, multiplied,* or *divided,* as well as whole numbers, but to perform these operations it is necessary to make changes in the form of the fraction.

Art. 3. REDUCTION OF FRACTIONS.

Obs. 1. *The process of changing the terms of a fraction without altering its value, is called* REDUCTION OF FRACTIONS.

CASE 1. *To reduce a fraction to its lowest terms.*

Obs. 2. *A fraction is reduced to its lowest terms, when the numerator and denominator are prime to each other.* (Sect. VII, Art. 2, Note 3, under the rule.)

Ex. 1. Reduce $\frac{12}{16}$ to its lowest terms.

Operation.

$4)\frac{12}{16} = \frac{3}{4}$ Ans.

To divide both terms of a fraction by the same number, does not alter its value. (Art. 2, Obs. 8.)

Therefore, dividing both 12 and 16 by 4, we obtain $\frac{3}{4}$ as our answer.

2, Reduce $\frac{192}{240}$ to its lowest terms,

First Method. *Second Method.*

$6)\frac{192}{240} = 4)\frac{32}{40} = 2)\frac{8}{10} = \frac{4}{5}$ Ans. Or, $48)\frac{192}{240} = \frac{4}{5}$ Ans.

By the first method we divide both 192 and 240 by 6, which gives $\frac{32}{40}$; next we divide both 32 and 40 by 4, which gives $\frac{8}{10}$; and dividing both these by 2, we obtain $\frac{4}{5}$ as our answer,

By the second method we divide both 192 and 240 by their greatest common divisor, (48,) found according to the rule in Sect. VII, Art. 2; this gives $\frac{4}{5}$ for our answer as before.

Hence—To reduce a fraction to its lowest terms:

Obs. 3. *Divide both terms of the fraction by any number that will divide both of them without a remainder; so continue to do, till no number greater than* 1 *will divide both numerator and denominator without a remainder.* Or,

a. Divide both terms of the fraction by their greatest common divisor.

NOTE 1. When the terms of the fraction are *small,* it is generally best to work according to the first method; but when the terms are *large,* the latter method is the most convenient.

Why? Can fractions be added, subtracted, multiplied and divided? What is it necessary to do in order to perform these operations? What is Reduction of Fractions? When is a fraction reduced to its lowest terms? When are numbers prime to each other? What is the first rule for reducing fractions to their lowest terms? The second rule? When is it best to use the first rule?

2. The learner will bear in mind that the value expressed by the fraction does not depend upon the *magnitude* of its terms, but upon the *relation* they bear to each other. Thus, $\frac{1000}{10000}$, is much less than $\frac{1}{2}$, although its terms are larger.

3. The value of a fraction is not altered by reducing it to its lowest terms, because the relation of the terms to each other remains the same. (Art. 2, Obs. 8.)

Reduce the following expressions to their lowest terms :

3. $\frac{14}{21}$.	Ans. $\frac{2}{3}$.	11. $\frac{288}{648}$.	Ans. $\frac{4}{9}$.
4. $\frac{35}{50}$.	Ans. $\frac{7}{10}$.	12. $\frac{864}{1152}$.	Ans. $\frac{3}{4}$.
5. $\frac{75}{150}$.	Ans. $\frac{1}{2}$.	13. $\frac{1140}{1728}$.	Ans. $\frac{95}{144}$.
6. $\frac{144}{256}$.	Ans. $\frac{9}{16}$.	14. $\frac{4536}{5832}$.	Ans. $\frac{7}{9}$.
7. $\frac{49}{63}$	Ans. $\frac{7}{9}$.	15. $\frac{4608}{5760}$.	Ans. $\frac{4}{5}$.
8. $\frac{27}{81}$.	Ans. $\frac{1}{3}$.	16. $\frac{1397}{1651}$.	Ans. $\frac{11}{13}$.
9. $\frac{44}{77}$.	Ans. $\frac{4}{7}$.	17. $\frac{1296}{1440}$.	Ans. $\frac{9}{10}$.
10. $\frac{225}{1000}$.	Ans. $\frac{9}{40}$.	18. $\frac{4369}{7469}$.	Ans. $\frac{17}{29}$.

CASE 2. *To reduce an improper fraction to a whole or mixed number.*

Obs. 4. A MIXED NUMBER *is the value of an improper fraction, expressed by integers.*

Ex. 1. Reduce $\frac{6}{3}$ to a whole number. Ans. 2.

The value of a fraction is the quotient of the numerator, divided by the denominator. (Art. 1. Obs. 9.) Therefore, $6 \div 3 = 2$. Ans.

2. Express the value of $\frac{21}{6}$ by integers. Ans. $3\frac{1}{2}$.

Operation.

$$6)21$$
$$3\tfrac{3}{6} = 3\tfrac{1}{2}.$$

In this case there was a remainder after dividing by 6 ; this we write over our divisor, and reduce the fraction to its lowest terms, as directed in Case 1. Hence—

To reduce an improper fraction to a whole or mixed number :

Obs. 5. *Divide the numerator by the denominator, and if there is a remainder, write it over the divisor, and annex it to the quotient.*

NOTE.—It is generally best to reduce the fraction, (in the result or answer,) to its lowest terms.

Upon what does the value expressed by the fraction depend? Give examples. Is the value of the fraction altered by reducing it to the lowest terms? Why not? What is a mixed number? To what is the value of a fraction equal? What is the rule for reducing an improper fraction to a whole or mixed number?

Reduce the following expressions to whole or mixed numbers :

3. $\frac{24}{6}$.	Ans. 4.	11. $\frac{1728}{57}$.	Ans. $30\frac{6}{19}$.
4. $\frac{31}{5}$.	Ans. $6\frac{1}{5}$.	12. $\frac{2136}{94}$.	Ans. $22\frac{34}{47}$.
5. $\frac{42}{9}$.	Ans. $4\frac{2}{3}$.	13. $\frac{1449}{176}$.	Ans. $8\frac{41}{176}$.
6. $\frac{276}{12}$.	Ans. 23.	14. $\frac{4567}{891}$.	Ans. $5\frac{112}{891}$.
7. $\frac{149}{11}$.	Ans. $13\frac{6}{11}$.	15. $\frac{10236}{1234}$.	Ans. $8\frac{182}{617}$.
8. $\frac{369}{7}$.	Ans. $52\frac{5}{7}$.	16. $\frac{48321}{4567}$.	Ans. $10\frac{2651}{4567}$.
9. $\frac{492}{8}$.	Ans. $61\frac{1}{2}$.	17. $\frac{81723}{1579}$.	Ans. $51\frac{1194}{1579}$.
10. $\frac{576}{15}$.	Ans. $38\frac{2}{5}$.	18. $\frac{624156}{345}$.	Ans. $1809\frac{17}{115}$.

Case 3. *To change a whole or mixed number to an improper fraction.*

Obs. 6. *In changing a whole or mixed number to an improper fraction, its value must not be altered.*

a. An integer is reduced to an improper fraction by writing a unit under it.

Thus, $6 = \frac{6}{1}$; $10 = \frac{10}{1}$; $19 = \frac{19}{1}$, &c.

Ex. 1. Chance 6 to a fraction, the denominator of which is 3. Ans. $\frac{18}{3}$.

Solution.—$6 = \frac{6}{1}$; multiplying both numerator and denominator by 3, we obtain $\frac{18}{3}$. Ans. Or, there are 3 thirds in a whole number, or unit, and therefore in 6 whole numbers are $6 \times 3 = 18$ thirds, or, $\frac{18}{3}$. Ans. Hence—

To reduce a whole number to a fraction with any given denominator :

Obs. 7. *Multiply the whole number by the given denominator : the product will be the numerator of the required fraction, under which write the denominator.*

2. Change 12 to a fraction, whose denominator shall be 9. Ans. $\frac{108}{9}$.

3. Change 15 to a fraction whose denominator shall be 12. Ans. $\frac{180}{12}$.

4. Change 24 to a fraction whose denominator shall be 36. Ans. $\frac{864}{36}$.

5. Change 48 to a fraction whose denominator shall be 114. Ans. $\frac{5472}{114}$.

6. Change $\frac{3}{4}$ to an improper fraction.

Solution.—$3 \times 4 = 12$ fourths, and 3 fourths make 15 fourths, or $\frac{15}{4}$. Ans. Hence—

What must be observed in changing a whole or mixed number to an improper fraction? How can we change a whole number to an improper fraction? Give examples. How do we reduce a whole number to a fraction with any given denominator?

To change a mixed number to an improper fraction:

Obs. 8. *Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the result over the denominator.*

Remark.—This rule is directly the reverse of Obs. 5, and each proves the other.

Reduce the following expressions to improper fractions:

7. $6\frac{1}{4}$. Ans. $\frac{25}{4}$.
8. $12\frac{1}{7}$. Ans. $\frac{85}{7}$.
9. $18\frac{7}{9}$. Ans. $\frac{169}{9}$.
10. $23\frac{4}{5}$. Ans. $\frac{119}{5}$.
11. $31\frac{5}{6}$. Ans. $\frac{191}{6}$.
12. $45\frac{7}{8}$. Ans. $\frac{367}{8}$.
13. $78\frac{4}{11}$. Ans. $\frac{862}{11}$.
14. $99\frac{11}{12}$. Ans. $\frac{1199}{12}$.
15. $45\frac{8}{13}$. Ans. $\frac{593}{13}$.
16. $85\frac{12}{17}$. Ans. $\frac{1457}{17}$.
17. $116\frac{28}{47}$. Ans. $\frac{5480}{47}$.
18. $240\frac{31}{50}$. Ans. $\frac{12031}{50}$.
19. $242\frac{123}{329}$. Ans. $\frac{79741}{329}$.
20. $356\frac{210}{446}$. Ans. $\frac{158986}{446}$.
21. $482\frac{365}{594}$. Ans. $\frac{286673}{594}$.
22. $1848\frac{9876}{12345}$. Ans. $\frac{22823436}{12345}$.

Case 4. *To change a compound fraction to a simple one:*

Obs. 9. *This process consists in finding a single expression equal in value to several expressions.*

Ex. 1. Change $\frac{2}{3}$ of $\frac{6}{7}$ to a simple fraction. Ans. $\frac{4}{7}$.

Solution.—$\frac{2}{3}$ of $\frac{6}{7}$ is twice as much as $\frac{1}{3}$ of $\frac{6}{7}$. $\frac{1}{3}$ of $\frac{6}{7}$ is $\frac{6}{21}$; (Art. 2. Obs. 4.) and $\frac{2}{3}$ being twice as much, is $\frac{12}{21}$; (Art. 2, Obs. 1.) $\frac{12}{21}=\frac{4}{7}$. Ans.

It will be perceived that the result is obtained by multiplying the numerators together, and the denominators. Thus, $\frac{2}{3}\times\frac{6}{7}=\frac{12}{21}=\frac{4}{7}$.

The same process is observed when the compound fraction consists of more than two terms. Hence—

To change a compound fraction to a simple one:

Obs. 10. *Multiply together all the numerators for a new numerator, and all the denominators for a new denominator.*

Compound fractions may be reduced to simple ones by cancelation, which is often much shorter, as it at once reduces the fraction to its lowest terms.

2. Change $\frac{4}{3}$ of $\frac{5}{6}$ of $\frac{12}{16}$ of $\frac{7}{8}$ of $\frac{6}{7}$ to a single fraction. Ans. $\frac{5}{8}$.

How do we change a mixed number to an improper fraction? Of what is this rule the reverse? What relation do they bear to each other? In what does the process of changing a compound fraction to a simple one consist? How is the result obtained in the solution of the first example?

Operation.

$$\begin{array}{r|l} \cancel{3} & \cancel{4} \\ \cancel{6} & 5 \\ \cancel{16} & \cancel{12} - 4 \\ 8 & \cancel{7} \\ \cancel{7} & \cancel{6} \\ & - \end{array}$$

Since the numerator of a fraction answers to the dividend, and the denominator to the divisor, (Art. 1. Obs. 9.,) we place the numerator at the right, and the denominator at the left. In all other respects, we proceed according to the General Rule, Sect. VIII., Art. 1. Hence—

Ans. $\frac{5}{8}$.

To work fractions by cancelation:

Obs. 11. *Place the numerators at the right, and the denominators at the left of the line; in all other respects proceed as usual in canceling. In the answer, the numerator is always at the right, aad the denominator at the left.*

Note.—When whole or mixed numbers occur, they must be reduced to improper fractions.

Reduce the following expressions to simple fractions:

3. $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{9}{10}$. Ans. $\frac{27}{40}$.
4. $\frac{8}{9}$ of $\frac{5}{6}$ of $\frac{10}{11}$. Ans. $\frac{200}{297}$.
5. $\frac{5}{7}$ of 6 times $\frac{4}{6}$ of $\frac{3}{8}$. Ans. $\frac{15}{14} = 1\frac{1}{14}$.
6. 9 times 15 times $\frac{14}{18}$ of $\frac{15}{16}$. Ans. $98\frac{7}{16}$.
7. $\frac{1}{5}$ of $\frac{2}{3}$ of $\frac{9}{14}$ of $\frac{3}{4}$. Ans. $\frac{9}{140}$.
8. $\frac{4}{9}$ of $\frac{7}{15}$ of 15 times 423. Ans. 1316.
9. $\frac{4}{7}$ of $\frac{12}{16}$ of $\frac{21}{86}$ of $\frac{32}{48}$ of $\frac{43}{64}$ of $24\frac{16}{129}$. Ans, $1\frac{45}{344}$.
10. $\frac{3}{10}$ of $\frac{5}{6}$ of $8\frac{2}{5}$ of $\frac{6}{17}$ of $\frac{2}{3}$ of $\frac{9}{12}$ of $6\frac{3}{4}$. Ans. $2\frac{341}{680}$.
11. $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{5}{14}$ of $\frac{1}{6}$ of $\frac{4}{5}$ of $\frac{11}{12}$ of $1\frac{1}{2}$. Ans. $\frac{11}{320}$.
12. $\frac{1}{17}$ of $5\frac{2}{3}$ of $21\frac{1}{6}$ of $\frac{44}{254}$ of $7\frac{1}{5}$ of $9\frac{1}{16}$ of $13\frac{1}{2}$. Ans. $1076\frac{5}{8}$.

Case 5.—*To change a complex fraction to a simple one.*

This process cannot be thoroughly understood before the learner has studied Division of Fractions. It is explained in Art. 6, Case 5.

Case 6.—*To reduce fractions to their least common denominator.*

Obs. 12. *Fractions have a common denominator, when their denominators are alike, and in reducing them their values must not be altered.*

Is the same process observed when the compound fraction consists of more than two terms? What then is the rule for reducing a compound fraction to a simple one? By what other method can the operation be performed? Why is this method preferable? How do we write the terms of the fraction by this method? Why are they thus placed? How do we otherwise proceed? What then is the rule? When whole or mixed numbers occur, what must be done with them? When have fractions a common denominator? What must be observed in finding their common denominator?

Ex. 1. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having a common denominator.

Solution.—If we multiply both terms of a fraction by the same number, its value will not be altered, (Art. 2. Obs. 8.) Therefore if we multiply both terms of the first fraction ($\frac{1}{2}$,) by the denominator of the second fraction, ($\frac{1}{3}$,) and both terms of the second fraction, ($\frac{1}{3}$,) by the denominator of the first fraction, ($\frac{1}{2}$,) we shall obtain $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$. Ans.

2. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having their least common denominator.

It is evident that each denominator must divide the common denominator; therefore, *the least common denominator must be the least common multiple of the denominators of the given fractions.*

Operation.

$$\cancel{2}\,..\,\cancel{3}\,..\,4\,..\,6$$
$$2$$
$$2\times 6=12.$$

We find the least common multiple of the denominators of the given fractions to be 12. Then

$$\left.\begin{array}{l}\frac{1}{2}\text{ of } 12=6,\\ \frac{1}{3}\text{ of } 12=4,\\ \frac{1}{4}\text{ of } 12=3,\\ \frac{1}{6}\text{ of } 12=2,\end{array}\right\}\quad\text{and the fractions are}\quad\left\{\begin{array}{l}\frac{1}{2}\times\frac{6}{6}=\frac{6}{12}.\\ \frac{2}{3}\times\frac{4}{4}=\frac{8}{12}.\\ \frac{3}{4}\times\frac{3}{3}=\frac{9}{12}.\\ \frac{5}{6}\times\frac{2}{2}=\frac{10}{12}.\end{array}\right.$$

Here we perceive that each result is found by multiplying both terms of each fraction by the quotient arising from dividing the least common denominator, by the denominator of this fraction.

The reason why we do this, is, because we wish to multiply both terms of each fraction by such a number as will produce an equivalent fraction, having the least common denominator. Now the denominator of each fraction must be one factor of this denominator, therefore the other factor must be the number by which both terms of the fraction are to be multiplied. Hence—

To reduce fractions to their least common denominator:

Obs. 13. *Find the least common multiple of the denominators of the given fractions, and multiply both terms of each fraction by the quotient arising from dividing the least common multiple by the denominator of this fraction.*

NOTE.—*Compound fractions* must be reduced to *simple ones* before finding their least common denominator.

The object of reducing fractions to a common denominator, *is to facilitate the addition and subtraction of fractions,* (Art. 4.); and as

---

Explain the solution of Ex. 1, and show why it is correct. What must the least common denominator of several fractions always be? Why so? How do we obtain each result in Ex. 2? Why do we proceed in this manner? How then do we reduce fractions to their least common denominator? What must be done with compound fractions when they occur?

it is generally considered most practical to find the least common denominator, we have given but one rule.*

3. Reduce $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{8}{10}$ to fractions having their least common denominator.

NOTE.—When the fractions are small, the pupil should be taught to reduce them *mentally*, and not be allowed the use of the slate or black-board at all. This will accustom him to thinking for himself.

In the last example we at once discover that 10 will contain 5, but will not contain 3, or any factor of 3; then 10×3, or 30, is the least common denominator of the fractions,

Now let the learner be questioned thus:

How much is $\frac{1}{5}$ of 30?

If 6 is $\frac{1}{5}$ of 30, how much is $\frac{4}{5}$?

Then if $\frac{4}{5}$ of 30 is 24, $\frac{4}{5} = \frac{24}{30}$.

Proceed in the same way with the other fractions. The answers are $\frac{2}{3} = \frac{20}{30}$; and $\frac{8}{10} = \frac{24}{30}$.

Reduce the following expressions to their least common denominators:

4. $\frac{1}{4}$, $\frac{3}{7}$, and $\frac{5}{12}$. Ans. $\frac{21}{84}$, $\frac{36}{84}$, and $\frac{35}{84}$.
5. $\frac{2}{7}$, $\frac{4}{9}$, and $\frac{1}{63}$. Ans. $\frac{18}{63}$, $\frac{28}{63}$, and $\frac{1}{63}$.
6. $\frac{2}{5}$, $\frac{1}{2}$, and $\frac{4}{7}$. Ans. $\frac{28}{70}$, $\frac{35}{70}$. and $\frac{40}{70}$.
7. $\frac{12}{25}$, $\frac{9}{40}$, $\frac{13}{60}$, $\frac{23}{90}$, and $\frac{37}{100}$.
Ans. $\frac{864}{1800}$, $\frac{405}{1800}$, $\frac{390}{1800}$, $\frac{460}{1800}$, $\frac{666}{1800}$.
8. $\frac{5}{8}$, $\frac{10}{12}$, $\frac{17}{24}$, and $\frac{4}{5}$ of $\frac{9}{16}$. Ans. $\frac{75}{120}$, $\frac{100}{120}$, $\frac{85}{120}$, and $\frac{54}{120}$.
9. $\frac{3}{4}$ of $\frac{4}{5}$, $\frac{6}{7}$ of $\frac{2}{3}$, $\frac{4}{9}$ of $\frac{1}{4}$, and $\frac{5}{6}$ of $\frac{7}{10}$ of $\frac{3}{8}$.
Ans. $\frac{6048}{10080}$, $\frac{5760}{10080}$, $\frac{1120}{10080}$, $\frac{2205}{10080}$.

For the convenience of the learner, we will now present at one view the following

GENERAL RULES FOR THE REDUCTION OF FRACTIONS.

To reduce a fraction to its lowest terms:

a. Divide both terms of the fraction by any number that will divide them without a remainder; so continue to do till no number greater than 1 *will divide them.* (Obs. 3.) Or,

b. Divide both terms of the fraction by their greatest common divisor. (Obs. 3, *a.*)

2d, To reduce an improper fraction to a whole or mixed number:

What is the object of reducing fractions to a common denominator? What is the advantage derived from solving questions mentally? How do we reduce a fraction to its lowest terms?

---

*Fractions may be reduced to a *common* denominator, by multiplying all their denominators together for a new denominator, and each numerator into all the denominators except its own for a new numerator; but this rule will not always give the *least* common denominator.

Divide the numerator by the denominator; write the remainder, (if any,) over the divisor, and annex it to the quotient. (Obs. 5.)

3d. To change a whole or mixed number to an improper fraction:

Multiply the whole number by the denominator of the fraction, to the product add the numerator, (if it is a mixed number,) and under the result write the denominator. (Obs. 7 and 8.)

4th. To change a compound fraction to a simple one:

a. Multiply together all the numerators for a new numererator, and all the denominators for a new denominator; then reduce the fraction to its lowest terms. (Obs. 10.)

Or, by cancelation:

b. Place the numerators at the right, and the denominators at the left of the line; then proceed as usual. (Obs. 11.)

5th. To reduce fractions to their least common denominator:

First, find the least common multiple of the denominators; then multiply both terms of each fraction by the quotient, arising from dividing this multiple by the denominator of the fraction. (Obs. 13.)

EXERCISES FOR THE SLATE.

1. Reduce $\frac{1152}{2592}$ to its lowest terms. Ans. $\frac{4}{9}$.
2. Reduce $\frac{4368}{6864}$ to its lowest terms. Ans. $\frac{7}{11}$.
3. Reduce $\frac{1555}{2177}$ to its lowest terms. Ans. $\frac{5}{7}$.
4. Change $\frac{1236}{48}$ to a whole or mixed number. Ans. $25\frac{3}{4}$.
5. Change $\frac{9876}{54}$ to a whole or mixed number. Ans. $182\frac{8}{9}$.
6. Change $144\frac{17}{23}$ to an improper fraction. Ans. $\frac{3329}{23}$.
7. Change $246\frac{23}{36}$ to an improper fraction. Ans. $\frac{8879}{36}$.
8. Reduce $\frac{4}{7}$ of $\frac{6}{11}$ of $\frac{15}{16}$ of $\frac{21}{32}$ of $\frac{12}{14}$ of $\frac{33}{44}$ to a simple fraction. Ans. $\frac{1215}{9856}$.
9. Reduce $\frac{4}{5}$ of $\frac{10}{11}$ of $\frac{22}{27}$ of $\frac{18}{44}$ of $\frac{9}{20}$ of $\frac{7}{8}$ of $\frac{9}{10}$ to a simple fraction. Ans. $\frac{189}{2200}$.
10. Reduce $\frac{5}{6}$, $\frac{7}{9}$, $\frac{11}{12}$, and $\frac{15}{16}$, to their least common denominator. Ans. $\frac{120}{144}$, $\frac{112}{144}$, $\frac{132}{144}$, and $\frac{135}{144}$.
11. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{12}$, $\frac{9}{16}$, and $\frac{12}{20}$ to their least common denominator. Ans. $\frac{180}{240}$, $\frac{210}{240}$, $\frac{100}{240}$, $\frac{135}{240}$, and $\frac{144}{240}$.
12. Reduce $\frac{6}{7}$ of $\frac{14}{18}$ of $\frac{9}{14}$ and $\frac{8}{11}$ of $\frac{33}{64}$ of $\frac{16}{27}$ to their least common denominator. Ans. $\frac{27}{63}$ and $\frac{14}{63}$.

How do we change an improper fraction to a whole or mixed number? How do we change a whole or mixed number to an improper fraction? How do we change a compound fraction to a simple one? How by cancelation? How do we reduce fractions to their least common denominator?

Article 4. Addition and Subtraction of Fractions.

Obs. 1. *Fractions are added or subtracted, by adding or subtracting their numerators.*

MENTAL EXERCISES.

1. James had $\frac{1}{6}$ of an apple, John $\frac{2}{6}$, and William $\frac{3}{6}$. How much had they all?

$\frac{1}{6}$, and $\frac{2}{6}$, and $\frac{3}{6}$ = how many sixths? Ans. $\frac{6}{6}=1$.

2. Susan had $\frac{3}{8}$ of an orange, Mary $\frac{4}{8}$, and Jane $\frac{1}{8}$. How much had they all?

3. A man gave one boy $\frac{2}{15}$ of a pie, another $\frac{4}{15}$, and another $\frac{6}{15}$. How much did he give them all?

4. One man planted $\frac{7}{21}$ of an acre of land, another $\frac{11}{21}$, and another $\frac{15}{21}$. How much did they all plant?

5. What is the sum of $\frac{4}{36}$, $\frac{5}{36}$, $\frac{14}{36}$, $\frac{9}{36}$, $\frac{19}{36}$, $\frac{12}{36}$, and $\frac{25}{36}$?

6. What is the sum of $\frac{2}{100}$, $\frac{6}{100}$, $\frac{19}{100}$, $\frac{40}{100}$, $\frac{75}{100}$, $\frac{125}{100}$, $\frac{36}{100}$, and $\frac{99}{100}$?

7. Matthew has $\frac{3}{4}$ of a pie, and John $\frac{1}{4}$. How much more has Matthew than John?

$\frac{1}{4}$ from $\frac{3}{4}$ leaves how many fourths? Ans. $\frac{2}{4}=\frac{1}{2}$.

8. One boy has $\frac{8}{9}$ of a dollar, and another $\frac{5}{9}$. How much has the one more than the other?

9. Henry has $\frac{5}{7}$ of a pound of raisins, and Thomas $\frac{2}{7}$. How much has Henry more than Thomas?

10. A. traveled $\frac{7}{8}$ of a mile, and B. traveled $\frac{5}{8}$. How much farther did A. travel than B.?

11. From $\frac{36}{47}$ take $\frac{29}{47}$.

12. From $\frac{75}{99}$ take $\frac{59}{99}$.

Method of adding and subtracting fractions having different denominators, mixed numbers, &c.

Ex. 1. John had $\frac{1}{2}$ of a dollar, and Frank $\frac{1}{3}$ of a dollar. How much had both?

Solution.—If we add $\frac{1}{2}$ to $\frac{1}{3}$, the result will neither be $\frac{2}{2}$ nor $\frac{2}{3}$. But $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$; and $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. Ans.

2. How much more had John than Frank in the last example?

Solution.—$\frac{1}{3}=\frac{2}{6}$; $\frac{1}{2}=\frac{3}{6}$; $\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$. Ans.

Remark.—We perceive in these examples, that the fractions have to be reduced to a *common denominator* before they can be either added or subtracted.

This principle is the same as that laid down in Sect. VI. Art. 1. Obs. 7 and 10. Hence—

Obs. 2. *No numbers, whether integral or fractional, can be added or subtracted, unless they are all of the same name or kind.*

How do we add or subtract fractions? Explain the solutions of Ex. 1 and 2.

3. Add together $\frac{2}{3}$ and $\frac{3}{4}$. Ans. $\frac{17}{12}=1\frac{5}{12}$.
4. Add together $\frac{4}{5}$ and $\frac{7}{8}$. Ans. $1\frac{27}{40}$.
5. Add together $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{3}{8}$. Ans. $2\frac{7}{120}$.
6. From $\frac{7}{8}$ take $\frac{3}{4}$. Ans. $\frac{1}{8}$.
7. From $\frac{11}{12}$ take $\frac{4}{5}$. Ans. $\frac{7}{60}$.
8. From $\frac{9}{11}$ take $\frac{13}{16}$. Ans. $\frac{1}{176}$.
9. Charles has $12\frac{7}{8}$ dollars, and Henry has $10\frac{3}{4}$ dollars. How many dollars have both?

1st *Operation.*

$12\frac{7}{8}=\frac{103}{8}$,
$10\frac{3}{4}=\frac{43}{4}=\frac{86}{8}$.
$\frac{103}{8}+\frac{86}{8}=\frac{189}{8}=23\frac{5}{8}$ dollars. Ans.

By the first operation we reduce the expressions to improper fractions, and then proceed as in Ex. 1. This method is preferable when the numbers ars small.

2d *Operation.*

$$\begin{array}{r} 12\frac{7}{8} \\ 10\frac{3}{4}=10\frac{6}{8} \\ \hline \text{Ans. } 23\frac{5}{8} \text{ dollars.} \end{array} \quad \frac{7}{8}+\frac{6}{8}=\frac{13}{8}=1\frac{5}{8}.$$

By the second operation, we add the fractions and integers separately. This method is to be preferred when the numbers are large.

10. In the last example, how much more had Charles than Henry?

1st *Operation.*

$12\frac{7}{8}=\frac{103}{8}$.
$10\frac{3}{4}=\frac{43}{4}=\frac{86}{8}$.
$\frac{103}{8}-\frac{86}{8}=\frac{17}{8},=2\frac{1}{8}$ dollars. Ans.

2nd *Operation.*

$$\begin{array}{r} 12\frac{7}{8}. \\ 10\frac{3}{4}=10\frac{6}{8}. \\ \hline \text{Ans. } 2\frac{1}{8} \text{ dollars.} \end{array}$$

Explanation—The same as above, except to read Ex. 2, instead of Ex. 1, and subtract instead of add.

11. Theren has $56\frac{1}{4}$ cents, and Clarence has $37\frac{1}{2}$ cents. How much has Theren more than Clarence?

Operation.

$$\begin{array}{r} 56\frac{1}{4} \\ 37\frac{1}{2}=37\frac{2}{4} \\ \hline \text{Ans. } 18\frac{3}{4} \text{ cents.} \end{array}$$

We cannot take $\frac{2}{4}$ from $\frac{1}{4}$; therefore we borrow 1 ($\frac{4}{4}$) from the 6, and add it to it the $\frac{1}{4}$, making $\frac{5}{4}$; then $\frac{5}{4}-\frac{2}{4}=\frac{3}{4}$. But as we borrowed 1 we must return it; therefore 1 to carry to 7 makes 8, and 8 from 16 leaves 8, &c.

12. From $78\frac{1}{4}$ take $39\frac{4}{5}$. Ans. $38\frac{9}{20}$.

What principle do we notice in these solutions? With what does this principle oincide? What inference is deduced from this? Explain the operations of Ex. 9 and 10? When is the first method to be preferred? The second method? Explain the operation of Ex. 11.

From the preceding remarks and illustrations, we derive the following

GENERAL RULE FOR THE ADDITION AND SUBTRACTION OF FRACTIONS.

Reduce the fractions to their least common denominator, (Rem.,) *and then add or subtract their numerators.* (Obs. 1.)

NOTE.—Compound fractions must be reduced to simple ones before they are added or subtracted.

To add or subtract mixed numbers:

Obs. 3. *Either reduce them to improper fractions, and proceed as above directed.* Or, *add or subtract the integral and fractional parts separately.*

When there are but two fractions to be added, and the numerator of each is 1, *we may add their denominators for a numerator, and multiply them for a denominator, and then reduce the fraction to its lowest terms.*

Thus: $\frac{1}{5}+\frac{1}{10}=\frac{5+10}{5\times 10}=\frac{15}{50}=\frac{3}{10}$.

If we wish to subtract fractions, when the numerator of each is 1, *we may subtract their denominators for a numerator, and multiply them for a denominator, and then reduce the fraction to its lowest terms.*

Thus: $\frac{1}{5}-\frac{1}{10}=\frac{10-5}{10\times 5}=\frac{5}{50}=\frac{1}{10}$.

This is in effect multiplying both terms of each fraction by the denominator of the other fraction, and then subtracting the numerators.

Thus: $(\frac{1}{5})\pm(\frac{1}{10})=(\frac{1}{5}\times\frac{10}{10})\pm(\frac{1}{10}\times\frac{5}{5})=(\frac{10}{50})\pm(\frac{5}{50})=\frac{15}{50}$ or $\frac{5}{50}=\frac{3}{10}$ or $\frac{1}{10}$.

EXERCISES FOR THE SLATE.

1. A man owning $\frac{3}{8}$ of a vessel, afterwards bought $\frac{4}{7}$ of the vessel more. How much did he then own? Ans. $\frac{53}{56}$ of the vessel.

2. William had $\frac{4}{5}$ of a dollar, and his father gave him $\frac{5}{8}$ of a dollar more. What part of a dollar had he then?
Ans. $\frac{57}{40}=1\frac{17}{40}$.

3. He afterwards spent $\frac{11}{16}$ of a dollar. How much had he left?
Ans. $\frac{59}{80}$ of a dollar.

What is the general rule for the addition and subtraction of fractions? What must be done with compound fractions when they occur? How do we add or subtract mixed numbers? When there are but two fractions to add, and the numerator of each is 1, how do we proceed? When we wish to subtract fractions, and the numerator of each is 1, how may we proceed? How, in reality, are these two last cases performed?

4. A man planted $4\frac{1}{6}$ acres to corn, $1\frac{4}{7}$ acres to potatoes, and $2\frac{3}{14}$ acres to beans. How many acres did he plant in all?

Ans. $7\frac{20}{21}$.

5. How much more did he plant to corn than to beans?

Ans. $1\frac{20}{21}$.

6. How much more to corn than to beans and potatoes both?

Ans. $\frac{8}{21}$ of an acre.

7. John has $\frac{2}{3}$ of $\frac{4}{5}$ of 3 dollars, Charles has $\frac{3}{4}$ of $\frac{5}{6}$ of 5 dollars, and Luke has $\frac{1}{2}$ of $\frac{8}{9}$ of 6 dollars. How much have they all?

Ans. $7\frac{47}{120}$ dollars.

8 How much more has Charles than John?

Ans. $1\frac{21}{40}$ dollars.

9 How much more has Luke than John?

Ans. $1\frac{1}{15}$ dollars.

10. A merchant has three pieces of cloth, one containing $24\frac{3}{8}$ yards, another $16\frac{3}{4}$ yards, and the other $14\frac{1}{2}$ yards. How many yards in all? Ans. $55\frac{5}{8}$.

11. How many yards in the first piece more than in the second?

Ans. $7\frac{5}{8}$.

12. How many yards in the first p'ece more than in the third?

Ans. $9\frac{7}{8}$.

13. How many yards in the second and third pieces together more than in the first? Ans. $6\frac{7}{8}$.

14. A. has traveled $28\frac{2}{3}$ miles, and B. $21\frac{5}{8}$ miles. How many miles have they both traveled? Ans. $50\frac{7}{24}$.

15. How much farther has A. traveled than B.?

Ans. $7\frac{1}{24}$ miles.

16. Thomas has $62\frac{1}{2}$ cents, and Henry has $43\frac{3}{4}$ cents. How many cents have both? Ans. $106\frac{1}{4}$.

17. How much more has Thomas than Henry?

Ans. $18\frac{3}{4}$ cents.

18. A. has $124\frac{7}{12}$ dollars, B. has $119\frac{5}{6}$ dollars, and C. has $214\frac{5}{8}$ dollars. How many dollars have they all? Ans. $459\frac{1}{24}$.

19. How much has A. more than B.? Ans. $4\frac{3}{4}$ dollars.

20. How much has C. more than A.? Ans. $90\frac{1}{24}$.

21. How much have A. and B. together, more than C.?

Ans. $29\frac{19}{24}$.

24. A little girl had $\frac{1}{5}$ of an orange, and her mother gave her $\frac{1}{10}$ of an orange more. How much had she then?

Ans. $\frac{3}{10}$ of an orange.

23. Add together $\frac{1}{15}$ and $\frac{1}{45}$. Ans. $\frac{4}{45}$.

24. Add together $\frac{1}{10}$ and $\frac{1}{470}$. Ans. $\frac{24}{235}$.

25. Add together $\frac{1}{40}$ and $\frac{1}{500}$. Ans. $\frac{27}{1000}$.

26. Add together $\frac{1}{39}$ and $\frac{1}{633}$. Ans. $\frac{224}{8229}$.

27. From $\frac{1}{11}$ take $\frac{1}{14}$. Ans. $\frac{3}{154}$.

28. From $\frac{1}{75}$ take $\frac{1}{300}$. Ans. $\frac{1}{100}$.

29. From $\frac{1}{66}$ take $\frac{1}{99}$. Ans. $\frac{1}{198}$.
30. From $\frac{1}{180}$ take $\frac{1}{270}$. Ans. $\frac{1}{540}$.

Article 5. Multiplication of Fractions.

Obs. 1. In multiplying by a *proper fraction*, the learner will always find that *the product is less than the multiplicand;* but this need not cause him any surprise, if he recollects that *the multiplier is less than unity, or* 1*; and therefore, he only repeats the multiplicand a part of* 1 *time.* (Sect. VI. Art. 1. Obs. 19. Rem.)

MENTAL EXERCISES.

1. If an apple costs $\frac{1}{3}$ of a cent, how much will 3 apples cost?

Solution.—3 apples will evidently cost 3 times as much as 1 apple; and 3 times $\frac{1}{3}$ equals $\frac{3}{3}$, or 1 cent. (Art. 2. Obs. 1.)

Ans. 1 cent.

2. If 1 marble costs $\frac{1}{6}$ of a cent, how much will 5 marbles cost? Ans. $\frac{5}{6}$ of a cent.

3. If 1 pear costs $\frac{3}{4}$ of a cent, how much will 8 pears cost? Ans. $\frac{24}{4} = 6$ cents.

3. At $\frac{1}{4}$ of a dollar per bushel, how much will 6 bushels of corn cost?

5. At $\frac{2}{3}$ of a dollar per bushel, how much would 9 bushels of apples cost?

6. At $\frac{2}{3}$ of a dollar apiece, how much would 9 books cost?

7. At 25 cents a pound, how much would $\frac{3}{5}$ of a pound of raisins cost?

Solution.—The learner will perceive that this question is just like the preceding, except that the multiplier and multiplicand have changed places. Now as it makes no difference which factor we use as the multiplier, (Sect. IV. Art. 2. Obs. 5. Rem.,) we may proceed as in Ex. 1, and say, 25 times $\frac{3}{5} = \frac{75}{5} = 15$ cents. Ans.

Or, we may prodeed thus:

If 1 pound cost 25 cents, $\frac{1}{5}$ of a pound will cost $\frac{1}{5}$ of 25 cents, or 5 cents, and $\frac{3}{5}$ will cost 3 times as much as $\frac{1}{5}$, and $5 \times 3 = 15$ cents. Ans.

It would be a good plan to solve such questions both ways.

8. A tree 60 feet high, had $\frac{5}{12}$ of its length broken off by the wind; what was the length of the broken piece? Ans. 25 feet.

9. The remaining part was $\frac{7}{12}$ of the length of the tree. Required—the length of this part.

What will always be noticed in multiplying by a proper fraction? Need this cause any surprise? Why not?

10. At 50 cents a yard, how much would $\frac{7}{10}$ of a yard of cloth cost?

11. At 96 cents a pound, how much will $\frac{7}{8}$ of a pound of tea cost?

12. There are 320 rods in a mile. How many rods in $\frac{9}{20}$ of a mile?

13. One-eighth of a dollar is $12\frac{1}{2}$ cents. How many cents are there in $\frac{7}{8}$ of a dollar?

Solution.—There are evidently 7 times as many cents as there are in $\frac{1}{8}$ of a dollar. Then 7 times $12 = 84$ cents, and 7 times $\frac{1}{2}$ a cent $= \frac{7}{2} = 3\frac{1}{2}$ cents, and 84 cents and $3\frac{1}{2}$ cents make $87\frac{1}{2}$ cents. Or, $12\frac{1}{2} = \frac{25}{2}$; 7 times $\frac{25}{2} = \frac{175}{2} = 87\frac{1}{2}$ cents, as before.

14. At $16\frac{2}{3}$ cents a yard, how much will 6 yards of calico cost?

15. At $18\frac{3}{4}$ cents apiece, how much will 8 slates cost?

16. At $6\frac{1}{4}$ cents apiece, how much will 12 lead pencils cost?

Solution.—Here the multiplier and multiplicand have changed places; but according to Sect. IV. Art. 2. Obs. 5. Rem., we can proceed as in the last three examples. Thus, $6 \times 12 = 72$; $\frac{1}{4} \times 12 = \frac{12}{4} = 3$; $72 + 3 = 75$ cents. Ans.

Or, we can say—

12 pencils, at 6 cents apiece, will cost 72 cents, and 12 pencils at $\frac{1}{4}$ of a cent apiece, will cost 3 cents; and 72 cents and 3 cents are 75 cents, as before.

It would be a good plan to solve such questions both ways.

17. How much would 10 pounds of coffee cost, at $12\frac{1}{2}$ cents per pound?

18. How much would 12 pounds of sugar cost, at $8\frac{1}{3}$ cents per pound?

EXERCISES FOR THE SLATE

CASE 1.—*To multiply a fraction and a whole number together.*

1. If a man earn $\frac{5}{6}$ of a dollar a day, how much can he earn in 6 days?

Solution.—6 times $\frac{5}{6} = \frac{30}{6} = 5$ dollars. (Art. 2. Obs. 1.) Or 6 times $\frac{5}{6} = \frac{5}{1} = 5$ dollars. (Art. 2. Obs. 5.) Ans. 5 dollars.

2. How much will $\frac{2}{3}$ of a pound of raisins cost, at 30 cents per pound?

Solution.—They will cost $\frac{2}{3}$ times 30 cents; but this is the same as 30 times $\frac{2}{3}$; (Sect. IV. Art. 2. Obs 5. Rem.) 30 times $\frac{2}{3} = \frac{60}{3} = 20$ cents. Ans. Or, $\frac{1}{3}$ of 30 is 10, and $\frac{2}{3}$ is twice 10, or 20 cents, as before.

It will be perceived that both these examples can be worked by one rule. Hence—

To multiply a fraction and whole number together:

Obs. 2. *Multiply the numerator of the fraction and the whole number together, and divide the product by the denominator.* Or,

a. Divide the denominator of the fraction by the whole number, when it can be done without a remainder, and divide the numerator by the quotient. (Art. 2. Obs. 7. *a.*)

Remark.—Compound fractions must of course be reduced to simple ones before the operation can be performed.

If we choose, we can work these sums by cancelation. Thus:

Ex. 1. *Operation*

$$\begin{array}{c|l} \cancel{6} & 5 \\ & \cancel{6} \\ \hline \end{array}$$

Ans. 5 dollars.

Ex. 2. *Operation.*

$$\begin{array}{c|l} & \cancel{30} \text{ -- } 10 \\ \cancel{3} & 2 \\ \hline \end{array}$$

Ans. 20 cents.

In both questions, we write the fractions according to Art. 3. Obs. 11., and the whole numbers according to the General Rule, Sect. VII. Art. 1., and cancel as usual. Hence—

To multiply fractions and whole numbers together by cancelation:

Obs. 3. *Write the fraction as directed by* Art. 3. Obs. 11., *and the whole number as directed by the General Rule,* Sect. VII., Art. 1., *and cancel as usual.*

By this method compound fractions may be reduced to simple ones by expression merely.

3. If $\frac{7}{9}$ of a cord of wood last a month, how much will it take to last 18 months? Ans. 14 cords.

4. If a bushel of wheat weigh $\frac{3}{5}$ of a hundred weight, how much will 24 bushels weigh? Ans. $14\frac{2}{5}$ hundred weight.

5. If 1 yard of cloth cost $\frac{13}{16}$ of a dollar, how much will 60 yards cost? Ans. $48\frac{3}{4}$ dollars.

6. 8 men spent each of them $\frac{3}{4}$ of a dollar. How many dollars did they all spend? Ans. 6.

7. If I pay $\frac{11}{14}$ of a dollar for a bushel of rye, how much must I pay for 28 bushels? Ans. 22 dollars.

8. If a family eat $\frac{9}{10}$ of a barrel of flour in a month, how many barrels will it take to last them a year, or 12 months? Ans. $10\frac{4}{5}$.

9. If a merchant sells tea at $\frac{7}{8}$ of a dollar per pound, how much will 32 pounds cost? Ans. 28 dollars.

How do we multiply a fraction and whole number together? Explain why this method is correct. By what other method can such questions be worked? How do we proceed according to this method?

10. If a man spends $\frac{15}{16}$ of a dollar a day, how much will he spend in 48 days? Ans. 45 dollars.

11. How much would $\frac{4}{6}$ of a pound of tea cost, at 96 cents per pound? Ans. 64 cents.

12. How much would $\frac{2}{3}$ of $\frac{6}{10}$ of $\frac{4}{2}$ of a bushel of wheat cost, at 90 cents per bushel? Ans. 72 cents.

13. How much would $\frac{3}{4}$ of $\frac{6}{2}$ of a yard of cloth cost, at 60 cents per yard? Ans. 135 cents.

14. How much would $\frac{9}{11}$ of an acre of land cost, at 30 dollars per acre? Ans. $24\frac{6}{11}$ dollars.

Case 2.—*To multiply a whole number and a mixed number together.*

1. How much would $132\frac{6}{7}$ acres of land cost, at 14 dollars an acre?

Operation.

$132\frac{6}{7}$
14
——
$12 = 14$ times $\frac{6}{7}$
528
132
——
Ans. 1860 dollars.

We first multiply by $\frac{6}{7}$ by 14, which gives 12. Next we multiply 132 by 14, and adding the several products together, we obtain 1860 dollars as our answer. The reason of our working this question thus, is the same as in Ex. 13, Mental Exercises.

2. How much would 48 yards of calico cost, at $18\frac{3}{4}$ cents per yard?

Operation.

48
$18\frac{3}{4}$
—— $48\times\frac{3}{4}=36$
$36 =$ cost at $\frac{3}{4}$ cents per yard.
384
48
——
Ans. 900 cents.

This question is just like the preceding one, except that the multiplier and multiplicand have changed places. In the operation, we first multiply 48 by $\frac{3}{4}$, according to Obs. 2 or 3, and then multiply 48 by 18, and adding the several products together, we obtain 900 cents as our answer. Hence—

To multiply a whole number and mixed number together:

Obs. 4. *Multiply the integral and fractional parts separtely, and add their products together.*

How do we multiply a whole number and a mixed number together? How may we proceed when the mixed number is small?

REMARK.—If the mixed number is small, we may, if we choose, reduce it to an improper fraction, and proceed according to Obs. 2 or 3.

3 How much would $26\frac{3}{8}$ yards of cloth cost, at 25 cents per yard? Ans. $659\frac{3}{8}$ cents.

4. How much will $47\frac{3}{4}$ acres of land cost, at 16 dollars an acre? Ans. 764 dollars.

5. If a man travel $144\frac{7}{15}$ miles in a week, how far can he travel in 48 weeks? Ans. $6934\frac{2}{5}$ miles.

6. How much will $37\frac{5}{12}$ yards of cloth cost, at 144 cents per yard? Ans. 5388 cents.

7. If a man read $288\frac{15}{16}$ pages in a month, how many pages can he read in 24 months? Ans. $6934\frac{1}{2}$.

8. At $73\frac{2}{3}$ dollars apiece, how much would 124 horses cost? Ans. $9134\frac{2}{3}$ dollars.

9. How much would 432 acres of land cost, at $36\frac{7}{8}$ dollars per acre? Ans. 15930 dollars.

10. How much would 216 pieces of broadcloth cost, at $97\frac{13}{18}$ dollars apiece? Ans. 21108 dollars.

11. How much would 86 pounds of tea cost, at $93\frac{3}{4}$ cents per pound? Ans, $8062\frac{1}{2}$ cents.

12. How much would 74 yoke of cattle cost, at $52\frac{7}{8}$ dollars a yoke? Ans. $3912\frac{3}{4}$ dollars.

CASE 3. *To multiply fractions together.*

Ex. 1. A man having $\frac{1}{2}$ a dollar, spent $\frac{1}{2}$ of it for his dinner. How much did his dinner cost him? Ans. $\frac{1}{4}$ of a dollar.

Solution. If we divide $\frac{1}{2}$ an apple into two equal parts, each part will evidently be $\frac{1}{4}$ of the apple. Also, if we divide $\frac{1}{2}$ a dollar into two equal parts, each part will be $\frac{1}{4}$ of the dollar. Or,

If he gives $\frac{1}{2}$ of all he has for his dinner, and has but $\frac{1}{2}$ a dollar, his dinner must cost him $\frac{1}{2}$ of $\frac{1}{2}$ a dollar; but $\frac{1}{2}$ of $\frac{1}{2}$ is a compound fraction and equals $\frac{1}{4}$. (Art. 3, Obs. 10.)

2. At $\frac{8}{9}$ of a dollar a yard, how much would $\frac{3}{4}$ of a yard of lace cost?

Solution.—If a yard cost $\frac{8}{9}$ of a dollar, $\frac{1}{4}$ of a yard will cost $\frac{1}{4}$ of $\frac{8}{9}$, which is $\frac{2}{9}$ of a dollar; (Art. 2. Obs. 2.); and $\frac{3}{4}$ of a yard will cost 3 times $\frac{2}{9}$, or $\frac{6}{9}=\frac{2}{3}$ of a dollar. (Art. 2. Obs. 1.) Or,

If 1 yard costs $\frac{8}{9}$ of a dollar, $\frac{1}{4}$ of a yard will cost $\frac{1}{4}$ of $\frac{8}{9}$, or $\frac{8}{36}$ of a dollar, (Art. 2. Obs. 4. and Obs. 7, *b.*;) and $\frac{3}{4}$ will cost $\frac{24}{36}$, or $\frac{2}{3}$ of a dollar, as before. Ans. $\frac{2}{3}$ of a dollar.

How do we multiply fractions together? What is the shortest method? Why so?

It will be perceived that the result in both these questions is obtained by multiplying the numerators together, and the denominators. Hence—

To multiply one fraction by another:

Obs. 5. *Multiply together the numerators for a new numerator, and the denominators for a new denominator.*

Note.—The learner will perceive that this is precisely like reducing compound fractions to simple ones.

Remark.—The shortest method is by cancelation, (Art. 3. Obs. 11.), as it at once reduces it to the lowest terms.

3. At $\frac{5}{6}$ of a dollar a yard, how much would $\frac{7}{8}$ of a yard of linen cost? Ans. $\frac{35}{48}$ of a dollar.
4. At $\frac{15}{16}$ of a dollar a bushel, how much would $\frac{17}{30}$ of a bushel of wheat cost? Ans. $\frac{17}{32}$ of a dollar.
5. At $\frac{16}{21}$ of a dollar a pound, how much would $\frac{42}{48}$ of a pound of tea cost? Ans. $\frac{2}{3}$ of a dollar.
6. How much would $\frac{30}{28}$ of a gallon of oil cost, at $\frac{14}{15}$ of a dollar per gallon? Ans. 1 dollar.
7. How much would $\frac{63}{85}$ of a yard of satinet cost, at $\frac{17}{21}$ of a dollar a yard? Ans. $\frac{3}{5}$ of a dollar.
8. Multiply $\frac{3}{7}$ by $\frac{7}{8}$ of $\frac{8}{9}$. Ans. $\frac{1}{3}$.
9. Multiply $\frac{4}{15}$ of $\frac{12}{17}$, by $\frac{16}{18}$ of $\frac{34}{36}$. Ans. $\frac{64}{405}$.
10. Multiply $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$, by $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{2}{3}$. Ans. $\frac{1}{4}$.

Case 4. *To multiply mixed numbers together.*

1. Multiply $14\frac{2}{3}$ by $12\frac{3}{4}$.

Solution.—$14\frac{2}{3} = \frac{44}{3}$; $12\frac{3}{4} = \frac{51}{4}$; $\frac{44}{3} \times \frac{51}{4} = 187$ Ans. Hence—

To multiply mixed numbers together:

Obs. 6. *Reduce the mixed numbers to improper fractions, and then proceed according to* Obs. 5.

2. How much would $12\frac{1}{2}$ yards of cloth cost, at $5\frac{2}{5}$ dollars a yard? Ans. $67\frac{1}{2}$ dollars.
3. How much must be paid for $18\frac{3}{4}$ acres of land, at $31\frac{1}{2}$ dollars per acre? Ans. $590\frac{5}{8}$ dollars.
4. How much must be paid for $64\frac{3}{8}$ yards of cloth, at $6\frac{2}{5}$ dollars a yard? Ans. 412 dollars.
5. Multiply $8\frac{1}{4}$ by $16\frac{2}{3}$. Ans. $137\frac{1}{2}$.
6. Multiply $18\frac{3}{4}$ by $24\frac{5}{6}$. Ans. $465\frac{5}{8}$.
7. Multiply $34\frac{7}{12}$ by $48\frac{15}{16}$. Ans. $1692\frac{27}{64}$.

How do we multiply mixed numbers together?

8. Multiply $37\frac{1}{2}$ by $62\frac{1}{2}$. Ans. $2343\frac{3}{4}$.
9. Multiply $45\frac{4}{7}$ by $46\frac{5}{11}$. Ans. 2117.
10. Multiply $50\frac{4}{9}$ by $75\frac{13}{27}$. Ans. $3807\frac{151}{243}$.

For the convenience of the learner we now present at one view the following

GENERAL RULES FOR THE MULTIPLICATION OF FRACTIONS

1st. To multiply fractions and whole numbers together:

a. Either multiply the numerator of the fraction and the whole number together, and divide the product by the denominator, (Obs. 2.) Or,

b. Divide the denominator by the whole number, when it can be done without a remainder, and divide the numerator by the quotient. (Obs. 2. *a.*) Or, by cancelation:

c. Write the fraction as directed by the 4th Rule, Art. 3, *and the whole number as directed by the General Rule,* Sect. VII., Art. 1, *and cancel as usual.* (Obs. 3.)

2nd. To multiply mixed numbers and whole numbers together:

a. Multiply the integral and fractional parts separately, and add their products together. (Obs. 4.)

3d. To multiply fractions together:

Multiply the numerators together for a new numerator, and the denominators for a new denominator. (Obs. 5.)

4th. To multiply mixed numbers together:

First reduce them to improper fractions, and then proceed as in Multiplication of Fractions. (Obs. 6.)

EXERCISES FOR THE SLATE.

1. How much would 24 bushels of wheat cost, at $\frac{10}{8}$ of a dollar per bushel? Ans. 30 dollars.
2. How much would 36 bushels of corn cost, at $\frac{2}{3}$ of a dollar per bushel? Ans. 24 dollars.
3. If a store is worth 25000 dollars, how much is $\frac{19}{35}$ of it worth? Ans. $13571\frac{3}{7}$ dollars.
4. How much would 60 bushels of wheat cost, at $\frac{11}{8}$ of a dollar per bushel? Ans. $82\frac{1}{2}$ dollars.

What is the first rule for multiplying fractions and whole numbers together? The second? What is the rule by cancelation? How do we multiply whole numbers and mixed numbers together? How do we multiply fractions together? How do we multiply mixed numbers together?

5. How much would 36 yards of cloth cost, at $\frac{17}{24}$ of a dollar per yard? Ans. $25\frac{1}{2}$ dollars.

6. How much would 66 books cost, at $\frac{7}{8}$ of a dollar apiece? Ans. $57\frac{3}{4}$ dollars.

7. How much would 96 bushels of corn cost, at $\frac{5}{8}$ of a dollar per bushel? Ans. 60 dollars.

8. When land is worth 12 dollars an acre, how much is $\frac{15}{16}$ of an acre worth? Ans. $11\frac{1}{4}$ dollars.

9. If powder is 75 cents a pound, how much is $\frac{11}{15}$ of a pound worth? Ans. 55 cents.

10. How much would $18\frac{3}{4}$ yards of cambric cost, at 16 cents per yard? Ans. 300 cents.

11. How much would $12\frac{7}{9}$ cords of wood cost, at 2 dollars per cord? Ans. $25\frac{5}{9}$ dollers.

12. At $87\frac{1}{2}$ cents a yard, how much must I pay for 12 yards of satinet? Ans. 1050 cents.

13. How much would 40 acres of land cost, at $15\frac{4}{5}$ dollars an acre? Ans. 632 dollars.

14. How much would 72 yards of cloth cost, at $4\frac{5}{9}$ dollars a yard? Ans. 328 dollars.

15. How much would $\frac{9}{4}$ a pound of coffee cost, at $\frac{1}{8}$ of a dollar per pound? Ans. $\frac{9}{32}$ of a dollar?

16. How much would $\frac{9}{10}$ of a yard of cloth cost, at $\frac{2}{3}$ of a dollar a yard? Ans. $\frac{3}{5}$ of a dollar.

17. How much would $412\frac{7}{8}$ bushels of corn cost, at $\frac{3}{10}$ of a dollar a bushel? Ans. $123\frac{69}{80}$ dollars.

18. How much would $12\frac{1}{2}$ bushels of apples cost, at $31\frac{1}{4}$ cents a bushel? Ans. $390\frac{5}{8}$ cents.

19. How much would $25\frac{15}{22}$ acres of land cost, at $16\frac{2}{15}$ dallars an acre? Ans. $414\frac{1}{3}$ dollars.

20. If a man travel $28\frac{3}{4}$ miles in one day, how far can he travel in $21\frac{3}{8}$ days? Ans. $614\frac{17}{32}$ miles.

Article 6. Division of Fractions.

Obs. 1. In dividing by a *proper fraction,* the learner will always find that *the quotient is larger than the dividend;* but this need not cause any surprise, if he recollects that *the divisor is less than unity, and consequently the number of parts into which the number is divided, must be greater than the dividend itself.* (Sect. VI. Art. 1. Obs. 26. *b.*)

What is always noticed iu dividing by a proper fraction? Need this cause any surprise? Why not? How do we divide a fraction by a whole number?

MENTAL EXERCISES.

If 3 pounds of coffee cost $\frac{6}{7}$ of a dollar, how much is that per pound?

Solution.—1 pound will evidently cost $\frac{1}{3}$ as much as 3 pounds, and $\frac{1}{3}$ of $\frac{6}{7}$ is $\frac{2}{7}$. (Art. 2. Obs. 2.)

2. If 5 yards of calico cost $\frac{10}{12}$ of a dollar, how much is that per yard?

3 If 8 pounds of lead cost $\frac{24}{50}$ of a dollar, how much is that per pound?

4. If 6 men own $\frac{12}{13}$ of a vessel, what part of it is owned by each?

5. If 12 lead pencils cost $\frac{36}{100}$ of a dollar, how much is that apiece?

6. At $\frac{1}{4}$ of a dollar a bushel, how many bushels of potatoes can you buy for 2 dollars?

Solution.—You can evidently buy as many bushels as $\frac{1}{4}$ is contained in 2. Now there 4 fourths in a unit, or 1; then in 2 there are twice 4 fourths, or 8 fourths; that is, 2 contains $\frac{1}{4}$ 8 times.

7. At $\frac{1}{2}$ a dollar a yard, how many yards of cloth can I buy for 6 dollars?

8. At $\frac{1}{5}$ of a cent apiece, how many marbles can be bought for 10 cents?

9. At $\frac{1}{3}$ of a dollar a pound, how many pounds of coffee can be bought for 5 dollars?

10. At $\frac{1}{4}$ of a cent apiece, how many marbles can be bought for $\frac{3}{4}$ of a cent? Ans. 3.

Solution.—There can evidently be as many marbles bought as $\frac{1}{4}$ is contained in $\frac{3}{4}$, and $\frac{3}{4}$ contains $\frac{1}{4}$, 3 times.

11. At $\frac{2}{5}$ of a dollar apiece, how many books can be bought for $\frac{6}{5}$ of a dollar?

12. At $\frac{3}{8}$ of a dollar a bushel, how many bushels of potatoes can be bought for $\frac{15}{8}$ of a dollar?

EXERCISES FOR THE SLATE.

CASE. 1. *To divide a fraction by a whole uumber.*

1. If 4 yards of cambric cost $\frac{8}{9}$ of a dollar, how much is that a yard?

Solution.—To find the cost of 1, we must divide the cost of the quantity by the quantity. (Sect. VI., Art. 1. Obs. 24.) Then $\frac{8}{9} \div 4 = \frac{2}{9}$; (Art. 2, Obs. 2,) or, $\frac{8}{9} \div 4 = \frac{8}{36}$; (Art. 2. Obs. 4.) $\frac{8}{36} = \frac{2}{9}$, as before.

Or, by cancelation:
We place our numbers according to Art. 5, Obs. 3, and cancel as usual.

$$\begin{array}{c|l} 9 & \cancel{8}\ 2 \\ \cancel{4} & \\ \hline 9 & 2 = \frac{2}{9} \end{array}$$

Ans. $\frac{2}{9}$ of a dollar.

The several results it will be perceived are alike. Hence—

To divide a fraction by a whole number:

Obs. 2. *Divide the numerator of the fraction by the whole number, when it can be done without a remainder; and under the quotient write the denominator.* Or,

a. Multiply the denominator of the fraction by the whole number, and over the product write the numerator.

b. Or, by Cancelation: *Proceed acording to Art. 5, Obs. 3.*

NOTE.—Compound fractions, both in this, and the following cases, must of course, be reduced to simple ones before the operation can be performed.—When the operation is performed by cancelation, however, we may reduce them by expression, merely.

2. A man divided $\frac{4}{5}$ of a dollar between his two children; how much did each receive? Ans. $\frac{2}{5}$ of a dollar.
3. If 5 boys can earn $\frac{10}{11}$ of a dollar, how much can 1 boy earn? Ans. $\frac{2}{11}$ of a dollar.
4. If 6 pounds of coffee cost $\frac{3}{4}$ of a dollar, how much is that a pound? Ans. $\frac{1}{8}$ of a dollar.
5. If 4 bushels of corn cost $\frac{10}{8}$ of a dollar, how much is that per bushel? Ans. $\frac{5}{16}$ of a dollar.
6. If 7 yards of cloth cost $\frac{14}{16}$ of a dollar how much is that per yard? Ans. $\frac{1}{8}$ of a dollar.
7. If 12 yards of ribbon cost $\frac{4}{5}$ of $\frac{10}{12}$ of $\frac{9}{15}$ of 10 dollars, how much is that a yard? Ans. $\frac{1}{3}$ of a dollar.
8. If 15 bushels of wheat cost $\frac{21}{160}$ of 100 dollars, how much is that per bushel? Ans. $\frac{7}{8}$ of a dollar.
9. If 16 pounds of nails cost $\frac{4}{5}$ of a dollar, how much is that per pound? Ans. $\frac{1}{20}$ of a dollar.
10. If 18 lemons cost $\frac{9}{10}$ of a dollar, how much is that apiece. Ans. $\frac{1}{20}$ of a dollar.

CASE 2. *To divide a whole number by a fraction.*

1. A teacher divided 8 apples among a class at school, giving

---

Why do we divide the fraction by the whole number in Ex. 1, Case 1?—How do we divide a fraction by a whole number? What must be done with compound fractions when they occur? If the operation is performed by cancelation how may we proceed?

each scholar $\frac{2}{3}$ of an apple. How many scholars were there in the class? Ans. 12.

Solution.—It is evident that there were as many scholars as $\frac{2}{3}$ is contained in 8, because each scholar receives $\frac{2}{3}$ of an apple, and there are 8 apples to be divided. We will first divide 8 by $\frac{1}{3}$. In a unit are 3 thirds, therefore in 8 units are $3 \times 8 = 24$ thirds; that is, 8 contains $\frac{1}{3}$ 24 times. Now $\frac{2}{3}$ is twice as much as $\frac{1}{3}$, consequently 8 will not contain $\frac{2}{3}$ but half as many times as it will $\frac{1}{3}$; that is, 12 times. (Sect. VI, Art. 1, Obs. 26, *a.*) It will be perceived in this example, that we multiply the whole number by the denominator of the fraction, and divide the product by the numerator. It makes no difference, however, whether we perform the multiplication or division first; because $8 \times 3 = 24$; $24 \div 2 = 12$, and $8 \div 2 = 4$; $4 \times 3 = 12$ as before. Hence—

To divide a whole number by a fraction:

Obs. 3. *Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

Or, when it can be done without a remainder,

a. Divide the whole number by the numerator, and multiply the quotient by the demominator.

Obs. 4. *The reciprocal of a number in the quotient arising from dividing a unit by that number.* Thus the reciprocal of 2 is $\frac{1}{2}$; of 3, $\frac{1}{3}$; of 5, $\frac{1}{5}$, &c. If we divide a unit by $\frac{1}{2}$, the quotient is $\frac{2}{1}$ or 2; if we divide a unit by $\frac{4}{5}$, the quotient is $\frac{5}{4} = 1\frac{1}{4}$. Hence—

Obs. 5. *The reciprocal of a fraction is the same fraction inverted.*

Invert, means to *turn upside down.* Thus, $\frac{3}{4}$ inverted becomes $\frac{4}{3}$, &c.

It will appear from these definitions, that the solution of the above example consists merely *in multiplying the whole number and the reciprocal of the fraction together,* according to Art. 5. General Rules, 1st.

Examples of this kind can also be performed by Cancelation.—Thus:

$$\begin{array}{r|l} & \cancel{8}\ldots 4 \\ \cancel{2} & 3 \\ & \overline{12} \end{array}$$

We proceed as directed, Art. 5, Obs. 3, except to use the reciprocal of the fraction, instead of the fraction itself. Hence—

Explain the solution of Ex. 1, Case 2. Does it make any difference whether we perform the multiplication or the division first? Why not? How do we divide a whole number by a fraction? What is the reciprocal of a number? Give examples. What is the reciprocal of a fraction? Show why this is correct. What does invert mean? In what does the solution of Ex. 1, Case 2 consist?

To divide a whole number by a fraction, by cancelation:

Obs. 6. *Place the numerator of the divisor at the left, and the denominator at the right; in other respects proceed as usual.*

2. At $\frac{1}{4}$ of a dollar a bushel, how much corn can be bought for 3 dollars? Ans. 12 bushels.

3. At $\frac{1}{2}$ a dollar a bushel, how much rye can be bought for 7 dollars? Ans. 14 bushels.

4. At $\frac{1}{8}$ of a dollar a yard, how many yards of muslin can be bought for 6 dollars? Ans. 48 yards.

5. At $\frac{2}{3}$ of a dollar apiece, how many books can be bought for 8 dollars? Ans. 12.

6. At $\frac{5}{7}$ of a dollar a bushel, how much barley can be bought for 12 dollars? Ans. $16\frac{4}{5}$ bushels.

7. A man gave 12 dollars to some destitute people, giving each $\frac{6}{7}$ of a dollar. How many were relieved? Ans. 14.

8. At $\frac{1}{6}$ of $\frac{3}{4}$ of 10 dollars a bushel, how much wheat can be bought for 18 dollars? Ans. $14\frac{2}{5}$ bushels.

In this example, the expression $\frac{1}{6}$ of $\frac{3}{4}$ of 10 dollars is our divisor.

9. At $\frac{2}{7}$ of a dollar apiece, how many hats may be bought for 6 dollars? Ans. 21.

10. At $\frac{1}{12}$ of $\frac{3}{4}$ of $\frac{1}{3}$ of 12 dollars apiece, how many slates can be bought for 9 dollars? Ans. 36.

Case 3. *To divide one fraction by another.*

1. At $\frac{2}{3}$ of a dollar a yard, how many yards of cloth can be bought for $\frac{8}{9}$ of a dollar? Ans. $1\frac{1}{3}$.

The result is evidently found by dividing $\frac{8}{9}$ by $\frac{2}{3}$.

Operation.

$\frac{2}{3}=\frac{6}{9}$. We first reduce the fractions to a common denomina-
$\frac{8}{9}\div\frac{6}{9}=1\frac{1}{3}$. tor, and then divide the numerator of the dividend, by the numerator of the divisor.

We can however, divide one fraction by another without reducing them to a common denominator.

We learn from the remark under Obs. 5, that to divide a whole number by a fraction, consists merely in multiplying the whole number by the reciprocal of this fraction. Hence, it is evident that to divide one fraction by another consists merely in *multiplying the frac-*

How do we divide a whole number by a fraction, by cancelation? Explain the solution of Ex. 1, Case 3. Is there any other method of performing such operations? In what do such operations consist?

tion which is the dividend, by the reciprocal of the fraction which is the divisor. That is, we invert our divisor and proceed according to Art. 3, Obs. 5, thus: $\frac{8}{9} \times \frac{3}{2} = \frac{24}{18} = \frac{4}{3} = 1\frac{1}{3}$. Ans.

Or, we may proceed by cancelation, according to Art. 3, Obs. 11, thus:

3 -- $\cancel{9}$	$\cancel{8}$ -- 4
$\cancel{2}$	$\cancel{3}$
3	4 $= 1\frac{1}{3}$. Ans.

The learner will perceive that we write our divisor as directed by Obs. 6. In all these operations the final result is the same. Hence—

To divide one fraction by another:

Obs. 7. *Invert the divisor and then proceed as in multiplication of fractions.* Or, by cancelation:

a. *Write the divisor as directed by Obs. 6, and the other numbers as directed by Art. 3, Obs. 11, and cancel as usual.*

2. If a man mow $\frac{7}{8}$ of an acre of grass per day, how many days will it take him to mow $\frac{49}{16}$ of an acre? Ans. $3\frac{1}{2}$.
3. If a man travel $\frac{3}{4}$ of a league in an hour, how many hours will it take him to travel $\frac{9}{14}$ of a league? Ans. $\frac{6}{7}$ of an hour.
4. At $\frac{6}{7}$ of a dollar a yard, how many yards of cloth can I buy for $\frac{11}{12}$ of a dollar? Ans. $1\frac{5}{72}$.
5. If I pay $\frac{4}{9}$ of a dollar a pound, how many pounds of tea can I buy for $\frac{16}{45}$ of a dollar? Ans. $\frac{4}{5}$ of a pound.
6. How many yards of satinet can I buy for $\frac{16}{33}$ of a dollar, at $\frac{8}{11}$ of a dollar a yard? Ans. $\frac{2}{3}$ of a yard.
7. Divide $\frac{2}{7}$ of $\frac{7}{8}$ of 4, by $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{5}{6}$ of 7. Ans. $\frac{4}{15}$.

The learner will observe that he must *invert all the terms of the divisor, when it is a compound fraction.*

8. Divide $\frac{1}{2}$ of $\frac{4}{9}$ of $\frac{3}{4}$ of $\frac{14}{17}$ of 52 by $\frac{13}{34}$ of $\frac{8}{9}$ of $\frac{3}{4}$ of 28. Ans. 1.
9. Divide $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{6}{11}$ of $\frac{48}{49}$ of 23 by $\frac{5}{7}$ of $\frac{22}{25}$ of $\frac{15}{16}$ of 46. Ans. $\frac{192}{605}$.
10. Divide $\frac{1}{3}$ of $\frac{4}{7}$ of $\frac{7}{12}$ of 20, by $\frac{3}{4}$ of $\frac{2}{9}$ of $\frac{5}{12}$ of 32. Ans. 1.

CASE 4. *Method of proceeding when mixed numbers occur.*

1. How many bushels of wheat can I buy for $5\frac{1}{7}$ dollars, at $1\frac{1}{8}$ dollars per bushel? Ans. $4\frac{4}{7}$.

Solution.—$5\frac{1}{7} = \frac{36}{7}$; $1\frac{1}{8} = \frac{9}{8}$; $\frac{36}{7} \div \frac{9}{8} = 4\frac{4}{7}$ bushels, Ans.

How do we divide one fraction by another? How by cancelation? When our divisor is a compound fraction, what must be observed in the operation?

Hence—When mixed numbers occur in both the dividend and divisor:

Obs. 8. *Reduce them to improper fractions, and then proceed according to Obs. 7.*

2. How much rye can I buy for $4\frac{12}{17}$ dollars, at $\frac{10}{17}$ of a dollar per bushel? Ans. 8 bushels
3. How many books can I buy for $1\frac{11}{19}$ dollars, at $\frac{15}{19}$ of a dollar apiece? Ans. 2.
4. At $5\frac{1}{4}$ dollars an acre, how many acres of land can I buy for $52\frac{1}{2}$ dollars? Ans. 10.
5. At $2\frac{1}{7}$ dollars apiece, how many sheep can I buy for $32\frac{1}{7}$ dollars? Ans. 15.
6. At $24\frac{4}{5}$ dollars per acre, how many acres of land can I buy for 620 dollars? Ans. 25.

Solution.—$24\frac{4}{5} = \frac{124}{5}$; $620 \div \frac{124}{5} = 25$ acres, Ans.

Hence—When only the divisor is a mixed number:

Obs. 9. *Reduce it to an improper fraction, and then proceed according to Obs. 3, or 6.*

7. At $52\frac{6}{7}$ dollars apiece, how many horses can I buy for 2220 dollars? Ans. 42.
8. At $17\frac{4}{9}$ dollars apiece, how many cows can I buy for 314 dollars? Ans. 18.
9. At $93\frac{3}{4}$ cents per bushel, how many bushels of wheat can I buy for 1875 cents? Ans. 20.
10. At $62\frac{1}{2}$ cents per bushel, how many bushels of apples can I buy for 3000 cents? Ans. 48.
11. At 3 dollars a yard, how many yards of cloth can I buy for $187\frac{1}{2}$ dollars? Ans. $62\frac{1}{2}$.

Solution.—$187\frac{1}{2} = \frac{375}{2}$; $\frac{375}{2} \div 3 = \frac{125}{2} = 62\frac{1}{2}$. Or, $187\frac{1}{2} \div 3 = 62$, and $1\frac{1}{2}$ remainder; $1\frac{1}{2} = \frac{3}{2}$; $\frac{3}{2} \div 3 = \frac{1}{2}$, which annexed to $62 = 62\frac{1}{2}$. Hence—

When only the dividend is a mixed number:

Obs 10. *Reduce it to an improper fraction, and then proceed according to Obs. 2.* Or,

a. Divide the integral part as in simple division; to the remainder (if any) annex the fractional part and proceed as above directed.

Note.—The first rule is the best when the dividend is small, but when the dividend is large, the latter method is preferable.

When both dividend and divisor are mixed numbers, how do we proceed?—When only the divisor is a mixed number, how do we proceed? When only the dividend is a mixed number, how do we proceed? When is the first rule preferable in this case? The second? What must be observed in all cases of reduction of fractions?

12. How many acres of land can I buy for $146\frac{2}{3}$ dollars, at 8 dollars an acre? Ans. $18\frac{1}{3}$.

13. How many lead pencils can I buy for $37\frac{1}{2}$ cents, at 3 cents apiece? Ans. $12\frac{1}{2}$.

14. At 15 dollars an acre, how many acres of land can I buy for $1234\frac{4}{5}$ dollars? Ans. $82\frac{8}{25}$.

15. At 45 cents a yard, how many yards of cloth can be bought for $843\frac{3}{4}$ cents? Ans. $18\frac{3}{4}$.

CASE 5. *Complex Fractions.*

NOTE.—This and the following case properly belong to Reduction of Fractions, but it was thought best to defer them until the learner had studied Division of Fractions.

REMARK.—The learner will recollect, that in all cases of reduction of fractions, the *value of the fraction* must not be altered. (Art. 3, Obs. 1.)

1. Reduce $\dfrac{2\frac{1}{2}}{4\frac{1}{4}}$ to a simple fraction. Ans. $\frac{10}{17}$.

Solution.—This expression is the same as $2\frac{1}{2} \div 4\frac{1}{4}$. (Art. 1, Obs. 9.) By Obs. 8, we find the quotient of $2\frac{1}{2}$ divided by $4\frac{1}{4}$ to be $\frac{10}{17}$. Hence—

To reduce a complex fraction to a simple expression:

Obs. 11. *Reduce both numerator and denominator to improper fractions, and then divide the former expression by the latter, as directed by Obs.* 7.

Change the following expressions to their simplest form:

2. $\dfrac{5\frac{1}{3}}{2\frac{2}{3}}$. Ans. $2\frac{2}{9}$

3. $\dfrac{8\frac{2}{5}}{3\frac{1}{2}}$ Ans. $2\frac{2}{5}$.

4. $\dfrac{7\frac{1}{4}}{6}$ Ans. $1\frac{5}{24}$.

5. $\dfrac{9\frac{4}{5}}{5\frac{1}{4}}$ Ans. $1\frac{13}{15}$.

6. $\dfrac{4\frac{2}{7}}{15}$ Ans. $\frac{2}{7}$.

7. $\dfrac{8\frac{1}{3}}{\frac{2}{6}}$ Ans. 25.

8. $\dfrac{7}{3\frac{1}{2}}$ Ans. 2.

9. $\dfrac{24\frac{3}{4}}{4\frac{1}{8}}$ Ans. 6.

How do we reduce a complex fraction to a simple expression?

10. $\dfrac{4}{7\frac{1}{6}}$ Ans. $\frac{24}{43}$.

11. $\dfrac{\frac{4}{9}}{12\frac{1}{7}}$ Ans. $\frac{28}{765}$.

12. $5\dfrac{6\frac{1}{8}}{3\frac{1}{4}}$ Ans. $6\frac{23}{26}$.

13. $\dfrac{1\frac{1}{5}}{1\frac{1}{3}}$ Ans. $\frac{9}{10}$.

REMARK 1.—Complex fractions may be either added, subtracted, multiplied or divided, by first reducing them to simple expressions.

14. Add together $\dfrac{4\frac{2}{9}}{1\frac{4}{6}}$, and $\dfrac{6\frac{1}{8}}{3\frac{1}{4}}$. Ans. $4\frac{163}{390}$.

15. Add together $\dfrac{4}{1\frac{1}{3}}$, $\dfrac{6\frac{1}{2}}{9}$, and $\dfrac{2\frac{2}{3}}{3\frac{3}{4}}$. Ans. $4\frac{39}{90}$.

16. Add together $\dfrac{1\frac{1}{5}}{2\frac{7}{10}}$, $\dfrac{1\frac{5}{6}}{3\frac{2}{3}}$, and $\dfrac{6\frac{4}{15}}{7\frac{11}{18}}$. Ans. $1\frac{9467}{12330}$.

17. From $\dfrac{7\frac{3}{11}}{9\frac{9}{14}}$ take $\dfrac{3\frac{1}{4}}{8\frac{11}{12}}$. Ans. $\frac{12385}{31779}$.

18. From $\dfrac{16\frac{1}{2}}{9}$ take $\dfrac{4}{10\frac{2}{5}}$. Ans. $1\frac{35}{78}$.

19. From $\dfrac{1\frac{3}{13}}{4}$ take $\dfrac{3\frac{7}{11}}{29\frac{1}{11}}$. Ans. $\frac{19}{104}$.

REMARK 2.—When complex fractions are multiplied or divided by *cancelation*, we may merely reduce them by expression, and proceed according to Obs. 7, or Art. 5, Obs. 5.

20. Multiply $\dfrac{7\frac{2}{9}}{9\frac{7}{18}}$ by $\dfrac{6\frac{2}{7}}{8\frac{9}{14}}$. Ans. $\frac{80}{143}$.

21. Multiply $\dfrac{9\frac{4}{5}}{5\frac{1}{4}}$ by $\dfrac{5\frac{1}{4}}{3\frac{1}{2}}$. Ans. $2\frac{4}{5}$.

Can complex fractions be added, subtracted, multiplied, or divided? How? When complex fractions are multiplied or divided by cancelation, how may we proceed?

22. Multiply $\dfrac{15\frac{3}{10}}{4\frac{1}{2}}$ by $\dfrac{11\frac{3}{5}}{6\frac{4}{9}}$. Ans. $6\frac{3}{25}$.

23. Multiply $\dfrac{2\frac{1}{4}}{5}$ by $\dfrac{7}{4\frac{2}{3}}$. Ans. $\frac{27}{40}$.

24. Multiply $\dfrac{1\frac{1}{2}}{3\frac{1}{4}}$ by $\dfrac{2\frac{1}{3}}{4\frac{1}{5}}$. Ans. $\frac{10}{39}$.

25. Divide $\dfrac{24\frac{3}{4}}{8\frac{1}{4}}$ by $\dfrac{4\frac{1}{6}}{8\frac{1}{3}}$ Ans. 6.

26. Divide $\dfrac{12}{4\frac{1}{2}}$ by $\dfrac{5\frac{1}{4}}{6}$. Ans. $3\frac{1}{21}$.

27. Divide $\dfrac{4\frac{2}{7}}{3\frac{3}{4}}$ by $\dfrac{8\frac{1}{3}}{1\frac{1}{4}}$. Ans. $\frac{6}{35}$.

28. Divide $\dfrac{1\frac{1}{4}}{2\frac{1}{2}}$ by $\dfrac{2\frac{1}{3}}{10}$. Ans. $1\frac{1}{2}$.

29. Divide $\dfrac{1\frac{1}{2}}{3\frac{1}{4}}$ by $\dfrac{2\frac{1}{3}}{4\frac{1}{5}}$. Ans. $\frac{54}{65}$.

30. Multiply $\dfrac{2\frac{2}{3}}{5\frac{3}{4}}$ by $\dfrac{6\frac{3}{5}}{14\frac{3}{10}}$ and divide product by $\dfrac{8\frac{8}{13}}{40\frac{1}{4}}$. Ans. 1.

Case 6.—*To change a fraction to a required denominator.*

This process consists merely in multiplying both the terms of the given fraction, by such a number as will give a resulting fraction having the required denominator. Hence—

Obs. 12. *The denominator of the given fraction, is a factor of the denominator of the required fraction.*

In this case, our multiplier (which must be the other factor,) is found by dividing the denominator of the required fraction, by the

---

In what does the process of changing of a fraction to any required denominator consist? What inference is deduced from this?

denominator of the given fraction. (Sect. VI. Art. 1. Obs. 16.)

1. Change $\frac{1}{4}$ to a fraction, the denominator of which is 8. Ans. $\frac{2}{8}$.

Solution.—Dividing 8 by 4, we obtain 2 as our multiplier, and multiplying both terms of the fraction ($\frac{1}{4}$) by 2, we obtain $\frac{2}{8}$ as our answer.

2. Change $\frac{3}{5}$ to twelfths. Ans. $\dfrac{7\frac{1}{5}}{12}$.

As the denominator of the required fraction is always given, our only trouble is in finding the numerator. In the last example, our numerator is found by dividing 12 by 5, and multiplying the quotient by 3. It is evident that by first performing the multiplication, and then the division, we shall obtain the same result, and thus avoid a fractional multiplier. Hence—

To change a fraction to any required denominator:

Obs. 13. ***Multiply*** *the numerator of the given fraction by the denominator of the required fraction, and divide the product by the denominator of the given fraction; the result will be the numerator of the required fraction under which write the required denominator.*

3. Change $\frac{5}{7}$ to thirty-fifths. Ans. $\frac{25}{35}$.

4. Change $\frac{3}{10}$ to fifteenths. Ans. $\dfrac{4\frac{1}{2}}{15}$.

5. Change $\frac{5}{12}$ to thirds. Ans. $\dfrac{1\frac{1}{4}}{3}$.

6. Change $\frac{2}{7}$ to fifths. $\dfrac{1\frac{3}{7}}{5}$.

7. Change $\frac{5}{6}$ to elevenths. Ans. $\dfrac{9\frac{1}{6}}{11}$

8. Change $\frac{2}{3}$ to fourths. Ans. $\dfrac{2\frac{2}{3}}{4}$.

In this case what must our multiplier be? How is it found? In what does the chief difficulty lie, in solving questions of this nature? How do we find the numerator in Ex. 2?

9. Change $\frac{7}{11}$ to ninths. $\frac{5\frac{8}{11}}{9}$.

10. Change $\frac{8}{15}$ to sevenths. $\frac{3\frac{11}{15}}{7}$.

For the convenience of the learner, we will now present at one view the following

GENERAL RULES FOR THE DIVISION OF FRACTIONS.

1st. To divide a fraction by a whole number:

a. Divide the numerator of the fraction by the whole number, when it can be done without a remainder, and under the quotient write the denominator. (Obs. 2.) Or,

b. Multiply the denominator of the fraction by the whole number, and over the product write the numerator. (Obs. 2, *a.*) or bycancelation:

c. Proceed in every respect as in multiplication of fractions.—(Obs. 2, *b.*)

2d. To divide a whole number by a fraction:

a. Multiply the whole number by the denominator of the fraction and divide the product by the numerator. (Obs. 3.) Or, when it can be done without a remainder.

b. Divide the whole number by the numerator, and multiply the quotient by the denominator. (Obs. 3. *a.*) Or, by cancelation.

c. Proceed in every respect as in multplication of fractions, except to write the numerator of the divisor at the left, and the denominator at the right. (Obs. 6.)

3d. To divide one fraction by another:

Invert the divisor and then proceed as in multiplication of fractions. (Obs. 7.) The rule by cancelation is the same. (Obs. 7.)

4th. When mixed numbers occur either in the dividend, or divisor, or both.

Reduce them to improper fractions, and proceed according to preceding rules. (Obs. 8, 9 and 10.)

5th. To reduce a a complex fraction to a simple one:

Consider the numerator a dividend, and the denominator a divisor, and then proceed according to the last rule. (Obs. 11.)

What is the rule for dividing a whole number by a fraction? The rule by cancelation? What is the rule for dividing one fraction by another? What is the rule when mixed numbers occur? What is the rule for reducing a complex fraction to a simple one? What is the rule for changing a fraction to any required denominator?

6th. To change a fraction to any required denominator.

Multiply the numerator of the given fraction, by the denominator of the required fraction, and divide the product by the denominator of the given fraction; the result will be the numerator of the required fraction, under which write the denominator. (Obs. 13.)

EXERCISES FOR THE SLATE.

1. If 6 slates cost $\frac{3}{4}$ of a dollar, how much is that apiece? Ans. $\frac{1}{8}$ of a dollar.

2. If 14 yards of ribbon cost $\frac{21}{25}$ of a dollar, how much is that per yard? Ans. $\frac{3}{50}$ of a dollar.

3. At $\frac{1}{4}$ of a dollar per bushel, how many bushel of corn can I buy for 10 dollars? Ans. 40.

4. At $\frac{3}{16}$ of a dollar per bushel, how many bushel of oats can be bought for 75 dollars? Ans. 400.

5. How many yards of cloth at $\frac{3}{8}$ of a dollar a yard, can be bought for 6 dollars? Ans. 16.

6. How many books at $\frac{5}{6}$ of a dollar apiece, can be bought for 10 dollars? Ans. 12,

7. How many tumblers at $\frac{5}{16}$ of a dollar apiece, can be bought for 15 dollars? Ans. 48.

8. How much wheat can be bought for 15 dollars, at $1\frac{1}{4}$ dollars per bushel? Ans. 12 bushels.

9. At $\frac{1}{3}$ of a dollar apiece, how many slates can I buy for $\frac{11}{3}$ of a dollar? Ans. 11.

10. At $\frac{1}{16}$ of a dollar a roll, how many rolls of tape can be bought for $\frac{7}{8}$ of a dollar? Ans. 14.

11. At $8\frac{1}{3}$ cents a pound, how many pounds of coffee can be bought for 87 $\frac{1}{2}$ cents? Ans. $10\frac{1}{2}$.

12. At $16\frac{2}{3}$ cents a pound, how many pounds of spice can be bought for $93\frac{3}{4}$ cents? Ans. $5\frac{5}{8}$.

13. How many boxes will it take to contain 1600 pounds of tea, each box containing $44\frac{4}{9}$ pounds? Ans. 36.

14. How many barrels of pork can be bought for $478\frac{4}{5}$ dollars, at $13\frac{3}{16}$ dollars per barrel? Ans. 36.

15. How many bales of velvet in $1166\frac{2}{3}$ yards, each bale containing $129\frac{17}{27}$ yards? Ans. 9

16. Add together $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}$, $\dfrac{3\frac{1}{4}}{5\frac{1}{5}}$ and $\dfrac{4\frac{1}{6}}{6\frac{3}{7}}$. Ans. $1\frac{1385}{1512}$.

17. Add together $\dfrac{2\frac{2}{3}}{3\frac{3}{4}}$, $\dfrac{4\frac{4}{5}}{6}$ and $\dfrac{15}{5\frac{5}{6}}$. Ans. $4\frac{26}{315}$.

18. From $\dfrac{3\frac{1}{4}}{4\frac{1}{5}}$ take $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}$. Ans. $\frac{11}{84}$.

19. From $\dfrac{15}{5\frac{5}{6}}$ take $\dfrac{4\frac{4}{5}}{6}$ Ans. $1\frac{27}{35}$.

20. Multiply $\dfrac{7\frac{1}{8}}{4\frac{2}{7}}$ by $\dfrac{3\frac{6}{7}}{8\frac{4}{9}}$. Ans. $\frac{243}{320}$.

21. Multiply $\dfrac{5}{4\frac{1}{2}}$ by $\dfrac{1\frac{4}{5}}{2\frac{3}{8}}$. Ans. $\frac{16}{19}$

22. Divide $\dfrac{2\frac{3}{11}}{\frac{4}{5}}$ by $\dfrac{\frac{5}{9}}{4\frac{5}{18}}$. Ans. $21\frac{7}{8}$.

23. Divide $\dfrac{5}{4\frac{1}{2}}$ by $\dfrac{10}{1\frac{1}{8}}$. Ans. $\frac{1}{8}$.

24. Change $\frac{4}{5}$ to thirteenths. Ans. $\dfrac{10\frac{2}{5}}{13}$.

25. Change $\frac{7}{8}$ to twenty-fifths. Ans. $\dfrac{21\frac{7}{8}}{25}$.

26. If $4\frac{1}{2}$ bushels of wheat cost $3\frac{3}{8}$ dollars, what will $6\frac{3}{4}$ bushels cost? Ans. $5\frac{1}{16}$ dollars.

First find the cost of 1 bushel, and then of $6\frac{3}{4}$

27. If 4 men spend 17 dollars in $5\frac{1}{2}$ days, how many dollars will 7 men spend in $16\frac{3}{4}$ days? Ans. $90\frac{53}{88}$.

Solution.—If 4 men spend 17 dollars in $5\frac{1}{2}$ days, 1 man will spend $17 \div 4 = 4\frac{1}{4}$ dollars in $5\frac{1}{2}$ days; and he will spend $4\frac{1}{4} \div 5\frac{1}{2} = \frac{17}{22}$ of a dollar per day. Then 7 men will spend 7 times $\frac{17}{22}$, or $\frac{119}{22}$ dollars per day, and $\frac{119}{22} \times 16\frac{3}{4} = 90\frac{53}{88}$ dollars in $16\frac{3}{4}$ days.

28. A man after spending $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ of his money, found he had 26 dollars left; how much had he at first? Ans. 120 dollars.

Solution.—Adding $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ together, we find, their sum to be $\frac{47}{60}$. This is what he spent. Then $1\frac{60}{60} - \frac{47}{60} = \frac{13}{60}$ is what he had left. Then 26 is $\frac{13}{60}$ of what number?

29. If a certain number is increased by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$ and $\frac{5}{6}$ of itself, and 33 more, the result will be 4 times the original number; what is this number? . Ans. 180.

Solution.—Adding together $\frac{1}{2}$. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{5}{6}$, we find their sum to be $\frac{169}{60}$; This added to 1, or $\frac{60}{60}=\frac{229}{60}=3\frac{9}{60}$. $4-3\frac{49}{60}=\frac{11}{60}$; then 33 is $\frac{11}{60}$ of what number?

MISCELLANEOUS EXERCISES FOR THE SLATE:

Involving the principles of Common Fractions.

1. At $\frac{5}{8}$ of a dollar a yard, how much would $\frac{3}{4}$ of a yard of cloth cost? Ans. $\frac{15}{32}$ of a dollar.

2. If $\frac{3}{4}$ of a yard of cloth cost $\frac{15}{32}$ of a dollar, how much is that per yard? Ans. $\frac{5}{8}$ of a dollar.

3. If 6 pounds of coffee cost $\frac{3}{4}$ of a dollar, how much is that per pound? Ans. $\frac{1}{8}$ of a dollar.

4. If $2\frac{1}{4}$ yards of cloth cost $\frac{5}{8}$ of a dollar how much would $6\frac{1}{2}$ yards cost? Ans. $1\frac{29}{36}$ dollars.

5. How much would $12\frac{6}{7}$ acres of land cost, at $8\frac{1}{6}$ dollars an acre? Ans. 105 dollars.

6. If $\frac{6}{13}$ of a ship cost 4728 dollars, how much is $\frac{15}{26}$ of her worth? Ans. 5910 dollars.

7. If $\frac{4}{5}$ of $\frac{9}{10}$ of a store is worth 2448 dollars, what is the worth of the store? Ans. 3400 dollars.

8. If $7\frac{1}{2}$ bushels of wheat cost $8\frac{3}{4}$ dollars, how much will $16\frac{2}{3}$ bushels cost? Ans. $19\frac{4}{9}$ dollars.

9. If a pole $9\frac{1}{2}$ feet high cast a shadow $12\frac{1}{4}$ feet, how high must that pole be that casts a shadow of 147 feet? Ans. 114 feet.

10. If 9 men can do a piece of work in $16\frac{2}{3}$ days, in what time can 14 men perform it? Ans. $10\frac{5}{7}$ days.

11. If it requires $3\frac{1}{2}$ yards of cloth to make a coat, when it is but $\frac{3}{4}$ of a yard wide, how much will it require when the cloth is 1 yard wide? How much when the cloth is $1\frac{1}{4}$ yards wide? Ans. to the last. $2\frac{1}{10}$ yards.

12. If 9 horses consume $5\frac{5}{8}$ tons of hay in 7 weeks, how many tons will 16 horses consume in 12 weeks. Ans. $17\frac{1}{7}$.

13. If 6 students spend $5\frac{1}{3}$ dollars in $12\frac{1}{2}$ days, how many dollars will 15 students spend in $23\frac{1}{8}$ days? Ans. $24\frac{2}{3}$.

14. If a family of 7 persons drink $18\frac{1}{3}$ gallons ot beer in $2\frac{1}{2}$ weeks, how many gallons will they drink in $12\frac{3}{4}$ weeks, if 7 persons more are added to the family? Ans. 187.

15. A man spent $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his money, and had 69 dollars left. how much had he at first? Ans. 180 dollars.

16. A man has an orchard in which $\frac{3}{8}$ of the trees bear apples, $\frac{1}{5}$

bear peaches, and 20 trees bear plums. How many trees are there in the orchard? Ans. 100.

17. In a certain school $\frac{2}{5}$ of the pupils study arithmetic, $\frac{1}{10}$ study grammar, $\frac{1}{8}$ geography, $\frac{1}{4}$ learn to write, and 15 learn to read.—How many pupils in the school? Ans. 120.

18. If a certain number be increased by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of itself, and 6 more, the sum will be double the number. Required—the number? Ans. 120.

19. A man being asked the time, answered, "If you increase it by $\frac{1}{2}$ of itself, it will be 12 o'clock. What time was it?

Ans. 8 o'clock.

20. I desire to know the time by knowing that if it is increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, it will be half past 12 o'clock.

Ans. 6 o'clock.

21. "If to my age there added be,
One-half, one-third, and three times three,
The whole will make six score and ten,—*
Pray tell my age now if you can." Ans. 66 years.

---

2. DECIMAL FRACTIONS.

Article 7. Definitions, &c.

Remark.—As we have said before, if a unit is divided into equal parts these parts are called Fractions. (Art. 1, Obs. 4.)

Obs. 1. A Decimal Fraction *is one in which the denominator is* 1 *with any number of ciphers annexed;* as $\frac{5}{10}$, $\frac{13}{100}$, $\frac{276}{1000}$, &c.

Note.— The word *decimal* is derived from the Latin word *decem*, which signifies ten.

Remark.—Decimal Fractions are generally written without the denominator being expressed, in which case a point (.) called a separatrix, or *separating point* is placed before it to distinguish it from whole numbers. Thus—.5, .17, .479, &c. are read 5 tenths, 17 hundredths, 479 thousandths, &c. Hence—

Obs. 2. *The denominator of a decimal fraction is* 1, *with as many ciphers annexed as there are figures in the numerator, or decimal.* Thus

$.1 = \frac{1}{10}$	$.132 = \frac{132}{1000}$.	$.12347 = \frac{12347}{100000}$.
$.12 = \frac{12}{100}$.	$.4376 = \frac{4376}{10000}$	$.476892 = \frac{476892}{1000000}$.

What is the denominator of .3? .7? .19? .54? .1008? .156? .98? .2067? .9?

Suppose it were required to write $\frac{4}{10}$ without the denominator.

---

*130 years.

In this case the numerator does not contain as many figures as there are ciphers in the denominator, but this is remedied by writing a cipher before the 4, thus, .04. In the same manner $\frac{6}{100} = .006$; $\frac{3}{10000} = .0003$, &c. Hence—

To write decimals, when the numerator does not contain as many significant figures as there are ciphers in the denominator:

Obs. 3. *Prefix ciphers to the significant figures of the numerator, until the number of decimal places is equal to the number of ciphers in the denominator.* Thus—

$\frac{6}{10}$ is written	.6	$\frac{6}{10000}$ is written	.0006
$\frac{6}{100}$ " "	.06	$\frac{6}{100000}$ " "	.00006
$\frac{6}{1000}$ " "	.006	$\frac{6}{1000000}$ " "	.000006

Obs. 4. From this it appears that the first place at the right of the separatrix is called TENTHS, *because a unit is divided into ten equal parts;* the second place is called HUNDREDTHS *from dividing tenths into ten equal parts, or a unit into a hundred equal* parts; &c. This can be easily shown from the following

DECIMAL NUMERATION TABLE.

Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	(Separatrix.)	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.	Billionths.	
					.	6									is read 6 tenths.
				9	.	4	3								is read 9 units. 43 hundredths.
			1	2	.	1	3	7							is read 12 units, 137 thousandths.
		4	3	7	.	0	3	4	2						is read 437 units, 342 ten thousandths.
	1	2	4	5	.	6	0	0	4	1					is read 1245 units, 60041 hundred thousandths.
6	0	0	0	0	.	0	0	0	0	0	4				is read 60000 units, 4 millionths.
		3	0	0	.	0	0	6	2	7	8	1			is read 300 units, 62781 ten millionths.
			1	2	.	8	9	4	7	3	2	1	3		is read 12 units, 89473213 hundred millionths.
		2	0	0	.	0	0	0	0	0	0	0	0	9	is read 200 units, 9 billionths.

Obs. 5. By examining this table attentively we notice the following considerations:

1st. *Decimals decrease from the left hand towards the right in a ten fold ratio.* Thus, .4 is only 4 tenths; .04 is 4 hundredths; .004 is 4 thousandths, &c. Therefore conversely, *Decimals increase from the right hand towards the left in a tenfold ratio.* Thus, .4 is ten times larger than .04; .04 is ten times larger than .004; &c. Hence—

Obs. 6. *Every removal of a decimal figure to the right decreases, and every removal to the left increases its value ten times.*

Remark.—The pupil will perceive that decimals increase and decrease in the same manner as whole numbers.

2d. *The value of every figure, whether a decimal, or an integer is determined by its place from units.* Therefore—

Obs. 7. *a. Prefixing one cipher to a decimal decreases its value ten times; two ciphers, a hundred times, &c.* Thus, $.3 = \frac{3}{10}$; $.03 = \frac{3}{100}$; $.003 = \frac{3}{1000}$, &c. But

b. Annexing a cipher to a decimal, however, does not alter its value. Thus, $.7 = \frac{7}{10}$; $.70 = \frac{70}{100} = \frac{7}{10}$; $\frac{700}{1000} = \frac{7}{10}$, as before.—Hence—

Obs. 8. *Decimals of different denominators may be reduced to a common denominator, by annexing ciphers until the number of decimal places in each are equal.* Thus, .4, .06, .037, are equal to .400, .060, .037, &c. Also—

Obs. 9. *Whole numbers may be reduced to decimals by annexing ciphers.* Thus, $16 = \frac{160}{10}, \frac{1600}{100}$, or 160 tenths, 1600 hundredths. (Art. 3, Obs. 7.)

Remark.—When the whole number thus reduced is written without the denominator, it is best to place the separatrix before the ciphers. Thus, 16 = 16.0 = 16.00, &c.

What are Fractions? What is a Decimal Fraction? From what is the term decimal derived? How are decimal fractions generally written? What do we use in this case? Where is this point placed? Why? What is the denominator of a decimal fraction? How do we write the decimal, when the numerator does not contain as many significant figures as there are ciphers in the denominator? What is the first place at the right of the separatrix called? Why? The second place? Why? How do decimals decrease? How increase? What inference is deduced from this? What is the difference between the increase and decrease of decimals and whole numbers? How is the value of every figure determined? What effect does it have to prefix a cipher to a decimal? Two ciphers? What effect does it have to annex a cipher to a decimal? How do we reduce decimals of different denominators to a common denominator? How may whole numbers be reduced to decimals? When the whole number is written without the denominator, how do we proceed? How may whole numbers and decimals be written together? What are such expressions called?

3d. *Whole numbers and decimals may be written together by placing the seperating point between them.* Thus, 9 units, 9 millionths is written 9 .000009.

a. A whole number and decimal written together is called a MIXED NUMBER. Thus, 4 .7, 6 .08, and 4.027 are mixed numbers.

4th *The units place is at the right of whole numbers, and at the left of decimals.* Hence—

The effect of annexing or prefixing ciphers to decimals, is the reverse of annexing or prefixing them to whole numbers.

5th. *The name of the order of the right hand figure of the decimal is given to the whole.* Hence—

To read decimals:

Obs. 10. *Read as in whole numbers, and to the right hand figure add the name of its order.*

NOTE.—When *mixed numbers* are read, it is preferable to place the word *units* after the whole number, to prevent ambiguity. Thus, 400. 0016 would be read by many pupils 416 ten thousandths. But by reading it 400 units, and 16 ten thousandths, all ambiguity is removed.

REMARK 1.—Expressing decimals by words is called NUMERATION OF DECIMALS.

Read the following decimals:

1. .5.	5. 12.00021.	9. 1200.000000016.	13. 4.07.
2. .27.	6. 217.162345.	10. 78.004.	14. 8.167.
3. 1.081.	7. 400.0000096	11. 120.3.	15. 11.4092.
4. 9.3006.	8. 210.3000009.	12. 100.006.	16. 700 .0075.

REMARK 2.—Expressing decimals by figures is called NOTATION OF DECIMALS.

To write decimals:

Obs. 11. *Write each figure in the order in which it belongs, and place a cipher in all vacant orders.*

Write the fractional parts of the following numbers in decimals:

Where is the units place in whole numbers? In decimals? What is the difference in the effect of annexing and prefixing ciphers to whole numbers, and to decimals? What name is given to the decimal? How then do we read decimals? How is it best to read mixed numbers? Why so? What is numeration of decimals? What is notation of decimals? How do we write decimals?

1. $7\frac{3}{10}$. Ans. 7. 3.
2. $27\frac{18}{100}$.
3. $14\frac{9}{100}$. Ans. 14. 09.
4. $130\frac{28}{1000}$.
5. $796\frac{12}{10000}$.
6. $14\frac{9}{10000}$.
7. $16\frac{1278}{10000000}$.
8. $77\frac{10008}{100000000}$.
9. $1008\frac{4}{10000000000}$.
10. $49\frac{16}{10000}$.
11. $132\frac{17}{1000}$.
12. $169\frac{18}{100}$.
13. $463\frac{333}{10000}$.
14. $8\frac{1}{10}$.
15. $1\frac{1}{100}$.
16. $400\frac{36}{1000}$.
17. $78\frac{9}{10}$.
18. $200\frac{9}{10000}$.
19. $60\frac{19}{1000}$.
20. $1476\frac{2}{10000000}$.

Write the following expr ssions in decimals:

21. 7 tenths
22. 4 hundreths.
23. 14 thousandths.
24. 6 millionths.
25. 18 hundredths.
26. 36 thousandths.
27. 9 tenths.
28. 112 ten thousandths.
29. 4002 ten millionths.
30. 3 units, 4 thousandths.
31. 7 units, 12 billionths.
32. 14 units, 2 thousandths.
33. 612 units, 25 hundredths.
34. 406 units, 406 thousanths.
35. 400 units, 6 thousandths.
36. 1000 units, 1 millionth.
37. 79 units,2001 ten thousandths
38. 976 ten billionths.
39. 1001 hundred trillionths.
40. 100 units, 111 milllonths.

Note.—The reading and writing of decimals is of great importance, and the pupil should be exercised at it until it is perfectly familiar to him.

Art. 8. Federal Money.

Obs. 1. *Federal Money is the national currency of the* United States, *as established* by Congress, *August* 8th, 1776. The denominations are Mill, Cent, Dime, Dollar and Eagle.

TABLE.

10 mills (*m.*)	make	1 cent	marked *ct.*
10 cents	"	1 dime	" *d.*
10 dimes	"	1 dollar	" *doll.* or $
10 dollars	"	1 eagle	" *E.*

Note.—This character ($) may be regarded as a contraction of U. S., and signifies United States money.

1st. The *Eagle* is a gold coin, and contains 10 pennyweights, 18

What is Federal Money? What are the denominations? Repeat the table? What are the national coin of the United States?

grains = 258 grains of standard gold.* Besides the eagle, we have the *half-eagle* and *quarter-eagle*, which are gold coins, and their value and weight accordingly.

2d. The *Dollar* is a silver coin, and contains 17 pennyweights, $4\frac{1}{2}$ grains = $412\frac{1}{2}$ grains of standard silver.† Besides the dollar there are *half-dollars, quarter-dollars, dimes,* and *half-dimes,* which are silver coins, and their value and weight accordingly.

3d. The *Cent* is a copper coin, and contains 7 pennyweights = 168 grains of pure copper.‡ The *half-cent* accordingly.

Mills are only imaginary, and are not coined.

Obs. 3. Pure gold is supposed to be divided into 24 equal parts called *carats*, and its fineness depends on the number of parts of some baser metal called *alloy* that it contains. Thus, if it contains 20 parts pure gold, and 4 parts alloy, it is said to be 20 carats fine; if it contains 6 parts of alloy, it is said to be 18 carats fine.

Obs. 4. Previous to 1837, the standard for gold was 22 parts of pure metal, to 2 parts of alloy, and it was said to be 22 carats fine. Now it is $21\frac{3}{5}$ carats fine.

Obs. 5. By Act of Congress, 1837, the legal standard for gold and silver coins in the UNITED STATES, *is* 900 *parts of pure metal, by weight, to* 100 *parts alloy.* The alloy of gold coin is composed of *silver* and *copper,* the silver not to exceed copper in weight. The alloy of silver coins is pure copper.

Obs. 6. *Accounts in the* UNITED STATES *are usually kept in dollars, cents, and mills;* eagles being expresssed as dollars, and dimes as cents. Thus, instead of saying 2 eagles, 5 dollars, we say 25 dollars; and instead of saying 7 dimes, 5 cents, we say 75 cents, &c. Five mills are often called $\frac{1}{2}$ a cent; thus 12 cents, 5 mills, are generally called $12\frac{1}{2}$ cents.

REMARK.—It will be perceived from the table that the denominations of Federal Money increase and decrease in a *ten fold ratio,* in the same manner as

---

What is the eagle? Its weight? What other gold coins have we? What is the dollar? Its weight? What other silver coins have we? What is the cent? Its weight? What are mills? How is pure gold divided? Upon what does its fineness depend? If it contains 4 parts alloy, how fine is it? What was the standard for gold previous to 1837? What is it now? What is the legal standard for gold and silver coins in the United States, by Act of Congress of 1837? Of what is the alloy of gold coins composed? Of silver coins? How are accounts in the United States generally kept? How are eagles expressed?—Dimes? What do we say instead of 2 eagles, 5 dollars? 7 dimes, and 5 cents? What are 5 mills often called? How is 12 cents, 5 mills, generally read? How do the denominations of Federal Money increase and decrease?

* Eagles coined before July 31st, 1834, contain 11 pennyweights, 6 grains = 270 grains of standard gold. These are worth $10.665. Half and quarter eagles accordingly.

† The dollar originally contained 17 pennyweights, 8 grains = 416 grains of standard silver.

‡ The cent originally contained 264 grains of pure copper.

whole numbers. Hence—the dollar being regarded as the unit, and the cents and mills as the fractional parts of the unit or dollar, it follows that

Obs. 7. *All operations in Federal Money can be performed precisely as in decimal fractions, the dollar being regarded as the unit, cents as tenths and hundredths,* (because 100 cents make a dollar) *and the mills as thousandths; the separating point being placed between the dollars and cents.*

Obs. 8. As cents are so many hundredths of a dollar, (it taking 100 to make a dollar,) it follows that *they must occupy the first two places at the right of dollars*, and if the cents are less than 10, *a cipher must be placed at the left, or in the tenth place.* Thus, 18 dollars, 5 cents is written $18 .05.

Likewise, as mills are so many thousandths of a dollar, (it taking 1000 to make a dollar,) *they must occupy the third place at the right of dollars, and if no cents are given, the place of cents must be supplied with ciphers.*

Thus, 9 cts. is written .09; 18 dolls. 6 cts. 7 m., is written $18 .067; 15 dolls. 37 cts. 5 m., is written $15 .375, or $15 .37½; 6 dolls. 9 m., is written $6 .009: and 7 m., is written $0 .007; &c.

Remark.—Business men often write cents as the fractional parts of a dollar. Thus, they write $10 . 46, 10\frac{46}{100}$. &c.

To read any sum in Federal Money:

Obs. 9. *Call the figures at the left of the separating point dollars, the first two places at the right cents, and the third place at the righ mills; the other places at the right are decimals of a mill.*

Thus, $56 .27516 is read 56 dolls, 27 cts, 5 m., and 16 hundredths of a mill.

Note.—The decimals at the right of mills are seldom counted, the mills being sufficiently exact for all business calculations. Some however, reckon in this way—if the mills exceed 5, they count another cent; if they are less than 5 they reject them entirely; 5 m. they count ½ a cent.

How can operations in Federal Money be performed? What is the dollar regarded? The cents? Why? The mills? Where is the separating point placed? By what are the first two places at the right of the separating point occupied? If the cents are less than 10 how do we proceed? What place do mills occupy? If no cents are given what must be done? How do business men often write cents? How do we read any sum in Federal Money? Why are not the decimals at the right of mills generally reckoned? How do some reckon mills?

Read the following sums in Federal Money :

1. $0.06.
2. $1.703.
3. $17.6954.
4. $243.009.
5. $78.805.
6. $204.73021.
7. 14.12\frac{1}{2}$.
8. $100.50.
9. $1086.375.
10. $12.1875.
11. $70.005.
12. $32671.09999.
13. $14.007.
14. $0.001.
15. $0.158.
16. $12.70.
17. 9.37\frac{1}{2}$.
18. 14.18\frac{3}{4}$.
19. 0.56\frac{1}{4}$.
20. $100.001.

Obs. 10. We write sums in Federal Money according to (Art. 7 Obs. 11.) dollars occupying the place of *whole numbers*, cents the place of *tenths* and *hundredths*, and mills the place of *thousandths*.

Thus, $18.37 cts., 5 m., is written $18.375, &c.

Write the following sums in Federal Money :

1. 27 dolls. 49 cts. 7 m.
2. 18 dolls. 18 cts. 8 m.
3. 20 dolls. 27 cts. 6 m.
4. 97 cts. 4 m.; 68 cts.
5. 78 cts. 9 m.; 47 cts. 1 m.
6. 1007 dolls. 87 cts. 6 m.
7. 947 dolls. 69 cts. 3 m.
8. 7 dolls. 5 cts. 9 m.
9. 18 dolls. 6 cts. 3 m.
10. 19 dolls. 9 cts. 1 m.
11. 34 dolls. 7 cts. 8 m.
12. 19 dolls. 2 cts. 3 m.
13. 10 dolls. 9 m.; 4 cts.
14. 74 dolls. 3 cts. 7 m.
15. 143 dolls. 9 m.
16. 9 m. and 12 hundredths of a mill.
17. 1 doll. 1 m.; 3 m.
18. 3 cts. 2 m.; 7 m.
19. 129 dolls. 8 cts. 1 m.
20. 6 m.; 7 m.; 4 m.
21. 8 m.; 1 ct. 1 m.
22. 200 dolls. 14 cts. 2 m.
23. 93$\frac{3}{4}$ cts.; 4 m.
24. 75 cts. 1 m.
25. 1 doll. 1 ct. 1 m.
26. 20 dolls. 2 m.
27. 79 dolls. 4 m.
28. 1 m.; 9 m. 5 tenths of a mill.
29. 25 hundredths of a mill.
30. 100 dolls. 1 ct. 75 hundredths of a mill.

ARTICLE. 9. REDUCTION OF FEDERAL MONEY AND DECIMAL FRACTIONS.

Obs. 1. As 10 mills make a cent, and 100 cents make a dollar, it follows that

a. Dollars are reduced to cents by annexing two ciphers, and to mills by annexing three ciphers. In either case we remove the sign of dollars ($).

How do we write sums in Federal Money? How are dollars reduced to cents?

Dollars and cents are reduced to cents, and dollars, cents, and mills are reduced to mills by erasing the separating point, and the sign of dollars.

b. Cents are reduced to dollars by pointing off two figures at the right and prefixing the sign of dollars. Cents are reduced to mills by annexing a cipher.

c. Mills are reduced to cents by pointing off three figures at the right and prefixing the sign of dollars.

EXERCISES FOR THE SLATE.

1. Reduce \$17 to cents. Ans. 1700 cts.
2. Reduce \$34 to mills.
3. Reduce \$18.73 to cents.
4. Reduce \$25.645 to mills.
5. Reduce \$1.01 to mills.
6. Reduce \$4.20 to cents.
7. Reduce \$27.06 to mills.
8. Reduce \$0.479 to mills.
9. Reduce \$12.06 to cents.
10. Reduce 478 cents to dollars.
11. Reduce 164 cents to mills.
12. Reduce 2080 cents to dollars.
13. Reduce 14000 cents to mills.
14. Reduce 120 mills to cents.
15. Reduce 14000 mills to dollars.
16. Reduce 1785 mills to cents.
17. Reduce 800 cents to dollars.
18. Reduce 1768 mills to dollars.
19. Reduce 12435 mills to cents and dollars.

Reduction of Decimal Fractions.

Obs. 2. The learner will remember that *in all cases of Reduction, whether of Common or Decimal Fractions, the value of the given number must not be altered.* (Art. 3, Obs. 1. and Art. 6, Case 5, Rem.)

Case 1. *To change a Decimal to a Common Fraction.*

Remark.—As we have said before, the denominator of a decimal fraction is 1 with as many ciphers annexed as there are places of decimals in the numerator. (Art. 7, Obs. 2.)

Hence—To change a decimal to a common fraction :

To mills? What is necessary in these cases? How are dollars and cents reduced to cents? How are dollars, cents, and mills reduced to mills? How are cents reduced to dollars? To mills? How are mills reduced to cents? To dollars? What must be observed in all cases of reduction? What is the denominator of a decimal fraction? How then do we change a decimal to a common fraction?

Obs. 3. *Erase the decimal point, and supply the denominator of the decimal:*

NOTE.—It is generally customary, after the decimal is reduced to a common fraction, to reduce it to its lowest terms. (Art. 3, Obs. 3.) This is the case with the following examples.

Ex. 1. Change .25 to a common fraction. Ans. $\frac{25}{100}=\frac{1}{4}$.

2. Change .125 to a common fraction. Ans. $\frac{1}{8}$.

3. Change .0625 to a common fraction. Ans. $\frac{1}{16}$.

4. Change .7425 to a common fraction. Ans. $\frac{297}{400}$.

5. Change .75 to a common fraction. Ans. $\frac{3}{4}$.

6. Change .875 to a common fraction. Ans. $\frac{7}{8}$.

7. Change .5625 to a common fraction. Ans. $\frac{9}{16}$.

8. Change .9375 to a common fraction. Ans. $\frac{15}{16}$.

9. Change .333 to a common fraction. Ans. $\frac{333}{1000}$.

10. Change .99 to a common fraction. Ans. $\frac{99}{100}$.

11. Change .1246 to a common fraction. Ans. $\frac{623}{5000}$.

12. Change .03125 to a common fraction. Ans. $\frac{1}{32}$.

CASE 2. *To change a Common Fraction to a Decimal.*

NOTE.—This case is exactly the reverse of the preceding one, and each proves the other.

Ex. 1. Change $\frac{2}{5}$ to a decimal.

Operation.

5)2.0

Ans. .4

Annexing a cipher to our numerator, we make it 20 tenths: 20 tenths divided by 5 (the denominator) equals 4 tenths.

The correctness of this operation may be shown as follows:

Annexing a cipher to the numerator, is, in reality multiplying it by 10; (Sect. IV, Art. 4, Case 1, Rem.) but multiplying the numerator is multiplying the value of the fraction; (Art. 2, Obs. 1.) hence, the quotient (after dividing the numerator by the denominator,) is 10 times too large, and we must therefore divide it by 10 to obtain a correct result. This is done by simply pointing off a figure at the right, or placing the decimal point before it. (Sect. V, Art. 4, Obs. 1.)

By the same course of reasoning, it follows that if we annex two

What relation does Case 1, and Case 2 bear to each other? Explain why the operation of Ex. 1 is correct. If we annex two ciphers to the numerator, what effect does it have on the quotient? If we annex three ciphers, what is the effect? Why so?

ciphers to the numerator we multiply it by 100, and must point off two decimals in the result; if we annex three ciphers to the numerator, we multiply it by 1000, and must point off three decimals in the result, &c. Hence—

To change a common fraction to a decimal:

Obs. 4. *Annex ciphers to the numerator, and divide by the denominator, pointing off a decimal in the quotient for every cipher annexed; and if there is not a sufficient number of figures in the quotient, supply the deficiency by prefixing ciphers.*

a. From this and the last case, the learner can often form rules for contracting the operation in multiplication and division. Thus, to multiply by .25, he can divide by 4, because $.25 = \frac{1}{4}$; to multiply by .125, divide by 8, because $.125 = \frac{1}{8}$; &c.; and conversely to divide by .25, multiply by 4, and to divide by .125, multiply by 8, &c., for the same reason.

Again, to multiply by .875, we multiply by 7, and divide by 8, be cause $.875 = \frac{7}{8}$, &c.; and conversely, to divide by .875 we multiply by 8, and divide by 7 for the same reason.

But when our multiplier is a whole number (and we contract the operation,) we must annex to the product 1, 2, 3, &c. ciphers, because our multiplier is a part of 10, 100, 1000, &c.; and conversely when our divisor is integral, we must point off from the right 1, 2, 3, &c., figures for the same reason. The figures cut off however, must be divided by our multiplier, because these are the remainder, and are as many times too large as there are units in this multiplier. (Sect. VI, Art 2, Rem. under Ex. 22 and 23.)

We can also abridge the operation when the multiplier or divisor is partly integral, and partly decimal; thus to multiply or divide by 1.25, we proceed exactly as if it were 125. In such cases we only annex as many ciphers, or point off as many decimals as there are integral numbers in the multiplier or divisor. When there are decimals in the dividend or multiplicand, we may proceed with them according to the rules in multiplication and division of decimals.

It may be proper to remark that the subject of abbreviations can only be useful to a person well acquainted with the properties and

---

How then do we change a common fraction to decimal? To what particular object can these two cases be applied? Give an example or two illustrating this point? When our multiplier is a whole number, how do we proceed? Why? When the divisor is integral, how do we proceed? Why? What must be done with the figures cut off? Why? When the multiplier or divisor is a mixed number, how do we proceed? What must be observed in such cases? When there are decimals in the dividend or multiplicand, how do we proceed? What is necessary in order to make the subject of abbreviations useful to a person?

relations of numbers,* and with such a few hints are generally sufficient. We will therefore only present the two following propositions, which are so evident that we omit the demonstration :

1. *If we wish to divide one number by another, it will produce the same effect to multiply the dividend by the reciprocal of the divisor.*— And

2. *If we wish to multiply one number by another, we can obtain the same result by dividing the multiplicand by the reciprocal of the multiplier.*

2. Change $\frac{3}{4}$ to decimal, Ans. .75
3. Change $\frac{3}{5}$ to a decimal. Ans. .6.
4. Change $\frac{1}{20}$ to a decimal, Ans. .05.
5. Change $\frac{6}{125}$ to a decimal. Ans. .048.
6. Change $\frac{3}{8}$ to a decimal. Ans. .375.
7. Change $\frac{5}{4}$ to a decimal. Ans. 1.25.
8. Change $\frac{9}{25}$ to a decimal. Ans. .36.
9. Change $\frac{5}{625}$ to a decimal. Ans. .008.
10. Change $\frac{7}{1000}$ to a decimal. Ans. .007.
11. Change $\frac{2}{6}$ to a decimal. Ans. .333333 +.
12. Change $\frac{52}{111}$ to a decimal. Ans. .468468 +.

In the last two examples, it will be perceived that the decimal will be continually repeated, the same remainder occuring after a certain number of divisions.

Obs. 5. *Decimals which consist of the same figure or set of figures continually repeated, are called* REPEATING DECIMALS, CIRCULATING DECIMALS, or REPETENDS.

ARTICLE 10. ADDITION AND SUBTRACTION OF DECIMALS AND FEDERAL MONEY.

Obs. 1. *Decimal Fractions can be added, subtracted, multiplied, or divided, the same as whole numbers.* The only difficulty is in knowing where to place the *separating point.*

Obs. 2. As *Federal Money* is based upon the *decimal* notation, it is evident that *it is subject to the same laws as decimal fractions, and therefore the same rules are applicable to both.*

Ex. 1. A man has several accounts to collect; the first is $12; the

If we wish to divide one number by another, by what other way can we obtain the same result? If we wish to multiply one number by another, by what other way can we obtain the same result? Demonstrate these propositions. What are circulating decimals or repetends What is the only difficulty in the addition, &c., of decimals? Upon what is Federal Money based? What is the inference deduced from this?

* The preceding rules and principles explain all that is necessary to understand this point.

second, \$9,375; the third, \$6,08; and the fourth, \$5,403. Required—the amount of the whole.

Operation.

In this example we place dollars under dollars, cents under cents, and mills under mills, and add in every respect as in Addition of Simple Numbers. (Sect. II. General Rule.)

$$\begin{array}{r} \$12.000 \\ 9.375 \\ 6.080 \\ 5.403 \\ \hline \text{Ans. } \$32.858 \end{array}$$

2. A man has in one bag 2.3 bushels of oats, in another 3.161 bushels, in another 1.34 bushels, and in a box 14.132 bushels. How much has he in all?

Operation.

In this example, we place whole numbers under whole numbers, tenths under tenths, hundredths under hundredths, &c., and add as usual.

$$\begin{array}{r} 2.300 \\ 3.161 \\ 1.340 \\ 14.132 \\ \hline \text{Ans. } 20.933 \text{ bushels.} \end{array}$$

NOTE—In both these examples, it will be perceived that we reduced all the decimals to the same denominations by annexing ciphers. This however, is not absolutely necessary if the pupil places all the figures of the same order directly under each other.

That these operations are correct, may be shown as follows:

3. Add together 5.846 and 7.9.

Operation.

$$\begin{array}{r} 5.846 \\ 7.9 \\ \hline \text{Ans. } 13.746 \end{array}$$

Or, $\left\{\begin{array}{l} 5.846 = 5\frac{846}{1000} \\ 7.9 = 7\frac{9}{10}. \end{array}\right.$ $\frac{9}{10} = \frac{900}{1000}$; $\frac{900}{1000} + \frac{846}{1000} = \frac{1746}{1000} = 1\frac{746}{1000}$. $7 + 5 = 12$, and $1\frac{746}{1000} = 13\frac{746}{1000}$. But $\frac{746}{1000} = .746$. Hence—$13\frac{746}{1000} = 13.746$, as before.

The same reasoning will apply to Federal Money, or to any number of decimals.

4. A man having \$18.25, spent \$9.625. How much had he left?

Operation.

In this example, we write dollars under dollars, cents under cents, and mills under mills, and subtract precisely as in Subtraction of Simple Numbers. (Sect. III. General Rule.)

$$\begin{array}{r} \$18.250 \\ 9.625 \\ \hline \text{Ans. } 8.625 \end{array}$$

Explain why the operations given are correct.

NOTE.—As there are no mills in the minuend, we write a cipher inthe mills' place; but this is not necsssary, as we always proceed as if the cipher was there, whether we express it or not.

5. A man having 18.4 yards of cloth, sold 12.694 yards. How much had he left?

Operation.

In this example, we write whole numbers under whole numbers, tenths under tenths, &c., and subtract as usual.

$$\begin{array}{r} 18.4000 \\ 12.694 \\ \hline \text{Ans. } 5.706 \text{ y'ds.} \end{array}$$

It will be perceived that we annex ciphers to the minuend until the number of decimal places are equal to those in the subtrahend.

The same example may also be solved thus:

$18.4 = 18\frac{4}{10}$. $18\frac{4}{10} = 18\frac{400}{1000} = 17\frac{1400}{1000}$. $17\frac{1400}{1000} - 12\frac{694}{1000} =$
$12.694 = 12\frac{694}{1000}$. $5\frac{607}{1000} = 5.706$.

This shows the operation to be correct. The same method of reasoning will apply to either Decimals or Federal Money.

Obs. 3. From the preceding operations, we notice two considerations:

1st. *The separating points, both in the given numbers and in the result, all fall directly under each other.*

2nd. *In Subtraction, that number is made the minuend, in which the whole number is the largest, without reference to the Decimals, because the Decimal is only a part of unity, and therefore merely a fraction.* Hence—

To add or subtract Decimals or Federal Money, we have this

RULE.—I. *Write the numbers so that the same orders, and also the separating points, shall all fall directly under each other.* (Sect. II. Art. 2. Obs. 4.)

II. *Then add or subtract as in Simple Numbers.* (Sects. II. and III., General Rules)

III. *Place the separating point in the result directly under the other points.*

NOTE.—In subtraction, of course we place the greater number at the top.

PROOF.—The same as in Simple Numbers. (Sect. II. Art. 2. Obs. 6. and Sect. III. Art. 2. Obs. 7.)

Where do the separating points fall in these examples? Which number do we make the minuend in subtracting? Why? What is the rule for adding or subtracting Decimals or Federal Money? What is the proof?

EXERCISES FOR THE SLATE.

1. A. has $18, B. has $14.06, C has $20.3275, D has $6.005, and E. has $12.3125. How much have they all?
Ans. $70.705.

2. How much has A. more than B. in the last example?
Ans. $3.94.

3. In the same example, how much has A. B. and C. more than D and E.? Ans. 34.07.

4. A farmer has grain as follows: of wheat 123.0993 bushels; of oats, 109.6 bushels; of corn. 83.97 bushels; of rye, 75.004 bushels; and of barley, 58.3267 bushels. How much grain has he?
Ans. 450 bushels.

5. How much more wheat has he than corn?
Ans. 39.1293 bushels.

6. How much more wheat and oats has he than all other kinds grain? Ans. 15.398 bushels.

7. A gentleman bought a coat for $15.75; a hat for $5.375; a pair of boots for $5.06; and an overcoat for 23.875. How much did he pay for the whole? Ans. $50.06.

8. How much did the coat cost more than the hat?
Ans. $10.375.

9. How much did the coat, hat, and boots cost, more than the overcoat? Ans. $2.31.

10. A merchant sold several pieces of cloth, as follows: the first piece contained 16.8 yards; the second, 19.006 yards; the third, 22.14 yards; and the fourth, 22.054 yards. How much was there in all? Ans. 80 yards.

11. How much did the third piece contain more than the fourth?
Ans. .086 yards.

12. How much did the second and third pieces contain, more than the first and fourth? Ans. 2.292 yards.

13. A merchant has due to him the following sums: 127 dollars, 7 cents; 96 dollars, $6\frac{1}{4}$ cents; 47 dollars, 5 mills; 57 dollars, $8\frac{1}{2}$ cents; 19 dollars, $18\frac{3}{4}$ cents; and 25 dollars. How much has he owing him in all? Ans. $371.41.

14. From $10 take $\frac{1}{2}$ a cent? Ans. 9.995.

15. From 81 take 2 thousandths. Ans. 80.998.

16. Add together 10 units, 3 hundredths; 5 ten thousandths; 123456 millionths; 762 units, 397844 millionths, and 12 units, 8972 ten thousandths. Ans. 785.449.

17. From 18 units take 4 billionths. Ans. 17999999996.

18. From 18 dollars, 75 cents, take $88\frac{3}{4}$ cents.
Ans. 17.86\frac{1}{4}$.

19. From 5 dollars take 3 mills. Ans. $4.997.

20. Add together 5 eagles, 5 dimes; 6 eagles, 3 dollars; 12

cents; 2 dollars, 4 mills; 218 mills; 57689 mills, and 15 eagles.
Ans. \$323.531.

21. From 9 eagles take 9 dimes. Ans. \$89.10.

ARTICLE 11. MULTIPLICATION OF DECIMALS, OR FEDERAL MONEY.

Ex. 1. Multiply 9.25 by 6.

Operation.

9.25	
6	
Ans. 55.50	

In this example, we multiply as usual, and point off two decimals in the product, because there are two decimals in the multiplicand. The correctness of this example may be shown by the following

Illustration.—$9.25 = 9\frac{25}{100} = \frac{925}{100}$; $\frac{925}{100} \times 6 = \frac{5550}{100} = 55\frac{50}{100} = 55.50$.

2. A gentleman bought 12 yards of cloth, at \$2.375 a yard. How much was the cost of the whole?

Operation.

2.375	
12	
Ans. \$28.500	

In this example, we multiply and point off exactly as in the first example. The correctness of this operation may be shown in the same manner as in the last.

3. Multiply 7.42 by 8.35.

Operation.

7.42	
8.35	
37 10	
222 6	
5936	
Ans. 61.9570	

In this example, we point off four decimals in the product, because there are two decimals in the multiplicand, and two in the multiplier, making 4 in both.

We prove this to be correct, thus: $7.42 = 7\frac{42}{100} = \frac{742}{100}$; $8.35 = 8\frac{35}{100} = \frac{835}{100}$; $\frac{742}{100} \times \frac{835}{100} = \frac{619570}{10000} = 61\frac{9570}{10000} = 61.9570$, as before.

4. Multiply .04 by .07.

Operation.

.04	
.07	
Ans. .0028	

In this example, there are but two figures in the product, therefore we prefix ciphers to obtain the requisite number of decimals. The correctness of this operation may be shown thus: $.04 = \frac{4}{100}$; $.07 = \frac{7}{100}$; (Art. 1. Obs. 2.) $\frac{4}{100} \times \frac{7}{100} = \frac{28}{10000} = .0028$, as before. (Art. 1. Obs. 3.)

This illustration shows why the ciphers should be prefixed, rather than annexed, to the product.

From these examples and illustrations, we derive the following

RULE FOR THE MULTIPLICATION OF DECIMALS, OR FEDERAL MONEY

I. *Write the numbers, and multiply as in Simple Numbers.* (Sect. IV., General Rule.)

II. *Point off as many decimals in the product, as there are decimals in both the multiplicand and multiplier.*

III. *If there are not as many decimals in the product as in both factors, prefix ciphers to it until the requisite number of decimals are obtained.*

PROOF.—*The same as in Simple Numbers.* (Sect. IV. Art. 2. Obs. 7.)

1. When the multiplier is a composite number, we may often contract the operation, the same as in Simple Numbers.

2. To multiply by 10, 100, 1000, &c., *we need only remove the separatrix* 1, 2, 3, *&c., places to the right, annexing ciphers if necessary.*

Thus: $1.6 \times 1000 = 1600.0$, or simply 1600.

EXERCISES FOR THE SLATE.

1. How much would 14.5 yards of cloth cost, at \$1.375 per yard? Ans. \$19.9375.

2. A farmer has 12 bags, each containing 2.75 bushels of wheat. How much has he in all? Ans. 33 bushels

3. If he sells it at \$1.173 per bushel, how much will it come to? Ans. \$38.709.

4. How much would 112 pounds of coffee cost, at 0.117 per pound? Ans. \$13.104.

5. How much would 216 pounds of sugar cost, at \$0.071 per pound? Ans. \$15.336.

6. How much grain in 16 boxes, each containing 77.93 bushels? Ans. 1246.88 bushels.

7. How many yards of cloth in 27 pieces, each containing 32.78 yards? Ans. 885.06.

8. How much would 1000 pounds of sugar cost, at \$0.07 per pound? Ans. \$70.

9. Multiply .0763 by 2.16. Ans. .164808.

10. Multiply 2.97 by .0042. Ans. .012474.

11. Multiply 12.62 by $81\frac{3}{4}$. Ans. $1031.68\frac{1}{2}$.

12. Multiply .276 by .000437. Ans. .000120612.

What is the rule for Multiplication of Decimals or Federal Money? Explain, by Ex. 1, 2, 3, and 4, why this rule is correct. How do we prove the operation? When the multiplier is a composite number, how may we proceed? How may we multiply by 10, 100, 1000, &c.?

13. How much would 97 pounds of pork cost, at \$3.50 per hundred? Ans. \$3.395.

Operation.

3.50
97
2450
3150
\$339.50

The result obtained is evidently the cost of 9700 pounds, (because the price is so much per hundred,) whereas we only wish the cost of 97 pounds. Now 97 is $\frac{1}{100}$ of 9700, therefore the cost of 97 pounds is but $\frac{1}{100}$ the cost of 9700 pounds. Then \$339.50 $\div$ 100 = \$3.3950. It will be perceived that the latter result is obtained by removing the separating point two additional places to the left.

The correctness of this last point may be shown thus: $339\cdot50 = 339\frac{50}{100} = \frac{33950}{100}$. $\frac{33950}{100} \div 100 = \frac{33950}{10000}$ (Art. 6. Obs. 2. *a.*) $\frac{33950}{10000} = 3\frac{3950}{10000} = 3.3950$, as before.

14. How much would 794 feet of boards cost, at \$5.875 per thousand? Ans. 4.664+.

Operation.

\$5.875
794
23 500
528 75
4112 5
\$4664.750

The result obtained is evidently the cost of 794000 feet, (as the price is so much per thousand,) whereas we only wish the cost of 794 feet. Now 794 feet is $\frac{1}{1000}$ of 794000, and therefore the cost of 794 feet is $\frac{1}{1000}$ of the cost of 794000. Then \$4664.750 $\div$ 1000 = \$4.664750. It will be perceived that the latter result is obtained by removing the separating point three additional places to the left.

Illustration.—$4664.750 = 4664\frac{750}{1000} = \frac{4664750}{1000}$; $\frac{4664750}{1000} \div 1000 = \frac{4664750}{1000000}$. (Art. 6. Obs. 2. *a.*) $\frac{4664750}{1000000} = 4\frac{664750}{1000000} = 4.664750$, as before.

REMARK—The sign of Addition placed at the right of an answer, signifies that the answer is not complete, or, that there is a remainder.

Hence—When buying or selling articles by the 100:

Obs. 1. *Point off two additional places at the right of the product.*

When buying or selling articles by the 1000:

Point off three additional places at the right of the product.

REMARK 1.—C. stands for 100, and M. stands for 1000; from two Latin words, *Centum* and *Mille*, which signify *hundred* and *thousand*.

2.—The learner will remember that in all cases of multiplication, whether of Simple, Compound, or Decimal numbers, the multiplier is to be considered an abstract number. (Sect. IV. Art. 2. Obs. 6. Rem. 1.)

How is the final result obtained in Exs. 13 and 14? Show why this is correct? What does the sign of addition, placed after an answer, signify? How do we proceed in buying and selling articles by the 100? By the 1000? For what does C. stand? M.? From what? What must be observed in all cases of Multiplication, whether of Simple, Compound, or Decimal Numbers?

15. How much would 18275 brick cost, at $4.37\frac{1}{2}$ per M.? Ans. $79.953+.

16. How much would 276 bunches of shingles cost, each bunch containing 1242, at $3.12\frac{1}{2}$ per M.? Ans. $1071.225.

17. How much would 4723 pounds of beef cost, at $3.75 per C.? $Ans. 177.1125.

18. How much would 1623 feet of lumber cost, at $4.62\frac{1}{2}$ per C.? Ans. $750.683+.

19. How much would 5273 pounds of pork cost, at $4.25 per C.? Ans. $224.1025.

20. How much would 2314 feet of mahogany cost, at $64.25 per M.? Ans. $148.6745.

Article 12. Division of Decimals, or Federal Money.

Ex. 1. Divide .84 by .7.

Operation.

.7).84
——
Ans. 1.2

Obs. 1. As we have said before, the dividend in Division answers to the product in Multiplication; (Sect. V. Art. 2. Obs. 7.) and as we point off as many decimals in the product as there are decimals in both factors, it follows that *the quotient in Division of Decimals must always contain as many decimal places, as the decimals in the dividend exceed those in the divisor;* the divisor and quotient being the factors which, multiplied together, produce the dividend. Thus, in the last example, the dividend has *two* decimal places, and the divisor but *one;* therefore we point off one decimal place in the quotient.

This point may also be illustrated as follows:

$.84 = \frac{84}{100}$; $.7 = \frac{7}{10}$; $\frac{84}{100} \div \frac{7}{10} = \frac{84}{100} \times \frac{10}{7} = \frac{12}{10} = 1\frac{2}{10} =$ 1.2, as before. (Art. 6. Obs. 7.)

The example may be proved thus:

quotient. 1.2
divisor. .7
——
divid'nd .84

2. Divide .1476 by 1.8. Ans. .082.

Operation.

1.8).1476).082
144
——
36
36
——
00

We perform this operation by Long Division. As the quotient does not contain as many significant figures, as the decimals in the dividend exceed those in the divisor, we supply the deficiency by prefixing ciphers.

Analytic Illustration.—$.1476 = \frac{1476}{10000}$; $1.8 = 1\frac{8}{10} = \frac{18}{10}$; $\frac{1476}{10000} \div \frac{18}{10} = \frac{1476}{10000} \times \frac{10}{18} = \frac{82}{1000} =$ .082. (Art. 7. Obs. 3.) This illustration shows why we prefix, rather than annex ciphers to the quotient.

How many decimal places should the quotient, in Division of Decimals contain? Why so?

3. Divide 2970 by .12.

Operation.

.12)2970.00

Ans. 24750

In this example, we annex two ciphers to the dividend to make the number of decimals equal to those in the divisor; and on account of this equality, our quotient is a whole number.

Illustration.—$.12 = \frac{12}{100}$; $2970 \div \frac{12}{100} = \frac{2970}{1} \times \frac{100}{12} = \frac{297000}{12}$, $= 24750$, as before.

4. A man bought 9 pair of boots for \$42.75. How much was that apiece?

Operation.

9)\$42.75

\$4.75

We proceed in Federal Money, in all cases, exactly as in Decimal Fractions. (Art. 10. Obs. 2.) Hence—

To perform operations in Division of Decimals, or Federal Money, we have this

RULE.—I. *Write the numbers, and divide as in Simple Numbers.* (Sect. V. General Rule.)

II. *Point off as many decimal places in the quotient, as the number of decimals in the dividend exceed those in the divisor.* (Obs. 1.)

III. *If there is not a sufficient number of significant figures in the quotient to point off for decimals, supply the deficiency by prefixing ciphers.* (Ex. 2.)

Proof.—The same as in Simple Numbers.

REMARK 1.—When the number of the decimals in the dividend is equal to those in the divisor, the result is a *whole number*. (Ex. 3.)

2.—When the number of decimal places in the dividend is not equal to those in the divisor, *annex ciphers to the right of the dividend until they are equal.* (Ex. 3) The result in this case is also a whole number.

NOTE.—In both these cases, the divisor is supposed to be less than the dividend, and also that there is no remainder.

3.—If there is a remainder after the division has been performed, ciphers may be annexed to the remainder, and the division continued; but *after the number of decimal places in the dividend are equal to those in the divisor, each cipher annexed produces an additional decimal in the quotient.*

This is evident from Art. 1, Obs. 7 and 9. For 1.4 = 1.40 = 1.400 = 1.4000, &c., and thus we may increase the decimals in the dividend without altering the decimals in the divisor, and in this way increase the decimals in the quotient.

What is the rule for Division of Decimals, or Federal Money?—What is the method of Proof? When the number of decimals in the dividend are equal to those in the divisor, what is the result? When the number of decimals in the dividend is not equal to those in the divisor, how do we proceed? What is the result in this case? If there is a remainder after the division has been performed, how may we proceed? What must be observed in this case? Explain why this is correct.

NOTE.—In common business calculations, it will be sufficiently exact to extend the quotient to three or four places of decimals; but when great accuracy is required, it should be extended farther.

4.—When the divisor is a composite number, we may often shorten the operation, as in whole numbers. (Sect. V. Art. 4. Obs. 3.)

5.—To divide by 10, 100, 1000, &c., we need only *remove the separating point as many places to the left, as there are ciphers in the divisor; and if there is not a sufficiency of figures, supply the deficiency by prefixing ciphers.*

Illustration.—Divide 1 by 1000. $.1 = \frac{1}{10}$; $\frac{1}{10} \div 1000 = \frac{1}{10000}$ (Art. 6. Obs. 2. *a.*) $\frac{1}{10000} = .0001$. (Art. 7. Obs. 3.)

EXERCISES FOR THE SLATE.

1. If 6 pounds of tea cost \$4.50, how much is that per pound? Ans. \$0.75.
2. If 12 pounds of coffee cost \$1.50, how much is that per pound? Ans. \$0.12½.
3. If 18 yards of cloth cost \$27, how much is that per yard? Ans. \$1.50.
4. If 26 acres of land cost \$232.44, how much is that per acre? Ans. \$8.94.
5. If 150 pounds of butter cost \$24, how much is that per pound? Ans. \$0.16.
6. If 124 bushels of wheat cost \$139.50, how much is that per bushel? Ans. \$1.125.
7. If 464 pounds of feathers cost \$116, how much is that per pound? Ans. \$0.25.
8. If 24 chairs cost \$60, how much is that apiece? Ans. \$2.50.
9. If it cost a man \$2.375 a week for board, how long will \$228 last him? Ans. 96 weeks.
10. A mechanic received \$144 for doing a piece of work, which took him 64 days. How much did he receive per day? Ans. \$2.25.
11. Divide 45 by .15. Ans. 300.
12. Divide 2.88 by 1.2. Ans. 2.4.
13. Divide .20736 by 288. Ans. .00072.
14. Divide 13.64589 by 2.19. Ans. 6.231.
15. Divide 1329.6 by .24. Ans. 5540.
16. Divide 1.4112 by 21. Ans. .0672.
17. Divide 1 tenth by 10. Ans. .01.
18. Divide 10 by 1 tenth. Ans. 100.

To how many places of decimals should the quotient be extended in common business calculations? When should it be extended farther? How may we proceed when the divisor is a composite number? How may we divide by 10, 100, 1000, &c.? Explain why this is correct?

19. Divide 1728 billionths by 288 thousandths.
Ans. .000006.

20. Divide 221 thousandths by 17 billionths.
Ans. 13000000.

MISCELLANEOUS EXERCISES FOR THE SLATE:

Involving the principles of Decimal Fractions and Federal Money

1. A man has owing to him as follows: from A., \$17; from B., \$6.12$\frac{1}{2}$; from C., \$27.06$\frac{1}{4}$; and from D., \$0.81$\frac{1}{4}$. How much has he owing to him in all? Ans. \$51.

2. How much does C. owe him more than A. and B.?
Ans. \$3.93$\frac{3}{4}$.

3. How much does C. owe him more than all the rest
Ans, \$3.12$\frac{1}{2}$.

4. A gentleman having \$100, spent \$25 for clothing, \$15 for books, \$6.62$\frac{1}{2}$ for riding in the stage, and \$40.87$\frac{1}{2}$ for jewelry; how much had he left? Ans. \$12.50.

5. How much more did he give for his jewelry than for his clothing? Ans. \$15.87$\frac{1}{2}$.

6. How much more did he spend for clothing than for riding in the stage? Ans. \$18.37$\frac{1}{2}$.

7. A farmer received for his marketing \$27.125; of this, he spent \$2.375 for sugar, \$1.1875 for coffee, \$2.25 for tea, \$0.75 for spice, \$14.0625 for cloth, and took the rest home. How much did he take home? Ans. \$6.50.

8. How much more did he spend for cloth than for all the other things? Ans. \$7.50.

9. Add together fourteen units, six-tenths; two-hundredths; nine units, forty-seven thousandths; seventy-six ten-thousandths; and one unit, two ten-thousandths. Ans. 24.6748.

10. Add together 7 units, 602 thousandths; 18 units; 9 hundredths; 43 units, 26 hundredths; 7 units, 8071 ten-thousandths; and 286 units, 4 tenths. Ans. 363.1591.

11. From 18 units, take 63 thousandths. Ans. 17.937.

12. From 1 unit take 3 millionths. Ans. .999997.

13. How much would 16 sacks of coffee, each containing 173 pounds, cost, at 7$\frac{1}{2}$ cents per pound? Ans. \$207.60.

14. How many bushels of wheat, at \$0.875 per bushel, would it take to pay for the coffee, in the last example?
Ans. 237$\frac{9}{35}$.

15. A merchant bought 46 bags of oats, each containing 2.5 bushels, at \$0.1875 per bushel. How much did they cost him?
Ans. \$21.5625.

16. He paid for them with coffee, at 10 cents per pound. How many pounds did it take? Ans. 215.625.

17. How much would 47398 feet of lumber cost, at $3.50 per C.? Ans. $1658.93.

18. How much would 8372 pounds of pork cost, at $3.875 per C.? Ans. $324.415.

19. How much would 2146 feet of boards cost, at $10 per M.? Ans. $21.46.

20. How much would 1623 shingles cost, at $6.125 per M.? Ans. $9.94+.

21. If a man earn $0.625 per day, how much can he earn in a year, (365 days.) Ans. $228.125.

22. If he spends $0.375 per day, how much will he have left at the end of the year? Ans. $91.25.

23. How many yards of cloth can he buy with this money, at $3.625 per yard? Ans. 25\frac{5}{29}$.

24. Multiply 62.7 by 100. Ans. 6270.

25. Divide 8.726 by 100. Ans. .08726.

26, Just fifteen pair of ladies' gloves,
For forty dimes had I:
How many pair of that same kind
Will forty eagles buy? Ans. 1090.

Article 13. Bills, Accounts, &c.

Obs. 1. A Bill, in dealings with merchants, and others, *is a written paper, containing a statement of the particulars, and total cost of the goods sold.*

To find the total cost of a Bill:

Obs. 2. *Find the cost of each particular at the price mentioned, and the sum of these will be the cost required.*

Required—the cost of the several articles, and the total sum in following bills:

(1.)

Columbus, Jan. 6th, 1847.

Peter Paywell,

Bought of *James Freeman, & Co.*

16	pounds	Coffee, ...	at	0.12\frac{1}{2}$	per pound,
18	"	Sugar, ...	at	.09	"
12	"	Tea, ...	at	.87$\frac{1}{2}$	"
26	"	Saleratus, ...	at	.04	"
25	"	Raisins, ...	at	.16$\frac{2}{3}$	"

Total cost, ... 19.32\frac{2}{3}$.

What is a bill? How do we find the total cost of a bill?

(2.)

CLEVELAND, Feb. 2d., 1848.

S. J. Arson,

Bought of *Charles Martin.*

6	yards	Broadcloth, ...	at	$4.62\frac{1}{2}$	per yard
4	"	Cambric,.............	at	.$12\frac{1}{2}$	"
$2\frac{1}{2}$	dozen	Buttons, ...	at	.25	per dozen
9	skeins	Silk, ...	at	.$06\frac{1}{4}$	per skein
$4\frac{1}{2}$	yards	Wadding,............	at	.09	per yard

Total cost,...................... $29.84\frac{1}{4}$.

(3.)

CINCINNATI, June 7th, 1848.

Henry Plyhard,

To *Lewis Anderson, & Co.* Dr.

For 4	copies	Davies' Bourdon,...........	at	$1.37\frac{1}{2}$	each
" 3	"	Anthon's Cæsar,............	at	1.00	"
" 4	"	Leverett's Dictionary,.......	at	$4.87\frac{1}{2}$	"
" 9	"	Greek Testament,...........	at	$1.12\frac{1}{2}$	"

Total cost,...................... $38.12\frac{1}{2}$.

(4.)

DELAWARE, March 1st, 1847.

Ambrose Plotner,

To *Augustus Reicharts, & Co.* Dr.

For	897 feet	Boards,............	at	$0.87\frac{1}{2}$	per C.
"	1247 "	Plank,............	at	$1.12\frac{1}{2}$	"
"	479 "	Scantling,..........	at	.75	"
"	2479 "	Flooring,..........	at	1.25	"
"	8762 "	Shingles,..........	at	$4.37\frac{1}{2}$	per M.

Total cost,.................... $94.788+.

Received Payment.

Augustus Reicharts, & Co.

(5.)

PITTSBURG, May 1st, 1848.

Thomas Thrifty,

Sold *Wm. Trader, & Co.*

786	bushels	Wheat, ...	at	$1.12½	per bushel
1423	"	Barley, ...	at	.56¼	"
4679	"	Corn, ...	at	.22	"
3716	"	Oats, ...	at	.25	"
2742	pounds	Cheese, ...	at	.08	per pound

In return, he received—

In Cash, ... $3000.00

144	yards	Satinet, ...	at	$1.25	per yard
68	"	Silk, ...	at	1.06¼	"
176	"	Muslin, ...	at	.11½	"
168	"	Calico, ...	at	.16⅔	"
1236	pounds	Coffee, ...	at	.11¾	per pound
1374	"	Sugar, ...	at	.08⅓	"

Required—the difference between the accounts, and in whose favor. Ans. $308.38¾ in favor of Thomas Thrifty.

SECTION IX.

COMPOUND NUMBERS.

ARTICLE I. DEFINITIONS, &c.

Obs. 1. When the *ratio of increase* is the same, numbers are called SIMPLE. Thus: 156, 28, 5 dollars; 13 yards; 147 miles, &c., are called simple numbers, because each order has ten times the value of the next lower order.

Obs. 2. When the ratio of increase varies in the different orders or numbers, they are called COMPOUND. Thus, 12 bushels, 3 pecks, 6 quarts; 3 miles, 30 rods, 14 feet, &c., are called compound numbers, because in some of these it takes *more*, and in some *less*, than ten units of one order to make one unit of the next higher order.

REMARK.—Compound numbers, by some authors, are called *Denominate Numbers.*

Obs. 3. The only difference between Simple and Compound

When are numbers called Simple numbers? Give examples. When are they called Compound numbeas? Give examples.

numbers, is this: *In Simple numbers, figures increase uniformly by* 10; that is, it takes 10 units of each order to make 1 unit of the next higher order; but *in Compound numbers figures increase differently, sometimes taking more, and sometimes less, than* 10 *units of one order, to make* 1 *unit of the next higher order.*

In the former one table alone is necessary; in the latter many tables are required.

Obs. 4. The Tables in Compound numbers *teach how many units it takes of one order to make a unit of the next higher order.* The *Numeration Table* teaches the same in Simple numbers.

REMARK.—The different orders in Compound numbers are called *denominations.*

Obs. 5. In *Simple numbers,* the units of any order can always be expressed by one figure; in *Compound numbers,* it sometimes takes two, three, or even four figures to express the units of a single order.

Likewise, the *sum,* or *difference* of any order, in Simple numbers, can always be expressed by one figure; but in Compound numbers, the *sum,* or *difference,* can sometimes be expressed by one figure, and sometimes it requires more than one figure to express it.

REMARK.—The learner will bear in mind, that we carry 1 to the next higher order, *as often as we obtain a sufficient number of units of the lower order, to make* 1 *unit of this higher order,* whether in Simple or Compound numbers Hence—

Obs. 6. *The principles of Simple and Compound numbers are the same.*

ARTICLE 2. REDUCTION.

Obs. 1. *Changing numbers from one denomination or kind to another, without altering their value, is called* REDUCTION. Thus, there are 4 pecks in a bushel; then in 2 bushels there are 8 pecks; therefore, 8 pecks, or 2 bushels, express the same quantity.

REMARK.—Reduction is of two kinds—*Descending* and *Ascending.*

What is the difference between Simple and Compound numbers? How many tables are necessary in the former? Are more tables than this required in the latter? What do the tables in Compound numbers teach? What does it teach in Simple numbers? What are the different orders in Compound numbers called? How many figures does it take to express a unit of a single order in Simple numbers? How many in Compound numbers? How many figures does it take to express the sum, or difference, of any order in Simple numbers? How many in Compound numbers? How often do we carry 1 to the next higher order? Is this the same, both in Simple and Compound numbers? What then is the difference between the principles of the two? What is Reduction? How is it divided? What is Reduction descending? What is Reduction ascending? Give an example illustrating each case.

Obs. 2. *When a number is changed from a denomination of greater value to a denomination of a less value, the process is called* REDUCTION DESCENDING.

When a number is changed from a denomination of less value to a denomination of greater value, the process is called REDUCTION ASCENDING.

Thus, to change bushels to pecks, we change a greater denomination to a less, and the process is called *Reduction Descending.* But to change pecks to bushels, we change a less denomination to a greater, and the process is called *Reduction Ascending.*

There are 4 pecks in a bushel; how many pecks in 2 bushels? In 3; 4; 6, 10; 12 bushels?

How many bushels in 4 pecks? In 8, 16; 20; 28, 39; 32; 44 pecks?

Obs. 3. From these examples, the pupil will perceive that
Reduction Descending is performed by Multiplication, and
Reduction Ascending is performed by division. Therefore—
Reduction Descending and Reduction Ascending mutually prove each other.

REMARK.—Compound numbers are chiefly confined to weights and measures.

WEIGHTS.

1. TROY WEIGHT.

Obs. 4. *This is used in weighing gold, silver, jewels, liquors, &c.*

The denominations are *Grain, Pennyweight, Ounce and Pound.*

TABLE.

24 grains (grs.)	make	1 pennyweight,	marked	pwt.
20 pennyweights	"	1 ounce,	"	oz.
12 ounces	"	1 pound.	"	lb.

REMARK 1.—The standard of Weights varies in different States in the Union In 1834 the Government adopted a uniform standard for the use of the several Custom-houses, and other purposes. It is very desirable that this standard should be adopted throughout the Union.

2.—The standard unit of *weight* adopted by the United States is the *Troy pound* of the *U. S. Mint*, which is the same as the Imperial Troy pound of England, established by Act of Parliament, A. D. 1826.

How is Reduction Descending performed? Reduction Ascending? What relation do they bear to each other? To what are compound numbers chiefly applied? For what is Troy weight used? What are the denominations? Repeat the Table. What is the standard unit of Weight in the United States?

MENTAL EXERCISES.

1. How many ounces in 2 pounds? 4; 9; 7; 12; 13; 6; 5; 11; 8?
2. How many pennyweights in 2 ounces? 4; 9; 6; 3; 5; 8; 10?
3. How many pounds in 24 ounces? 36; 72, 96; 120; 132?

EXERCISES FOR THE SLATE.

5. Reduce 3 lbs. 7 oz. 15 pwts. 18 grains to grains.

Operation.

```
lbs.  oz.  pwts.  grs.
 3---7---15---18
12
——
36
 7 oz. added.
——
43
 20
———
860
 15 pwts. added.
———
875
 24
————
3500
1750
—————
21000
  18 grs. added.
—————
21018 grs. Ans.
```

We first multiply by 12, to reduce the lbs. to oz., and add in the 7 oz., making 43 oz. We multiply this by 20, to reduce it to pwts., and add in the 15 pwts., making 875 pwts. This we multiply by 24 to reduce it to grs., and add in the 18 grs., making 2018 grs., as our answer.

If we choose, we can add in the numbers given of the different denominations mentally, and thus shorten the operation.

Thus: $3 \times 12 = 36$; $36 + 7 = 43$; set down 43, &c.

6. Reduce 21018 grs. to pounds

Operation.

```
24)21018
   ————
   2|0)87|5 + 18 grs. rem.
        ——
      12)43 + 15 pwts. rem.
         ——
          3 + 7 oz. rem.
```

Ans. 3 lbs. 7 oz. 15 pwts. 18 grs.

We first reduce the 21018 grs. to pwts. by dividing by 24; the result is 875 pwts., and 18 grs. remaining. We next reduce the 875 pwts. to oz., by dividing by 20; the result is 43 oz., 15 pwts. We reduce the 43 oz. to lbs. by dividing by 12; the result is 3 lbs. 7 oz., and the other remainders make 3 lbs., 7 oz., 15 pwts., 18 grs. as our answer.

NOTE.—It would be a good plan to write the name of the denominator to which each number belongs, over the number, (as in the 5th example,) to prevent mistakes.

From these examples, we derive the following

GENERAL RULES FOR REDUCTION.

For Reduction Ascending:

I. *Multiply the highest denomination by that number which it takes of the next less to make 1 of this higher, and add to the product the number given, (if any,) of the lower denomination.*

II. *Proceed in the same manner with the remaining denominators, until the whole is reduced to the denomination required.*

NOTE.—If any denomination is wanting, supply its place with a cipher.

For Reduction Ascending:

I. *Divide the given quantity by that number which it takes of this denomination to make one of the next higher.*

II. *Proceed in this manner until the whole is reduced to the denomination required. The last quotient, together with the several remainders, (if any,) annexed, will be the answer sought.*

NOTE.—If, after any division, there is no remainder, a cipher should be written in the place of that denomination.

7. Reduce 3 lbs. 8 oz. 10 pwts. to grains.

8. Reduce 21360 grs to pounds

9. Reduce 6 lbs. 16 pwts. to grains.

10. Reduce 34968 grains to pounds.

11. Reduce 7 lbs. 19 grs. to grains.

12. Reduce 40339 grains to pounds.

13. Reduce 2 lbs. to pennyweights.

14. Reduce 480 pennyweights to pounds.

2. AVOIRDUPOIS WEIGHT.

Obs. 5. *This is used in weighing the articles of a coarse, heavy nature, such as tea. coffee, sugar, flour, hogs, grain, &c., and all metals except gold and silver.* It is also used in buying and selling medicines. The denominations are DRAM, OUNCE, POUND, QUARTER, HUNDRED-WEIGHT and TON.

Why do we write the name of the denominations over the numbers? What is the rule for Reduction Descending? For Reduction Ascending? If any denomination is wanting, how do we proceed? If after any division there is no remainder, how do we proceed? For what is Avoirdupois weight used? What are the denominations? Repeat the Table.

TABLE.

16 drams (drs.)	make	1 ounce	marked	oz.
16 ounces	"	1 pound	"	lb.
25 pounds	"	1 quarter	"	qr.
4 quarters	"	1 hundred weight	"	cwt.
20 hundred weight	"	1 ton	"	T.

REMARK 1.—The *Avoirdupois Pound* of the United States is determined from the standard *Pound Troy*,* and is in the ratio of 5760 to 7000. That is, 1 pound Troy contains 5760 grains; 1 pound Avoirdupois contains 7000 grains Troy.

2.—In this weight the words *gross* and *net* occur. *Gross weight* is the weight of goods, together with that of the *boxes, casks*, or *bags* that contain them. *Net weight* is the weight of the goods alone.

3.—Formerly it was customary to allow 112 pounds for the hundred weight, and 28 pounds for the quarter; but this practice has become nearly, or quite, obsolete. In buying and selling all articles of commerce estimated by weight, the laws of most of the States, as well as general usage, call 100 pounds a hundred weight, and 25 pounds a quarter. In the U. S. Custom-house, and also in invoices of English goods, and of coal from the Pennsylvania mines, 28 lbs. are called a quarter, and 2240 lbs. a ton.

1. How many drams in 2 ounces? 3; 5; 4; 6?
2. How many ounces in 2 pounds? 6; 4; 3; 5?
3. How many quarters in 2 cwt.? 4; 9; 3; 6; 5; 7; 12?
4. How many pounds in 32 ounces? 48; 80, 64; 76?
5. How many cwt. in 8 quarters? 20; 27; 36; 24; 48?

6. Reduce 7 lbs. 9 oz. to drams.
7. Reduce 1936 drs. to ounces.
8. Reduce 7 cwt. 3 qrs. 22 lbs. to ounces.
9. Reduce 12752 oz. to hundred weight.
10. Reduce 3 tons to drams.
11. Reduce 1536000 drs. to tons.
12. Reduce 7 T. 14 cwt. 23 lbs. to ounces.
13. Reduce 246768 oz. to tons.
14. Reduce 5 T. 18 lbs. 9 oz. to drams.
15. Reduce 2564752 drs. to tons.

3. APOTHECARIES WEIGHT.

Obs. 6. *This is used by apothecaries and physicians in mixing medicines.*

How is the Avoirdupois pound determined? What is the ratio between the two? How many grains in a pound Troy? In a pound Avoirdupois? What is gross weight? Net weight? For what is Apothecaries weight used?

* A pound Avoirdupois is heavier than a pound Troy, but an ounce Troy is heavier than an ounce Avoirdupois.

The denominations are *Grain, Scruple, Dram, Ounce,* and *Pound.*

TABLE.

20 grains (grs.)	make	1 scruple,	marked	sc. or	℈.
3 scruples	"	1 dram,	"	dr. or	ʒ.
8 drams	"	1 ounce,	"	oz. or	℥.
12 ounces	"	1 pound,	"		℔.

REMARK.—The *pound* and *ounce* are the same in this weight as they are in *Troy weight;* but the other denominations are different.

1. How many scruples in 2 drams? 4; 6; 9; 8; 5; 7; 12?
2. How many drams in 2 ounces? 6; 5; 7; 4; 8; 10; 12?
3. How many ounces in 2 pounds? 4; 6; 8; 12; 5; 7; 9?
4. How many drams in 6 scruples? 24; 18; 36; 15; 27?
5. How many ounces in 16 drams? 32; 56; 40; 72; 96?
6. How many pounds in 24 ounces? 72; 60; 96; 144; 108?

7. Reduce 1 ℔. 7 ℥. 2 ʒ. 1 ℈. 16 grs. to grains.

8. Reduce 9276 grains to pounds.

9. Reduce 7 ℔. 6 ʒ. to scruples.

10. Reduce 2034 scruples to pounds.

11. Reduce 6 ℔. 1 ℈. to grains.

12. Reduce 34580 grains to pounds.

MEASURES OF CAPACITY.

1. DRY MEASURE.

Obs. 7. *This is used to measure grain, fruit, salt, &c.*

The denominations are *Pints, Quarts, Pecks and Bushels.*

TABLE.

2 pints (pts.)	make	1 quart,	marked	qt.
8 quarts	"	1 peck,	"	pk.
4 pecks	"	1 bushel,	"	bu.

REMARK 1.—A quart Dry Measure contains $67\frac{1}{5}$ cubic, or solid inches.

2—A Winchester bushel is $18\frac{1}{2}$ inches in diameter, (that is, across the top,) and 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches.

By this is meant a *compact bushel,* as wheat, oats, salt, shelled corn, &c. A

What are the denominations? Repeat the Table. To what other weights are the pound and ounce, Apothecaries weight, equal? For what is Dry measure used? What are the denominations? Repeat the the Table. How many cubic inches in a quart, dry measure? What is the size of a Winchester bushel? How many cubic inches does it contain? What kind of a bushel is meant by this?

dry bushel, as apples, peaches, coal, potatoes, contains 2688 cubic, or solid inches. The former is an even bushel, and the latter is a heaped bushel.

3.—The measure used varies in different States. In *Connecticut,* 2198 cubic inches make a bushel. In *New York,* 2218.192 cubic inches make a bushel. This is the imperial bushel of Great Britain, and weighs 8 lbs. Avoirdupois, of distilled water, at 62° Fahrenheit, and 30 inches of the barometer.

The *Winchester bushel* is the *United States* standard unit of *Dry Measure,* and contains 77.627413 lbs. Avoirdupois distilled water, maximum density, (which is about 41° Fahrenheit,) and weighed in air at 30 inches of the barometer. Its true contents are 2150.42 cubic inches, nearly, although 2150.4 is usually given.

It is desirable that the United States standard should be adopted by the different States, the same as our currency. This would create uniformity, and thus destroy much trouble and confusion usually attendant upon a variation of standards.

4. Many persons purchase grain by *weight,* instead of by *measure.* In Ohio, and also in a majority of the States, the weight of grain is established by law, as follows:

1 bushel of	Wheat.............	weighs	 60	lbs.	Avoirdupois.
1 "	Rye or Indian Corn,	"	 56	"	"
1 "	Barley.............	"	 48	"	"
1 "	Oats,..............	"	 33	"	"

Hence—To find the number of bushels of grain in a given quantity:

a. Divide the weight of the grain by 60, 56, 48, *or* 33, *according as it is Wheat, Rye, Corn, Barley, or Oats.*

1. How many pecks in 2 bushels? 5; 8; 10; 15; 18; 23; 30?
2. How many quarts in 2 pecks? 3; 4; 7; 5; 6; 9; 8; 11?
3. How many pints in 2 quarts? 5; 4; 6; 3; 9; 7; 8?
4. How many bushels in 8 pecks? 16; 32; 24; 48; 36?
5. How many pecks in 16 quarts? 32; 64; 48; 72; 96?
6. How many quarts in 4 pints? 12; 24; 16; 36; 18;
7. How many quarts in 1 bushel? 2; 4; 6; 3; 5; 8; 10?
8. How many pints in 1 peck? 2; 6; 4; 10; 5; 8; 3?
9. How many pints in 1 bushel? 2; 4; 6; 3; 5; 8; 10?
10. How many bushels in 32 quarts? 64; 96; 128; 160; 192?
11. How many pecks in 16 pints? 32; 48; 80; 64; 96; 112?
12. How many bushels in 64 pints? 128; 256; 192: 320?

Each of the following examples proves the one opposite:

13. Reduce 3 bu. 2 pks. to pecks.

14. Reduce 14 pks. to bushels.

15. Reduce 6 bu. 1 pk. 7 qts. to pints.

16. Reduee 414 pts. to bushels.

17. Reduce 10 bu. 3 pks. 6 qts. 1 pt. to pints.

18. Reduce 701 pts. to bushels.

How many cubic inches does the dry, or heaped bushel, contain? What is the United States standard unit of Dry Measure?

19. Reduce 37 bu. 1 pk. 5 qts. 1 pt. to pints.

20. Reduce 2395 pts. to bushels.

21. Reduce 68 bu. 1 pt. to pints.

22. Reduce 4353 pts. to bushels.

28. Reduce 121 bu. 2 qts. to pints.

24. Reduce 7748 pts. to bushels.

2. WINE MEASURE.

Obs. 8. *This is used for measuring all liquids, ale, beer, and milk excepted.*

The denominations are *Gill, Pint, Quart, Gallon, Barrel and Hogshead.*

TABLE.

4	gills (gi.)	make	1 pint,	"	pt.
2	pints	"	1 quart,	"	qt.
4	quarts	"	1 gallon,	"	gal.
$31\frac{1}{2}$	gallons	"	1 barrel,	"	bbl.
63	gallons, or 2 bbls.	"	1 hogshead,	"	hhd.

ALSO:

42	gallons.	make	1 tierce,	marked	tier.
84	gallons, or 2 tier.	"	1 puncheon,	"	pun.
126	gallons, or 2 hhd.	"	1 pipe, or butt,	"	P.
2	pipes	"	1 tun,	"	T.

REMARK.—The standard unit of *Liquor Measure* adopted by the United States is the *wine gallon,* containing 231 cubic inches, equal to 8.339 lbs. Avoir. of distilled water at the maximum density; i. e., 41° Fahrenheit.

1. How many gills in 2 pints? 3; 5; 9; 7: 12; 8; 18; 4; 11?
2. How many gills in 2 quarts? 3; 7; 9; 5; 12; 10. 6; 8; 11?
3. How many pints in 1 gallon? 2; 5; 3; 9; 6; 5; 7; 12; 10?
4. How many gills in 1 gallon? 2; 4; 3; 5?
5. How many pints in 8 gills? 16; 48; 40; 12; 32; 20; 36?
6. How many quarts in 8 gills? 16, 48; 96; 40; 20; 36; 28?
7. How many gallons in 8 pints? 16; 48; 60; 96; 56; 88; 40?
8. How many gallons in 32 gills? 96; 160; 64; 138?

9. Reduce 10 gal. 2 qts. to gills.

10. Reduce 336 gills to gallons.

11. Reducc 1 bl. to pints.

12. Reduce 252 pints to barrels.

13. Reduce 2 hhds. 27 galls. 3 qts. 1 pt. 3 gi. to gills.

14. Reduce 4927 gills to hogsheads.

For what is Wine measure used? What are the denominations? Repeat the Table. What is the standard unit of Liquid Measure adopted by the United States? What does it contain?

15. Reduce 1 T. 1 P. 1 hhd. 32 galls. 1 pt. to pints.

16. Reduce 3785 pts. to tuns.

17. Reduce 7 hhds. 3 gi. to gills.

18. Reduce 14115 gills to hogsheads.

3. ALE, OR BEER MEASURE.

Obs. 9. *This is used for measuriug ale, beer, and milk.*

The denominations are *Pint, Quart, Gallon, Barrel, and Hogshead.*

TABLE.

2 pints (pts.)	make	1 quart,	marked	qt.
4 quarts	"	1 gallon,	"	gal.
36 gallons	"	1 barrel,	"	bar.
54 gallons, or $1\frac{1}{2}$ barrels,	"	1 hogshead,	"	hhd.

REMARK.—A gallon, beer measure, contains 282 cubic inches.

1. How many pints in 2 quarts? 4; 9; 5; 7; 6; 12; 8; 10; 9?
2. How many quarts in 2 gallons? 3; 5; 4; 9; 12; 6; 8; 11?
3. How many pints in 1 gallon? 2; 4; 5; 3; 7; 12; 9; 10?
4. How many quarts in 8 pints? 4; 16; 12; 24; 14; 18; 22?
5. How many gallons in 8 quarts? 16; 20; 12; 48; 40; 36?
6. How many gallons in 8 pints? 24; 16; 48; 32; 56; 40; 96?

7. Reduce 18 galls. 2 qts. 1 pt. to pints.

8. Reduce 149 pts. to gallons.

9. Reduce 3 hhds. 3 qts. to pints.

10. Reduce 1302 pts. to hogsheads.

11. Reduce 6 bar. 1 pt. to pints.

12. Reduce 1729 pts. to barrels.

13. Reduce 2 hhds. 3 qts. to quarts.

15. Reduce 435 qts. to hogsheads.

MEASURES OF EXTENSION.

REMARK 1.—The three kinds of measures we have just given are used for measuring *liquids, grain, fruit,* &c., for which purpose we have different kinds of vessels, as the half bushel, peck measure, pint cup, quart cup, gallon measure, &c.

The kinds of measure we are now about to mention are used to measure *extension;* that is, *distances, surfaces, &c.* For this purpose we have the Sur-

---

For what is Beer measure used? What are the denominations? Repeat the Table? How many cubic inches does a gallon, Beer measure, contain? For what are the preceding measures used? What do we have for this purpose? For what are the other measures used! What do we have for this purpose?

veyor's chain, the square, or rule, yard measure, tapes, and lines of differen lengths, &c.

2.—Extension has three dimensions: *length, breadth,* and *thickness.*

1. LONG MEASURE.

Obs. 10. *This measure is used when length or distance is considered, without regard to breadth or thickness. It is frequently called Linear, or Lineal measure.*

The denominations are *Inch, Foot, Yard, Rod, Furlong,* and *Mile.*

TABLE.

12	inches (in.)	make	1 foot,	marked	ft.
3	feet	"	1 yard,	"	yd.
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet,	"	1 rod, perch, or pole,	"	rd.
40	rods, or 220 yards,	"	1 furlong,	"	fur.
8	furlongs, or 320 rods,	"	1 mile,	"	M.

ALSO:

3	barley corns, (bar. c.)	"	1 inch, (but little used.)
4	inches	"	1 hand, used in measuring the height of horses.
6	points	"	1 line; and
12	lines	"	1 inch, used in measuring the length of clock pendulums.
6	feet	"	1 fathom, used in measuring the depth of water.
3	miles	"	1 league, used in measuring distances at sea.
60	geographical, or $69\frac{1}{2}$* statute miles,	"	1 degree, " deg. or °.
360	°	"	the circumference, or distance round the earth.

REMARK.—The standard unit of *length* adopted by the *United States* is the *Yard* of 3 feet, or 36 inches, and is the same as the Imperial Yard of Great Britain. It is made of brass, and is determined at the temperature of 60° Fahrenheit, from the scale of Troughton, a celebrated English artist.

For what is Long measure used? What is it frequently called? What are the denominations? Repeat the Table. What is the standard unit of length adopted by the United States? How determined?

* This is not exactly correct, although it is what is usually given. On the equator, 69 and one-sixth statute miles make 1°, or 60 geographical miles, nearly; and on the meridian, at a mean, $69\frac{1}{2}$ statute miles make 1°.

1. How many inches in 2 feet? 3; 5; 7; 12; 8; 6; 9; 11; 15;
2. How many feet in 3 yards? 4; 9; 12; 6; 5; 8; 22?
3. How many furlongs in 2 miles? 4; 6; 9; 11; 8; 12?
4. How many inches in 1 yard? 2; 5; 3; 4?
5. How many feet in 24 inches? 48; 108; 36; 84; 60?
6. How many yards in 6 feet? 12; 36; 24; 15; 33; 18?
7. How many miles in 16 furlongs? 32f 72; 96; 48; 64?
8. How many yards in 36 iuches? 108; 180; 72; 144?
9. How many barley-corns in a foot? 2; 4; 5; 3?
10. How many feet in 3 hands? 4; 7; 8; 5; 9; 6; 12?
11. How many points in 1 inch? 2; 3; 4?
12. How many feet in 2 fathoms? 3; 5; 9; 12; 8; 6; 7?
13. How many feet in 36 barley-corns? 72; 144; 80; 108?
14. How many miles in 2 leagues? 4; 12; 8; 5; 7; 8?
15. How many hands in 12 inches? 1 ft. 4 in.; 3 ft; 2 ft. 8 in.
16. How many inches in 72 points? 144; 216; 288?
17. How many fathoms in 12 feet? 24; 72; 28; 60, 24?
18. How many leagues in 6 miles? 12; 36; 24. 32; 48?
19. How many inches in 2 ft. 3 in.? 3 ft. 6 in.; 3 ft. 9 in.; 5 ft.?
20. How many feet in 25 inches? 33; 40; 44; 57; 70?

21. Reduce 85 miles to inches.

22. Reduce 5385600 inches to miles.

23. Reduce 7 M. 6 fur. 27 rds. 3 yds. 2 ft. to inches.

24. Reduce 496518 inches to miles.

25. Reduce 7°. 49 M. 7 fur. 16 rds. 12 ft. 9 in. 2 bar. c. to barley corns.

NOTE.—Statute miles are understood in this example.

26. Reduce 101964125 barley corns to degrees.

27. How many barley corns would it take to reach around the globe, it being 360 degrees.

NOTE.—Multiply by 69½ to reduce it to miles.

28. How many degrees is it around the globe, it being 4755-801600 barley corns?

29. How many times will a carriage wheel turn over in going 360 miles, it being 16 ft. 6 in. in circumference, that is, around the outside?

30. How many miles does a carriage wheel, 16 ft. 6 in in. circumference, proceed, in turning over 115200 times.

2. CLOTH MEASURE.

Obs. 11. *This is used in measuring cloth, tape, lace, and all kinds of goods bought and sold by the yard.*

The denominations are *Nail, Quarter, Yard, and Ell.*

TABLE.

4 nails (na.)	make	1 quarter,	marked	qr.
4 quarters	"	1 yard,	"	yd.
3 quarters, or $\frac{3}{4}$ of a yd.	"	1 Flemish ell,	"	Fl. e.
5 quarters, or $1\frac{1}{4}$ yards	"	1 English ell,	"	E. e.
6 quarters, or $1\frac{1}{2}$ yards	"	1 French ell,	"	F. e.

REMARK.—*Cloth measure* is a species of *Long measure.* The yard is the same in both; that is, it contains 3 feet, or 36 inches. Therefore, a quarter contains 9 inches, and a nail $2\frac{1}{4}$ inches. Cloths, &c., are bought and sold according to their length, without regard to their width. The nail is but little used, eighths and sixteenths being used in its place.

1. How many nails in 2 quarters? 4; 8; 12; 16; 11; 9?
2. How many quarters in 2 yards? 9; 9; 7; 5; 12; 11?
3. How many nails in 2 yards? 3; 5; 4; 6?
4. How many quarters in 2 Flemish ells? 4; 8; 9; 6; 12?
5. How many quarters in 2 English ells? 8; 4; 7; 5; 9; 12?
6. How many quarters in 2 French ells? 12; 9; 8; 7; 5; 6?
7. How many quarters in 8 nails? 16; 48; 32; 12; 44; 36?
8. How many yards in 8 quarters? 48; 12; 16; 32; 36; 44?
9. How many yards in 6 nails? 64; 32; 80; 48; 96?
10. How many Flemish ells in 6 quarters? 36; 18; 30; 27; 52?
11. How many English ells in 10 quarters? 44; 16; 50; 20?
12. How many French ells in 12 quarters? 24; 42; 54; 72; 48?

13. Reduce 6 yds. 3 qrs. to nails.

14. Reduce 108 nails to yards.

15. Reduce 15 yds. 1 qr. 3 na. to inches.

16. Reduce $555\frac{3}{4}$ inches to yards.

17. Reduce 8 F. e. 3 qrs. to nails.

18. Reduce 204 nails to French ells.

19. Reduce 10 E. e. 1 qr. 2 na. to nails.

20. Reduce 206 nails to English ells.

21. Reduce 12 F. e. to Fl. e.

First reduce the French ells to quarters, and then to Flemish ells.

22. Reduce 24 Fl. e. to F. e.

NOTE.—First reduce the Flemish ells to quarters, and then to French ells.

23. Reduce 25 F. e. to English ells.

24. Reduce 30 E. e. to French ells.

25. Reduce 18 Fl. e. to French ells.

26. Reduce 9 F. e. to Flemish ells.

For what is Cloth measure used? What are the denominations? Repeat he Table. What kind of measure is Cloth measure? What measure is the ame in both? How many feet and inches does the yard contain? How are loths, &c., bought and sold?

27. Reduce 15 Fl. e. to French ells.

28. Reduce 9 E. e. to French ells.

29. Reduce 36 E. e. to Flemish ells.

30. Reduce 60 Fl. e. to English ells.

3. LAND, OR SQUARE MEASURE.

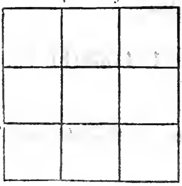
Obs. 12. *This is used for measuring land, flooring, or anything in which length and breadth are considered, without regard to depth or thickness.*

The denominations are *Square Inch, Square Foot, Square Yard, Square Rod, Rood, Acre* and *Square Mile.*

A. Obs. 13. *When any two lines meet together, the opening at their place of meeting is called an* ANGLE. When they meet so as to form a square corner, like the corner A., the angle is called a RIGHT ANGLE.

1 square yard.

3 ft. wide.



9 square feet.

3 ft. long.

Obs. 14. *A figure having four equal sides, and its angles right angles, is called a* SQUARE.

A *Square inch* is a square, each side of which measures an inch in length.

A Square yard is a square, each side of which is a yard, or 3 feet in length, and contains 9 square feet. A square foot contains $12 \times 12 = 144$ square inches.

TABLE.

144	square inches (sq. in.)	make	1 square foot,	marked	sq. ft.
9	square feet	"	1 square yard,	"	sq. yd.
$30\frac{1}{4}$	square yards, or $272\frac{1}{4}$ square feet,	"	1 square rod, perch, or pole,	"	sq. rd.
40	square rods	"	1 rood,	"	R.
4	roods, or 160 sq. yds.	"	1 acre,	"	A.
640	acres	"	1 square mile, or section,	"	sq. M.

For what is Land, or Square measure used? What are the denominations? What is an angle? A right angle? A square? A square inch? A square yard? How many square feet does a square yard contain? How many square inches does a square foot contain? Repeat the Table.

In measuring land, surveyors use a chain 4 rods long, and containining 100 links. Hence—

$7\frac{92}{100}$	inches	make	1 link,	marked	l.
25	links	"	1 rod,	"	rd.
4	rods or 66 feet,	"	1 chain,	"	ch.
80	chains	"	1 mile,	"	M.

These, the learner will observe, are all *linear* measure. But

1 square chain	makes	16 perches,	marked	P.
10 square chains	make	1 acre,	"	A.

This chain is generally called GUNTER'S CHAIN, from the name of its inventor.

1. How many square feet in 2 square yards? 3; 6; 12; 9; 7?
2. How many roods in 2 acres? 4; 9; 5; 7; 9; 12; 10?
3. How many square yards in 18 square feet? 27; 54; 90?
3. How many acres in 8 roods? 44; 32; 24: 48; 36; 49?

5. Reduce 1 sq. M. 326 A. 3 R. 27 sq. rd. 16 sq. yds. to square inches.

6. Reduce 6065153964 sq. in. to square miles.

7. Reduce 16 A. 2 R. 17 sq. rd. 124 sq. ft. 127 sq. in. to square inches.

8. Reduce 104183011 sq. in. to acres.

9. Reduce 7 A. 120 sq. rd. 212 sq. ft. 114 sq. in. to square inches.

10. Reduce 48643602 sq. in. to acres.

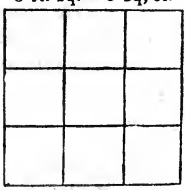
11. If a man's farm measures 120 chains in length, how many rods long is it? How many miles?

The student will observe that the mile is Long measure.

12. If a man's farm measures $1\frac{1}{2}$ miles in length, how many rods long is it? How many chains?

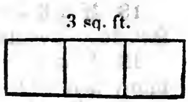
Obs. 15. Although a *square foot* and a *foot square* are the same, there is a difference between *square feet* and *feet square*.

3 ft. sq.=9 sq, ft.



It will be perceived from the figure, that 3 feet square measures 3 feet on each side, and contains $3 \times 3 = 9$ square feet.

What is used in measuring land? How long is this chain? How many links does it contain? Repeat the Table. What name is usually given to this chain? Why?



On the other hand, 3 square feet measures 3 feet in length, and only 1 foot in width, and contains $3 \times 1 = 3$ square feet. Therefore, there is a difference of 6 sq. ft. between 3 ft. sq. and 3 sq. ft.

Between 2 sq. ft. and 2 ft. sq. there is a difference of 2 sq. ft.

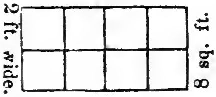
Between 4 sq. ft. and 4 ft. sq. there is a difference of 12 sq, ft.

13. What is the difference between 8 sq. ft. and 8 ft. sq.?
14. What ts the difference between 9 sq. ft. and 9 ft. sq,?
15. What is the difference between 12 sq. ft. and 12 ft. sq.?
16. What is the difference between 25 sq. ft. and 25 ft. sq.

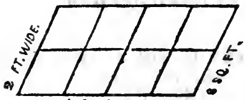
Ans. 600 sq. ft.

Besides the square, we have other four-sided figures, which have different names, according to their forms. The most common are the RECTANGLE, PARALLELOGRAM, and RHOMBUS.

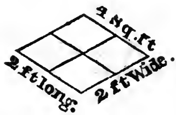
Rectangle.



Parallelogram.



Rhombus.



Obs. 16. The space enclosed by the lines which bound a figure, is called its AREA, or *superficial contents*.

It will be perceived that the area of the above figures, as well as as that of the square, is found by multiplying together their length and breadth. Hence—

To find the area of a square, rectangle, parallelogram, or rhombus:

Obs. 17. *Multiply together their length and breadth* :

REMARK.—Recollect that feet multiplied by feet produce *square feet*, and so of all the other denominations of *linear measure*.

17. What is the difference in the size of two rooms, one being 15 feet square, and the other containing 196 square feet?

Ans. The one 15 feet square contains 29 sq. ft. the most.

What is the difference between a square foot and a foot square? Between 3 sq. ft. and 3 ft sq.? Show why this is the case. What four other figures have we besides the square? What is the area of a figure? How do we find the area of a square, rectangle, parallelogram, or rhombus? What do the denominations of linear measure, multiplied by the same, produce?

18. If a floor is 18 feet long, and 12 feet wide, how many square feet does it contain? Ans. 216.

19. If a piece of land is 45 chains long, and 80 rods wide, how many acres does it contain? Ans. 90.

Reduce both to the same denomination.

20. How many square feet in a board 18 inches wide, and 24 feet long? Ans. 36.

Reduce both to the same denomination.

21. How many square feet in a board 32 feet long, and 16 inches long? Ans. 42 sq. ft. 96 sq. in.

22. How many acres in a square field, each side of which measures 120 rods? Ans. 90.

23. How many acres in a field 46 rods long, and 20 rods wide? Ans, 5 A. 120 sq. rds.

24. The largest of the Egyptian pyramids is a square at the base, each side being about 693 feet in length. How many acres does it cover? Ans. 11 A. 4 sq. rds.

25. What is the difference between a floor 25 feet square, and two others, each 16 feet square? Ans. 113 sq. ft.

26. If a room is 30 feet long, how wide must it be to contain 240 sq. ft.? Ans. 8 feet.

NOTE.—The length and breadth are the factors, which, multiplied together, produce the area. Therefore, the area divided by either factor, will give the other. (Sect. VI. Art. 1. Obs. 16.)

27. If a board is 18 inches wide, how long must it be to contain 18 sq. ft.? Ans. 12 ft.

Reduce both to the same denomination.

28. If a board is 15 feet long, how wide must it be to contain 20 sq. ft.? Ans. 16 in.

29. If a room is 12 ft. long, how wide must it be to contain 216 sq. ft.? Ans. 18 ft.

30. How many yards of muslin, $\frac{3}{4}$ of a yard wide, will it take to line 3 yds. satinet, $1\frac{1}{4}$ yds. wide? Ans. 5 yds.

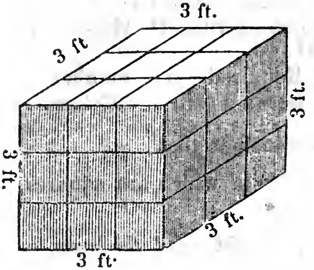
4. SOLID, OR CUBIC MEASURE.

This is used to measure wood, stone, or any thing in which length, breadth, and thickness, or depth, are considered.

The denominations are *Solid Inch, Solid Foot, Solid Yard, Cord,* and *Ton.*

Obs. 19. A solid body having six equal square faces, is called a CUBE, or HEXÆDRON.

When we have the area, and either the length or breadth given, how do we find the other? Why is this correct? For what is Solid or Cubic measure used? What are the denominations?



If each side of a cube measures an inch in length, it is called a *cubic*, or *solid inch*. If each side measures a yard in length, it is called a *cubic*, or *solid yard*, and contains 27 solid feet; that is, 3 feet in length 3 feet in width, and 3 feet in thickness.

A solid foot contains $12 \times 12 \times 12 = 1728$ solid inches.

NOTE.—This can best be explained by a number of small cubical blocks, with which any teacher can supply himself.

TABLE.

1728 solid inches (s. in.)	make	1 solid foot,	marked	s. ft.
27 solid feet	"	1 solid yard,	"	s. yd.
50 feet of round, or 40 feet of hewn timber.	"	1 ton,	"	T.
128 solid ft. $= 8 \times 4 \times 4$, that is 8 ft. long, 4 ft. wide, and 4 ft. high,	"	1 cord of wood or bark,	"	C.

REMARK 1.—A pile of wood 1 foot long, 4 feet wide, and 4 feet high, is called a *cord foot*, and contains 16 solid feet. 8 cord feet make 1 cord.

2.—In estimating the *tonnage* of ships, 42 solid feet are allowed for a ton.

3.—By a ton of round timber, is meant such a qnantity of timber in its rough, or natural state, as when hewn will make 40 cubic feet.

4.—A cubic foot contains 7.48 wine gallons, and 6.127 beer gallons.

5.—A cubic foot of distilled water weighs about 1000 oz. Avoirdupois, or very nearly $62\frac{1}{2}$ lbs., at 40° temperature. At 60°, which is generally used, it weighs only 62.353 lbs., which is less than 1000 oz. The foot weighs 911.458 oz. Troy, or .5274 oz. per cubic inch.

1. Reduce 6 solid yards to solid inches.

2. Reduce 279936 solid inches to solid yards.

3. Reduce 12 tons of round timber to solid inches.

4. In 1036800 solid inches, how many tons of round timber?

What is a cube? A cubic inch? A cubic yard? How many solid feet does a solid yard contain? How many solid inches in a solid foot? Repeat the Table. What is a cord foot? How many solid feet does it contain? How many cord feet make a cord? How many solid feet are allowed for a ton in estimating the tonnage of vessels? What is meant by a ton of round timber? How many wine gallons does a cubic foot contain? Beer gallons? What is the weight of a cubic foot distilled water? How do we find the contents of a solid, the sides of which are squares, rectangles, &c.?

5. In 5 cords of wood, how many cord feet? How many solid feet?

6. In 540 solid feet of wood, how many cord feet? How many cords?

7. A ship contains 12600 solid feet. Required—the tonnage.

8. Required—the number of solid feet in a ship of 300 tons.

From the remarks under Obs. 17, we conclude that

To find the contents of a solid, the sides of which are squares, rectangles, &c.:

Obs. 20. *Multiply together its length, breadth, and thickness, or depth.*

9. How many solid feet in a room 18 feet long, 16 feet wide, and 12 feet high? Ans. 3456.

10. How many solid feet in a box 3 feet long, 2 feet wide, and 60 inches deep? Ans. 8.

Reduce all to the same denomination.

11. How many cubic feet in a pile of wood 14 feet long, 4 feet wide, and $3\frac{1}{2}$ feet high? Ans. 196.

12. How many cords of wood in a pile 186 feet long, 6 feet wide, and $4\frac{1}{2}$ feet high? Ans. 39 cords, $1\frac{7}{8}$ cord feet.

5. TIME.

Obs. 21. *Time is the measure of duration.* It is naturally divided into *days* and *years*, the former being caused by the revolution of the earth around its axis, and the latter by revolution around the sun.

Other divisions have been made by man; the *day* being divided into *hours, minutes,* and *seconds;* and the *year* into *months, weeks,* and *days.*

TABLE.

60 seconds (sec.)	make	1 minute	marked	min.
60 minutes	"	1 hour	"	h.
24 hours	"	1 day	"	d.
$365\frac{1}{4}$ days	"	1 year	"	yr.
100 years	"	1 century	"	cen.

ALSO:

7 days	makes	1 week	marked	wk.
4 weeks	"	1 month	"	mo.

13 months, 1 day, and 6 hours, (nearly,) or $365\frac{1}{4}$ days, makes 1 common or Julian year.

How is Time naturally divided? What causes the day? The year? How is the day divided? The year? Repeat the Table.

REMARK 1.—A *Solar* year is the exact time in which the earth revolves around the sun, and contains 365 d. 5 h. 48 min. 48 sec.

2. The 6 hours, or ¼ of a day is not added to each year, but reserved until every fourth year, when a whole day is gained. Therefore, every fourth year must contain 366 days. This year is called *Bissextile*, or *Leap Year.*

3. *Leap Years* are those which can be divided by 4 without a remainder; as 1840, 1844, 1848, &c.

4. *Exceptions.*—It will be perceived that in counting 365¼ days to the year, we reckon 11 min. and 12 sec. too much. This in 100 years amounts to 18 hrs. 40 min.; dropping the 40 min., it being too small to be noticed, we find that each 100 years we count 18 hrs. or ¾ of a day too much, and in 400 years we increase 3 days in time. Therefore, to produce the exact time, we count every fourth centurial year as leap year, and the intermediate centurial years as common years, (365 days.) Hence—*If the centurial years can be divided by* 400 *they are leap years, otherwise they are not.*

Examples.—1200, 1600, 2000, &c., are leap years, but 1500, 1700, 1800, &c. are not.

5. The year is also divided into 12 calendar months, the order of which, and the number of days in each, are as follows:

January,	(Jan.)	1st	month	has	31	days.
February,	(Feb.)	2d	"	"	28	"
March,	(Mar.)	3d	"	"	31	"
April,	(Apr.)	4th	"	"	30	"
May,		5th	"	"	31	"
June,		6th	"	"	30	"
July,		7th	"	"	31	"
August,	(Aug.)	8th	"	"	31	"
September,	(Sept.)	9th	"	"	30	"
October,	(Oct.)	10th	"	"	31	"
November,	(Nov.)	11th	"	"	30	"
December,	(Dec.)	12th	"	"	31	"

6. The odd day which is added to every fourth year, is added to the month of February. Therefore, every fourth year February has 29 days.

7. The number of days in each month may be easily remembered by committing to memory the following lines:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February alone;
Which hath twenty-eight in store,
Till Leap year gives it one day more.

1. How many days in 2 weeks? 5? 6? 4? 7? 3? 9? 8? 12?
2. How many weeks in 2 months? 4? 6? 3? 7? 5? 8? 9? 12?
3. How many minutes in 2 hours? 3? 6? 4? 7? 5? 8? 12? 9?

What is a solar year? What is its length? What is done with the 6 hours or ¼ of a day? How many days has every fourth year? What is this year called? Which are leap years? Give examples. How is the year otherwise divided? Repeat the names of the months together, with the number of days in each. What is done respecting the centurial years? To which month is the odd day added? How many days then has February in leap year? Repeat the lines for remembering the number of days in each month?

4. How many weeks in 14 days? 29? 42? 35? 56? 84?
5. How many months in 8 weeks? 24? 48? 16? 40? 32? 44?
6. How many hours in 120 minutes? 300? 720? 360? 480?

7. Suppose a person's age to be 18 yrs., 217 d., 16 h., 43 min., 37 sec , how many seconds old is he?

8. Reduce 586845817 seconds to years.

9. How many seconds in 3 centuries, 57 yrs., 7 mo., 3 wks. 1 d., 18 h., 33 min.?

10. Reduce 10382754780 seconds to centuries.

11. How many seconds between July 4th., 1776, and July 4th., 1848, it being 72 years?

12. Reduce 2272147200 seconds to years.

6. CIRCULAR MEASURE, OR MOTION.

Obs. 22. *This is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies.*

All the calculations by this measure are made in *circles*, every circle, whether large or small, being supposed to be divided into 360 equal parts called *degrees*. Each degree is divided into 60 equal parts called minutes, and each minute into 60 equal parts called seconds.

TABLE.

60 seconds (″)	make	1 minute	marked	′
60 minutes	"	1 degree	"	°
30 degrees	"	1 sign	"	s.
12 Signs, or 360°	"	1 circle	"	c.

REMARK 1.—The Sign is but little used, calculations by this measure being chiefly made in degrees, minutes, and seconds.

2. The minute is the same as the geographical mile, 60 of which make a degree.

3. It must not be inferred, however, that 60 geographical, or 69½ satute miles (table long measure,) make a degree in every circle. This is only the case *when the circumference of the circle is the same as the circumference of the earth.* If the circle is larger or smaller, the distance is greater or less in proportion.

For what is circular measure used? How are the calculations made? How is the circle divided? The degree? The minute? Repeat the Table. What denominations are used principally in calculating by this measure? To what does the minute correspond? How large must a circle be to have 60 geographical, or 69½ statute miles make a degree?

1. How many signs in 2 circles? 4? 6? 10? 8? 5? 7? 12?
2. How many degrees in 2 signs? 4? 5? 7? 9? 8? 12?
3. How many minutes in 2 degrees? 4? 8? 5? 7? 6?
4. How many circles in 24 signs? 48? 60? 36? 72? 144?
5. How many signs in 60 degrees? 120? 210? 180? 240?
6. How many degrees in 120 minutes? 180? 300? 540? 720?

7. Reduce 17° 43′ 57″ to seconds.

8. Reduce 63837″ to degrees.

9. Reduce 8s. 29° 39″ to seconds.

10. Reduce 968439″ to signs.

11. Reduce 1c. 355° 59′ 49″ to seconds.

12. Reduce 2577589″ to circles.

MISCELLANEOUS TABLE.

12 things	make	1 dozen	marked	doz.
12 dozen	"	1 gross	"	gr.
12 gross	"	1 great gross	"	g. gr.
20 things	"	1 score.		
56 pounds	"	1 firkin of butter	"	fir.
112 pounds	"	1 quintal of fish	"	quin.
196 pounds	"	1 barrel of flour.		
200 pounds	"	1 barrel of pork, or beef.		
14 pounds of iron or lead	"	1 stone.		
21½ stone	"	1 pig.		
8 pigs	"	1 fother.		
24 sheets of paper	"	1 quire.		
20 quires	"	1 ream.		

BOOKS.

A sheet	folded in	two	leaves	is called	a folio.
A sheet	"	four	"	"	a quarto, or 4 to.
A sheet	"	eight	"	"	an octavo, or 8 vo.
A sheet	"	twelve	"	"	a duodecimo, or 12 mo.
A sheet	"	eighteen	"	"	an 18 mo.

MISCELLANEOUS EXERCISES FOR THE SLATE,

Involving the principles of Reduction.

1. In 15 bars of silver, each weighing 6 lbs. 8 oz. 15 pwts. 11 grs., how many grains? Ans. 581565.

2. How many thimbles, each weighing 8 pwts. 16 grs. can be made from 1 lb. 8 oz. 16 pwts. of silver? Ans. 48.

Repeat the Miscellaneous Table. What is a sheet folded in two leaves called? In four leaves? In eight leaves? In 12 leaves? In 18 leaves?

3. How many rings, each weighing 5 pwts. 7 grs., can be made from 2 lbs. 8 oz. 16 pwts. 4 grs. of gold? Ans. 124.

4. How many pounds of hay in 6 loads, each weighing 14 cwt. 2 qrs. 18 lbs.? Ans. 8808.

5. How many nails, each weighing 6 drams, can be made from 29 lbs. 3 oz. 4 drs. of iron? Ans. 1246.

6. How many casks, each holding 68 lbs., can be filled from 16 cwt. 1 qr. 7 lbs of sugar? Ans. 24.

7. If a family consume 4 lbs. 12 oz of coffee in a month, how long will 1 cwt. 3 qrs. 9 lbs. 8 oz. last them? Ans. $38\frac{16}{19}$, months.

8. In 45 pounds avoirdupois, how many pounds troy? Ans. $54\frac{11}{16}$.

9. In 36 pounds troy, how many pounds avoirdupois? Ans. $29\frac{109}{175}$.

10. A physician made 136 pills, each containing 4 grains, how much did they all contain? Ans. 1℥. 1ʒ. 4 grs.

11. How many doses of calomel each containing 8 grs., can be made from 1℔. 2℥. 3ʒ. 2℈. 12 grs. Ans. 869.

12. What quantity of rhubarb will it take to put up 324 doses, each containg 24 grs. Ans. 1℔. 4℥. 1ʒ. 1℈. 16 grs.

13. A merchant wishes to put 675 bushels of clover seed into casks, each holding 6 bushels, 3 pecks. How many casks are required? Ans. 100.

14. At $\frac{1}{4}$ of a dollar a peck, what would 18 bu. 1 pk. of wheat cost? Ans. $18\frac{1}{4}$ dollars.

15. How many bottles will it take, each holding $\frac{1}{2}$ a pint, (2 gills,) to put up a hogshead of wine? Ans. 1008.

16. A certain cistern holds 66 barrels of water, how many times will it fill a pail holding 3 galls. 1 qt. 1 pt. Ans. 616 times.

17. How many casks, each holding 7 galls. 3 qts. 1 pt., can be filled from a tun of wine? Ans. 32.

18. How many dozen bottles, each dozen containing 3 gal. 3 qts. 1 pt. 2 gi., can be filled from a hogshead of cider? Ans. 16.

19. How many gallon, quart, pint, and gill bottles, of each an equal number, can be filled from 210 galls. 3 qts. 1 pt. 2 gi. of wine? Ans. 150.

20. How many pints in a hogshead of ale? Ans. 432.

21. How many gallon, quart, and pint bottles will it take to put up a barrel of beer, having the same quantity in each kind of bottles?

Ans. 12 gallon bottles, 48 quart bottles, and 96 pint bottles.

22. How many casks, each holding 6 galls. 3 qts. 1 pt., will it take to hold 19 hhds., 19 galls of ale? Ans. 152.

23. If a man drink 1 pint of beer each day for 24 years, how much would it amount to? Ans. 20 hhds., 15 galls., 3 qts.

24. How many quarters in 12 pieces of cloth, each containing 15 yds., 2 qrs. Ans. 744 = 186 yds.

25. How many pieces of cloth each containing 12 yds. 3 qrs. 2 na., are there in 579 yds., 1 qr., 2 na? Ans. 45.

26. How many suits of clothes, each containing 4 yds. 1 qr., 3 na., can be made from 53 yds., 1 qr. of cloth? Ans. 12.

27. How many pair of pantaloons, each containing 1 yd. 2 qrs. 3 na., can be made from 18 Flemish ells of broadcloath?

Ans. 8.

28. How many times will a carriage wheel 16 ft. 6 in. in circumference, turn over in going 18 miles? Ans. 5760.

29 How many steps of 2 ft. 5 in. each, will it a man take in traveling 1 m. 7 fur. 33 rods. 3 yds. 1 ft.? Ans. 4326.

30. If a man travel 20 m. 6 fur. 26 rds. 11 ft. in a day, how long will it take him to travel around the earth, it being about 25000 miles? Ans. 1200 days = 3 yrs., 104 d., 6 hrs.

31. How many times will a ship, 133 ft. 4 in. long, sail her length in crossing the Atlantic ocean, it being 3000 miles.

Ans. 118800.

32. How many yards of carpeting 1 yard wide, will it take to cover the floor of a room 16 feet long, and 12 feet wide?

Ans. $21\frac{1}{3}$.

33. A land owner has in one place 329 acres, in another place 870 acres, in another place 1236 acres, and in another place 1405 acres; if it was all together, how many square miles would it make? Ans. 6.

34. A farmer wishing to know the length of his farm, found it to be 146 chains, $66\frac{2}{3}$ links. Required its length in rods? In miles?

Ans. $586\frac{2}{3}$ rods = $1\frac{5}{6}$ miles.

35. The surface of the planet Venus contains about 9327776422-03545600 square inches; how many square miles?

Ans. 232352736.

36. How many blocks 3 inches long, and 2 inches wide, can be cut from a board 3 feet in length, and 2 feet in width?

Ans. 144.

37. A piece of land is 144 rods long, and 95 rods wide; how many acres does it contain? Ans. $85\frac{1}{2}$.

38. A.'s farm is 66 chains, 50 links in length, and 45 chains, 75 links in width; required the number of acres owned by A.?

Ans. $304\frac{17}{80}$.

39. If a room is 16 feet log, and 12 feet wide, how many square feet does it contain? Ans. 192.

40. A board is 14 feet long, and 16 inches wide at one end, and 8 inches wide at the other end; required the number of square feet in the board? Ans. 14.

The width at one end is 16 inches, and at the other end 8 inches; therefore the width at both ends is $16 + 8 = 24$, and the half of this or 12 inches, must be the mean or average width. $12 \div 12 =$ 1 foot. $14 \times 1 = 14$ Ans. Hence—

To find the mean width of a board, or any other surface whose edges are straight, but the width at the ends are different:

Obs. 23. *Add together the width of the two ends, and take half their sum.*

41. If a board is 24 feet long, and 32 inches wide at one end, and 18 inches wide at the other end, how many square feet in the board? Ans. 50.

42. How many square feet in the surface of a rafter $19\frac{1}{2}$ feet long, 3 inches thick, and the width at the ends 4 and 3 inches? Ans. 21 sq. ft., 18 sq. in.

43. There is a room 18 feet in length, 16 feet in width, and 8 feet in heighth; how many rolls of paper, 2 feet wide, and containing 11 yards in each roll will it take to cover the walls. Ans. $8\frac{8}{33}$.

44. How many blocks, 2 inches long, 2 inches wide, and 2 inches thick, can be cut from a cubical block measuring 3 feet on each side, allowing no waste for cutting? Ans. 5832.

45. How many cords of wood in a pile 56 feet long, 12 feet wide, and 8 feet high? Ans. 42.

46. A cubic post of oak weighs 950 ounces avoirdupois; how many pounds will 6 cords of oak weigh? Ans. 45600.

47. If a clock tick 60 times a minute, how many times will it tick in a year? Ans. 31557600.

48. How many seconds longer was March than February in the year 1847. Ans. 259200.

49. What are the leap years between 1846 and 1862? Ans. 1848, 1852, 1856, 1860.

50. If a person wastes $\frac{3}{4}$ of an hour each day, how much time will he lose in 28 years. Ans. 319 d. 14 h. 15 min.

51. How much time will a person gain in 50 years by rising $\frac{1}{2}$ an hour (30 min.) earlier than usual, each day? Ans. 1 yr. 15 d. 5 h. 15 min.

52. Sound moves at the rate of 1142 feet in a second of time;

How do we find the number of square feet in a board, or any other surface, whose edges are strait, but the width of the ends different?

required the time it would take sound to pass from the sun to the planet Uranus, it being distant 1800000000 miles?

Ans. 263 yrs. 261 d. 11 hr. 48 min.+

NOTE.—The learner will observe that the remainder after dividing by $365\frac{1}{4}$ is fourths of days, not whole days.

53. If a person spends 8 hours of the day in sleeping and eating, how many minutes per day does he devote to other business.

Ans. 960.

54. If a vessel sails 10 miles an hour, how far will she sail in 4 wks. 5 d. 16 hrs. Ans. 8030 miles.

55. The precise length of the tropical year is 365 d. 5 h. 48 min. 48 sec.; how many such years in 1104492480 seconds?

Ans. 35.

56. How many seconds from the commencement of the Christian Era, until Oct 7th, 1849, 4 o'clock, P. M., allowance being mode for leap year? Ans. 58341398400.

57. If a planet move through an area of 3° 36′ in one day, how long will it take it to move through an area of 9s. 160°?

Ans. $79\frac{4}{9}$ days.

58. How long would it take a planet to move through a quadrant (90°) at the rate of 59′ 8″ per minute? Ans. 1 hr., $31\frac{283}{887}$ min.

59. How long would it take a comet to move through a semicircle, (180°) at the rate of 8° 16′ daily? Ans. $21\frac{24}{31}$ days.

60. What cost 3 dozen dozen yards of cloth, at 2 dollars a yard?

Ans. 864 dollars.

61. What would 6 gross of buttons cost, at 6 cents per dozen?

Ans. 432 cents.

62. What would 3 great gross of combs cost, at 3 cents apiece?

Ans. 15552 cents.

63. What would 4 score of hogs cost, at 2 dollars apiece?

Ans. 160 dollars.

64. A merchant purchased 8 fothers of iron ; how many pounds was this? Ans. 19264.

65. What cost 2 reams of paper, at $1\frac{1}{2}$ cents per sheet?

Ans. 1440 cents.

66. A farmer started to market with 12 dozen dozen eggs ; he broke a half a dozen dozen by the way, and then returned and got 6 dozen dozen more ; how many did he take to market at last?

Ans. 2520.

Article 3. Fractions of Compound Numbers.

Case 1. *To reduce a fraction of a higher, to a fraction of a lower denomination, and fractions of lower to fractions of higher denominations.*

Ex. 1. Reduce $\frac{1}{48}$ of a bushel to the fraction of a pint.

Solution.—We learn from Art. 2, Obs. 3, that we reduce integers from larger to small denominations by multiplication. Fractions are reduced in the same manner as whole numbers.—Thus:—$\frac{1}{48} \times 4 = \frac{1}{12}$; $\frac{1}{12} \times 8 = \frac{2}{3}$; $\frac{2}{3} \times 2 = \frac{4}{3}$. Ans.

Or, by cancelation:

Sect. VIII, Art. 5, Obs. 2 and 3.

$$\begin{array}{r|l} \cancel{48} & 1 \\ \cancel{12} & \cancel{4} \\ 3\_\cancel{6} & \cancel{8}\_4 \\ & \cancel{2} \\ \hline 3 & 4 = \frac{4}{3} \text{ Ans.} \end{array}$$

We multiply by 4, 8 and 2, because 4 pecks make a bushel, &c.

2. Reduce $\frac{4}{3}$ of a pint to the fraction of a bushel.

Solution.—Integers are reduced from a lower to a higher denomination by division. (Art. 2, Obs. 3). Fractions are reduced in the same manner as whole numbers. Thus—$\frac{4}{3} \div 2 = \frac{2}{3}$; $\frac{2}{3} \div 8 = \frac{1}{12}$; $\frac{1}{12} \div 4 = \frac{1}{48}$ Ans.

Or, by cancelation:

Sect. VIII, Art. 6, Obs. 2.

$$\begin{array}{r|l} 3 & \cancel{4} \\ 2 & \\ 8 & \\ \cancel{4} & \\ \hline 48 & 1 = \frac{1}{48} \text{ Ans.} \end{array}$$

We divide by 2, 8, and 4, because 2 pints make a quart, 8 quarts make a peck, &c.

3. Reduce $\frac{1}{72}$ of a bushel to the fraction of a pint.

4. Reduce $\frac{8}{9}$ of a pint to the fraction of a bushel.

5. Reduce $\frac{1}{3024}$ of a hogshead, wine measure, to the fraction of a gill.

6. Reduce $\frac{2}{3}$ of a gill to the fraction of a hogshead.

7. Reduce $\frac{1}{504}$ of a barrel, wine measure, to the fraction of a pint.

8. Reduce $\frac{1}{2}$ a pint to the fraction of a barrel, wine measure.

9. Reduce $\frac{1}{336}$ of a barrel, beer measure, to the fraction of a pint.

10. Reduce $\frac{6}{7}$ of a pint, beer measure, to the fraction of a barrel.

11. Reduce $\frac{1}{40}$ of a French ell to the fraction of a nail.

12. Reduce $\frac{3}{5}$ of a nail to the fraction of a French ell.

How do we reduce fractions of higher, to fractions of lower denominations? How do we reduce fractions of lower, to fractions of higher denominations?

13. A cucumber grew $\frac{7}{42240}$ of a mile in length; what fraction was that of a foot?

14. A cucumber grew $\frac{7}{8}$ of a foot in length; what fraction is that of a mile?

15. Reduce $\frac{1}{47190}$ of an acre to the fraction of a foot.

16. Reduce $\frac{12}{13}$ of a foot to the fraction of an acre.

17. Reduce $\frac{1}{61236}$ of a solid yard to the fraction of a solid inch.

18. Reduce $\frac{16}{21}$ of a solid inch to the fraction of a solid yard.

19. Reduce $\frac{1}{288}$ of a pound Troy, to the fraction of a penny-weight.

20. Reduce $\frac{5}{6}$ of a penny-weight to the fraction of a pound.

21. Reduce $\frac{1}{33600}$ of a ton to the fraction of an ounce.

22. Reduce $\frac{20}{21}$ of an ounce to the fraction of a ton.

23. Reduce $\frac{1}{384}$ of a pound to the fraction of a dram.

24. Reduce $\frac{2}{3}$ of a dram to the fraction of a pound.

25. Reduce $\frac{3}{928}$ of a pound to the fraction of a scruple.

26. Reduce $\frac{27}{29}$ of a scruple to the fraction of a pound.

27. Reduce $\frac{1}{4320}$ of a day to the fraction of a minute.

28. Reduce $\frac{1}{3}$ of a minute to the fraction of a day.

29. Reduce $\frac{1}{28800}$ of a circle to the fraction of a minute.

30. Reduce $\frac{3}{4}$ of a minute to the fraction of a circle.

CASE 2. *To reduce fractions of compound numbers to integers of the same, and also to reduce the integers back to fractions.*

Ex. 1. Reduce $\frac{1}{2}$ a bushel to pecks.

Solution.—As 4 pecks make a bushel, it is evident that $\frac{1}{2}$ a bushel contains $\frac{1}{2} \times 4 = 2$ pecks, Ans.

2. What part of a bushel is 2 pecks?

Solution.—As 4 pecks make a bushel, it is evident that 2 pecks are $\frac{2}{4}$ or $\frac{1}{2}$ a bushel.

3. Reduce $\frac{3}{4}$ of a pint to gills.

4. What part of a pint is 3 gills?

5. Reduce $\frac{1}{4}$ of a peck to quarts.

6. What part of a peck is 2 quarts?

7. Reduce $\frac{5}{8}$ of a bushel to pecks and quarts.

Solution.—Multiply $\frac{5}{8}$ by 4, we obtain $\frac{20}{8} = 2\frac{4}{8}$ pecks; multiplying $\frac{4}{8}$ by 8, we obtain $\frac{32}{8} = 4$ qts. Ans. 2 pks. 4 qts.

8. Reduce 2 pks. 4 qts. to the fraction of a bushel.

Solution.—We first reduce the 4 qts. to the fraction of a peck by dividing by 8. ($4 \div 8 = \frac{1}{2}$.) We then have $2\frac{1}{2} = \frac{5}{2}$ pecks, which

---

How do we reduce fractions of compound numbers to integers of the same? Explain the operation of Ex. 7 and 8, Case 2.

This operation may be shown thus—

$$\begin{array}{rl} & 5 \text{ numerator.} \\ & 4 \text{ pecks in a bushel.} \\ \text{denom. } 8) & \overline{20} \\ \text{pks. } & \overline{2+4} \\ & 8 \text{ qts. in a peck.} \\ \text{denom. } 8) & \overline{32} \\ & \overline{4} \text{ qts.} \end{array}$$

Or, the following form may be adopted if preferred by the student.

$$\text{denom.}\ \begin{array}{c|l} \cancel{8} & 5\times\cancel{4}=2 \text{ pks. \& 1 rem.} \\ \cancel{2} & 1\times 8=4 \text{ qts.} \end{array}$$

We cancel 4 into 8, leaving 2; dividing 5 by 2 we obtain 2 pks. and 1 remainder, which we place below the 5. We multiply 1 by 8, which cancels against the 2 leaving 4, and $1\times4=4$.

Ans. 2 pks. 4 qts.

It will be perceived that the number at the left is not canceled unless it divides the numbers at the right without a remainder, although we divide the numbers at the right by the number at the left.

This process is the same as in Art. 2, General Rule for Reduction Descending, except that we divide by the denominator after each multiplication.

we reduce to the fraction of a bushel by dividing by 4. ($\frac{5}{2}\div 4=\frac{5}{8}$.) This gives $\frac{5}{8}$ of a bushel as our answer.

The operation may be shown, thus—

$$\begin{array}{c|l} \cancel{8} & \cancel{4}=\frac{4}{8}=\frac{1}{2} \\ \hline 4 & 2\frac{1}{2}=\frac{5}{\cancel{2}}=\frac{5}{8} \text{ of a bushel Ans.} \end{array}$$

The reasoning is the same as above. Hence—

To reduce integers of compound numbers to fractions of the same, we have this

RULE.—I. *Divide the lowest denomination by that number which it takes of this to make a unit of the next higher denomination, and prefix to the result the number given of this higher denomination.*

II.—*Divide this in the same manner as the first; so continue to do until it is reduced to the fraction re ui d*

REMARK.—If any denomination is wanting, place a cipher in its stead.

9. Reduce $\frac{3}{10}$ of a hhd., wine measure, to its proper value.

10. Reduce 18 galls. 3 qts. $1\frac{1}{5}$ pts. to the fraction of a hogshead.

11. Reduce $\frac{2}{5}$ of a bl., beer measure, to its proper value.

12. Reduce 14 galls. 1 qt. $1\frac{1}{5}$ pts. to the fraction of a barrel.

13. What is the value of $\frac{2}{3}$ of a yard, cloth measure?

14. Reduce 2 qrs. $2\frac{2}{3}$ na. to the fraction of a yard.

How do we proceed in canceling? To what does this process correspond? How do we reduce integers of compound numbers to fractions of the same?

15. What is the value of $\frac{4}{7}$ of a mile?

17. What is the value of $\frac{7}{8}$ of an acre?

19. What is the value of $\frac{2}{3}$ of a cord?

21. What is the value of $\frac{5}{9}$ of a pound Troy?

23. What is the value of $\frac{6}{13}$ of a ton?

25. What is the value of $\frac{5}{11}$ of a pound, Apothecaries weight?

27. $\frac{4}{5}$ of a month is how many days, hours, &c.

29. What is the value of $\frac{2}{7}$ of a circle?

CASE 3. *To reduce the decimal of a compound number to its proper value.*

Ex. 1. Reduce .625 of a bushel to its proper value; that is, to pecks and quarts.

Operation.

```
       .625
          4
       ----
pks. 2.500
          8
       ----
qts. 4.000
```

Ans. 2 pks. 4 qts.

$.625 = \frac{625}{1000}$. (Sect. VIII, Art. 7, Obs. 2.) Reducing this as in Case 2, Ex. 7, we obtain 2 pks. 4 qts. as our answer. But since the denominator of a decimal fraction is 1 with ciphers annexed (Sect. VIII., Art. 7, Obs. 2,) we may dispense with the divis-

16. Reduce 4 fur. 22 rds. 4 yds. 2 ft. $1\frac{5}{7}$ in. to the fraction of a mile.

18. Reduce 3 R. 20 sq. rds. to the fraction of an acre.

20. Reduce 85 sq. ft. 576 sq. in. to the fraction of a cord.

22. Reduce 6 oz. 13 pwt. 8 grs. to the fraction of a pound.

24. Reduce 9 cwt. 23 lbs. 1 oz. $3\frac{9}{13}$ drs. to the fraction of a ton.

26. Reduce 5℥. 3ʒ. 1℈. $18\frac{2}{11}$ grs to the fraction of a pound.

28. 3 wks. 1 d. 9 hrs. 36 min. is what fraction of a month?

30. Reduce 102° 51′ $25\frac{5}{7}$″ to the fraction of a circle.

CASE 4.—*To change numbers of different denominations to a decimal of one denomination.*

2. Reduce 2 pks. 4 qts. to the decimal of a bushel.

Operation.

```
8 | 4.0
  | -----
4 | 2.500
  | -----
Ans. .625
```

As we have before said, (Sect. VIII, Art. 7, Obs. 9) a whole number can be reduced to a decimal by annexing ciphers. Hence, 4 qts.=4.0 qts., and this divided by 8, (because 8 qts., make 1 peck,) gives .5 of a peck. We then have 2.5 pks. which divi-

---

If any denomination is wanting, how do we proceed? How do we reduce the decimal of a compound number to integers? How do we reduce the integers back to decimals?

ion, and point off as in multiplication of decimals. (Sect. VIII, Art. 11. Rule.)

REMARK.—The learner will perceive that the numbers at the left of the decimal point are not multiplied, they not being decimals, but whole numbers.—Hence—

To change a decimal of a compound number to its proper value, we have this

RULE I. — *Multiply the given decimal by that number which it takes of the next less denomination to make 1 of this higher, and point off as in Multiplication of Decimals.* (Sect. VIII, Art. 11, Rule.)

II. *Proceed in the same manner, multiplying the decimals only, of each succeeding product, until no decimals remain, or it is reduced to the lowest denomination of the kind to which it belongs. The numbers at the left of the different separating points, will form the required answer.*

NOTE.—If after any multiplication there is no whole number at the left of the separating point, a cipher may be put in its stead.

ded by 4, (because 4 pks. make 1 bushel,) gives .625 of a bushel as our answer. Hence—

To change numbers of different denominations to a decimal of one denomination, we have this

RULE I.—*Begin with the least denomination: annex ciphers to the right and divide it by that number which it takes of this to make a unit of the next higher, and point off a decimal for every cipher annexed.*

II. *To the decimal thus obtained prefix the number given (if any,) of this higher denomination, and divide as before, pointing off as in Division of Decimals.* (Sect. VIII, Art. 12, Rule, and Rem. 3.) *So continue to do until it is reduced to the denomination required.*

NOTE 1.—If any denomination is wanting, a cipher may be put in its place at the left of the decimals.

2. Ciphers must be prefixed to the several quotients, (not to the integral parts,) when it is necessary, to complete the decimal places.

3. Reduce .75 of a bushel to its proper value.

4. Reduce 3 pks. to the decimal of a bushel.

5. Reduce .25 of a hhd., wine measure to its proper value.

6. Reduce 15 galls. 3 qts. to the decimal of a hhd., wine measure.

7. Reduce .8 of a gallon to its proper value.

8. Reduce 3 qts. 1.6 gi. to the decimal of a gallon.

9. Reduce .5625 of a yard to quarters and nails.

10. Reduce 2 qrs. 1 na. to the decimal of a yard.

11. Reduce .8125 of a mile to furlongs and rods.

12. Reduce 6 fur. 20 rds. to the decimal of a mile.

13. Reduce .5 of a yard to feet and inches.

14. Reduce 1 ft. 6 in. to the decimal of a yard.

15. Reduce .9375 of an acre to its proper quantity.

16. Reduce 3 R. 30 sq. rds. to the decimal of an acre.

17. Reduce .73125 of a pound Troy, to its proper value.

18. Reduce 8 oz., 15 pwts., 1 grs. to the decimal of a pound.

19. Reduce .7896875 of a ton to its proper value.

20. Reduce 15 cwt. 3 grs. 4 lbs. 6 oz., to the decimal of a ton.

21. Reduce .3984375 of a pound Apothecaries' weight, to its proper value.

22. Reduce 4℥. 6ʒ. 15 grs. to the decimal of a pound.

23. Change .5625 of a year to months and weeks.

24. Reduce 6 mos. 3 wks. to the decimal of a year.

REMARK.—There are two other cases of Reduction of Decimals of Compound numbers but they are not deemed of sufficient importance to present them in this work. We give below, an example of each, with their solution, by the aid of which the observing pupil can perceive their application and use.

1. Reduce .5 of a pint to the decimal of a gallon.

Operation.

```
2 | .50.
  |------
4 | .2500
  |------
Ans. .0625.
```

We annex ciphers to our decimal, which does not alter its value, (Sect. VIII, Art. 1, Obs 7. *b.*) and divide by 2 and 4 because 2 pts. make a quart, &c. We point off as in Division of Decimals.—(Sect. VIII, Art. 12. Rule.)

2. Reduce .0625 of a gallon to the decimal of a pint.

Operation.

```
     .0625
         4
     -----
     .2500
         2
     -----
Ans. .5000
```

We multiply by 4 and 2, because 4 qts. make a gallon, &c., and point off as in Multiplication of Decimals. (Sect. VIII, Art. 11, Rule.)

ARTICLE 4. ADDITION OF COMPOUND NUMBERS.

Ex. 1. There are two bushels of wheat in one bag, one bushel in another, and 2 bu., 3 pks. in another. How much in all?

Ans. 5 bu. 3 pks.

2. Thomas has 1 lb. 4 oz. of raisins, James has 2 lbs. 6 oz., and William has 2 lbs. 5 oz. How many pounds have they all?

Ans. 5 lbs. 15 oz.

3. A man sold at one time 18 bu. 3 pks. 6 qts. of oats, at another time 23 bu. 2 pks. 4 qts., and at another time 12 bu. 1 pk. 7 qts. How much did he sell in all?

Operation.

bu.	pks.	qts.
18	3	6
23	2	4
12	1	7
Ans. 55	0	1

We first write down the numbers, placing those of the same kind under each other.—We commence at the right hand, or lowest denomination, and add it as in simple numbers; $7+4+6=17$ qts. $=2$ pks. 1 qt.; setting down the 1 qt., we carry the 2 pks. to the next column; $1+2+3=6$ pks. and 2 pks to carry make 8 pks.; 8 pks $=2$ bu. and no remainder; setting down a cipher in the place of pecks, (because we had no pecks remaining,) we carry the 2 bu. to the column of bushels, and add as in simple numbers, setting down the entire sum.

From this illustration we derive the following

RULE FOR THE ADDITION OF COMPOUND NUMBERS.

I. *Set down the numbers, placing those of the same denomination under each other.* (Sect. II, Art. 2, Obs. 4.)

II. *Begin with the lowest denomination; add together the numbers given of this denomination, as in Addition of Simple numbers, and divide their sum by that number which it takes of this denomination to make a unit of the next higher; place the remainder under the column added, and carry the quotient to the next denomination.*

III. *Add the remaining denominations in the same manner as the first.*

PROOF.—*The same as in Addition of Simple numbers.* (Sect. II, Art. 2 Obs. 6.)

REMARK 1.—If any denomination is wanting, supply its place with a cipher.

2. The principles of Addition of compound and simple numbers are the same. (Art. 1, Obs. 6.) This can be shown by the following example:

Add together 2436, 1589, 1612, and 2849.

Operation.

thou.	hund.	tens.	units.
2	4	3	6
1	5	8	9
1	6	1	2
2	8	4	9
Ans. 8	4	8	6

$9+2+9+6=26$; $26\div10=2$ and 6 remainder. $4+1+8+3=16$ and 2 to carry $=18$; $18\div10=1$ and 8 rem. $8+6+5+4=23$ and 1 to carry $=24$, $24\div10=2$ and 4 rem. $2+1+1+2=6$ and 2 to carry $=8$.

What is the rule for addition of compound numbers? What is the proof? If any denomination is wanting, how do we proceed? What is the difference between the principles of addition of simple and compound numbers?

It will be perceived that we divide, &c., the same as in the above example, but as the remainder is always the right hand figure, this formal process of dividing may be dispensed with.

4. A merchant bought at one time 36 bu. 3 pks. 6 qts of wheat; at another time 25 bu. 2 pks. 3 qts.; at another time 30 bu. 1 pk. 7 qts.; at another time 28 bu. 4 qts.; and at another time 17 bu. 2 pks.; how many did he buy in all? Ans. 138 bu. 2 pks. 4 qts.

5. Bought 5 loads of oats; the first contained 27 bu. 8 qts.; the second, 37 bu.; the third, $26\frac{3}{4}$ bu. $6\frac{1}{2}$ qts.; the fourth, $36\frac{1}{2}$ bu. 1 pk. 6 qts.; and the fifth, 35 bu. $3\frac{7}{8}$ pks.; how many bushels in all?

Ans. 164 bu. $3\frac{1}{2}$ qts.

6. A wine merchant has 1 hhd. 27 galls. 3 qts. of Madeira; 2 hhd. 30 galls. $3\frac{1}{2}$ qts of Port; $61\frac{3}{10}$ galls. Champaign, and 3 hhds. $1\frac{2}{3}$ bls. of Claret wine; besides these he has $2\frac{7}{8}$ hhds. of Brandy; and 1 hhd. 24 galls. $2\frac{3}{4}$ qts. of other liquors. How much has he in all? Ans. 12 hhds. 1 bl. 31 galls. 2 qts. 1 pt. $3\frac{3}{5}$ gi.

NOTE.—The learner will bear in mind that the remainder, after dividing by $31\frac{1}{2}$ is half gallons, not whole gallons.

7. A wine merchant sold at one time 45 galls. 3 qts. 1 pt. 3 gi. of wine; at another time, 36 galls. 2 qts. 1 gi.; at another time 49 galls 1 pt.; at another time 57 galls. 3 gi.; and at another time 38 galls.; how much did he sell in all?

Ans. 3 hhds. 37 galls. 2 qts. 1 pt. 3 gi.

8. A brewer sold 5 casks of beer: the first contained 33 galls. 2 qts. 1 pt.; the second 28 galls. 1 pt.; the third, $34\frac{3}{4}$ galls.; the fourth, 26 galls. $3\frac{1}{2}$ qts.; the fifth, 18 galls. 2 qts.; how much was there in all? Ans. 2 hhds. 33 galls. 3 qts. 1 pt.

9. Bought 4 pieces of cloth; the first contained 22 yds. 3 qrs. 2 na.; the second, $21\frac{4}{5}$ yds.; the third, 18 yds. $2\frac{3}{4}$ qrs.; the fourth, 33 yds. 2 qrs. $2\frac{1}{5}$ na.; how many yards in all? Ans. 97.

10. If it takes 3 yds. $3\frac{3}{4}$ qrs. of cloth to make a coat, 2 yds. 2 qrs. 3 na. to make a pair of pants, and $2\frac{3}{4}$ qrs. to make a vest how many yards will it take for the whole? Ans. 7 yds. 1 qr. 1 na.

11. Thomas walked due north 43 miles, 7 fur. 29 rds. 12 ft., and James walked due south 56 miles, 5 fur. 34 rds. 14 ft.; how far apart are they? Ans. 100 M. 5 fur. 24 rds. $9\frac{1}{2}$ ft.

12. A gentleman traveling, rode the first week 167 M. 7 fur. 38 rds.; the second week, 180 M. 6 fur. 27 rds.; the third week, 173 M. 5 fur. 36 rds.; and the fourth week 77 M. 3 fur. 19 rds.; how many miles did he travel? Ans. 600.

Show by the example given why this is the case? Why do we not divide in this manner in addition of simple numbers?

13. A man has 4 farms; the first contains 120 A. 3 R. 27 sq. rds.; the second, 112 A. $2\frac{3}{4}$ R.; the third, $94\frac{4}{5}$ A.; and the fourth, $87\frac{9}{10}$ A.; how many acres does he own in all?

Ans. 416 A. 1 R. 9 sq. rds. or perches.

14. There are 6 piles of wood; the first contains 3 C. 64 sq. ft. 1126 s. in.; the second, 4 C. 112 s. ft. 1692 s. in.; the third, 5 C. 127 s. ft. 1724 s. in.; the fourth, $5\frac{2}{3}$ C.; the fifth, 6 C. $109\frac{3}{4}$ sq. ft.; and the sixth contains 3 C. 123 s. ft. 498 s. in.; how many cords in the whole? Ans. 30 C. 7 c. ft.

15. A silversmith purchased 4 bars of silver: the first weighed 2 lbs., 9 oz. 15 pwts. 22 grs; the second, $1\frac{4}{5}$ lbs.; the third, 3 lbs. $1\frac{5}{6}$ oz.; and the fourth, 2 lbs. 11 oz. 9 grs.; how much did they all weigh? Ans. 10 lbs. 8 oz. 4 pwts. 23 grs.

16. Sold at one time 1 cwt. 2 qrs. 23 lbs. $14\frac{1}{4}$ oz. of sugar; at another time $2\frac{7}{8}$ cwt.; at another time 2 cwt. 1 qr. $16\frac{3}{4}$ lbs.; and at another time 2 cwt. 3 qrs. 19 lbs. 13 oz. 8 drs.; how much was sold in all? Ans. 9 cwt. 3 qrs. 23 lbs.

17. Bought 4 stacks of hay; the first contained 3 T. 19 cwt. 2 qrs. 15 lbs.; the second, 2 T. 14 cwt. $3\frac{4}{5}$ qrs.; the third, $3\frac{3}{10}$ T.; and the fourth, 4 T. $19\frac{2}{5}$ cwt., how many tons in all?

Ans. 15.

18. An Apothecary mixed one compound with 5 simples: the first weighed 1 lb. 2 ℥. 4ʒ. 1 ℈.; the second weighed 9 ℥. 7ʒ. 2 ℈. 16 grs.; the third, 6 ℥. 5ʒ. 19 grs.; the fourth, 3 ℥. 15 grs., and the fifth, 7 ʒ. 1 ℈.; what was the weight of the whole?

Ans. 2 ℔. 11 ℥. 1ʒ. 10 grs.

19. A gentleman attended school until he was 21 yrs. 212 da. of age; he then traveled 15 yrs. 84 da.; he then married and lived with his wife 14 yrs. 176 da., when his wife died; after this he lived 23 yrs. 219 da.; what was his age when he died?

Ans. 75 yrs. 240 da. 12 hrs.

20. Upon what day of the year does Christmas come in leap year?

Ans. the 360th.

21. On what day of a common year does the fourth of July happen? Ans. the 185th.

22. Boston is 71° 3′ west longitude from Greenwich, Washington 6° 40′ farther west, Cincinnati 6° 44′ still farther, and the eastern line of Vancouvers' Island 38° 33′ still farther west. Required the longitude of Vancouvers' Island from Greenwich?

Ans. 123°.

23. Bought 4 casks of butter: the first contained 1 fir. 29 lbs.; the second, 2 fir. 43 lbs.; the third, $1\frac{3}{4}$ fir.; and the fourth, $2\frac{7}{8}$ fir.; how much was there in all? Ans. 8 fir. 51 lbs.

24. Bought 6 boxes of eggs: the first contained 2 dozen dozen; the second. 7 dozen dozen; the third, 12 dozen dozen; the fourth, half a dozen dozen; the fifth, 1 great gross; and the sixth, 12 score. How many eggs in all, and how many dozen?

Ans. 5064 eggs = 422 dozen.

25. Bought 1 ream, 18 quire, 9 sheets of paper at one time; 2 reams, 10 sheets at another time; $1\frac{4}{8}$ reams at another time; $2\frac{5}{6}$ reams at another time; and 1 ream, 8 quire, 13 sheets at another time; how much did I buy in all? Ans. 10 reams.

26. To $\frac{3}{4}$ of a bushel, add $\frac{1}{2}$ a peck. Ans. 3 pks. 4 qts.

27. To $\frac{7}{8}$ of a hhd., wine measure, add $\frac{3}{4}$ of a barrel.

Ans. 1 hhd. 15 galls. 3 qts.

NOTE.—Reduce both to their value in integers, and then add them.

28. To $\frac{5}{6}$ of a mile, add $\frac{2}{3}$ of a rod.

Ans. 6 fur. 27 rds. 5 ft. 6 in.

29. To $\frac{7}{8}$ of a cwt., add $\frac{3}{4}$ of a pound.

Ans. 3 qr. 13 lbs. 4 oz.

30. To $\frac{1}{2}$ a year, add $\frac{2}{3}$ of a day. Ans. 183 da. 7 hr.

ARTICLE 5. SUBTRACTION OF COMPOUND NUMBERS.

Ex. 1. There were 2 bu. 3 pks of oats in a box, and 1 bu. 2 pks. were taken out; how much was left? Ans. 1 bu. 1 pk.

2. From a box containing 8 bu. 2 pks. 6 qts., 5 bu. 1 pk. 2 qts. were taken, how much remained? Ans. 3 bu. 1 pk. 4 qts.

3. A man having 64 bu. 1 pk. 3 qts. of wheat, sold 45 bu. 3 pks. 6 qts.; how much had he left? Ans. 18 bu. 1 pk. 5 qts.

Operation.

	bu.	pks.	qts.
	64	1	3
	45	3	6
Ans.	18	1	5

We cannot take 6 qts. from 3 qts., but we can borrow 1 pk. (8 qts.) from the next order; then 8 qts. + 3 qts. = 11 qts., 11 qts. —6 qts. = 5 qts.; we set down the 5 qts., and as we borrowed 1 pk. we must carry 1 to the column of pecks, to make up for that we borrowed 1 + 3 = 4 pks.; we cannot take 4 pks. from 1 pk., but we can borrow 1 bu. (4 pks.) from the column of bushels: 4 pks. + 1 pk. = 5 pks.—4 pks. = 1 pk. We set down the 1 pk., and carry 1 (to pay for that we borrowed) to 5 making 6; 6 from 14 = 8, &c.

This process may also be shown, thus—

bu.	pks.	qts.		bu.	pks.	qts.
64	1	3	=	63	4	11
				45	3	6
			Ans.	18	1	5

This operation is so simple that it requires no explanation.

From these illustrations we derive the following

RULE FOR THE SUBTRACTION OF COMPOUND NUMBERS.

I. *Write the numbers, the less under the greater, placing those of the same denomination under each other.*

II. *Commence with the lowest denomination: take successively each denomination from the one above it, placing the result below as in simple numbers.* (Sect. III, General Rule.)

III. *If the lower number should be the largest, add to the upper number a unit of the next higher denomination, after which, subtract as usual, and carry 1 to the next denomination for that which was borrowed.*

PROOF.—*The same as in Subtraction of Simple Numbers.* (Sect. III, Art. 2, Obs. 7.)

REMARK 1. If any denomination is wanting, supply its place with a cipher.

2. The principles of subtraction of simple and compound numbers are the same. (Art, 1, Obs. 6.) This may be shown by the following example:

From 356 take 289.

1st, Operation.

hund.	tens.	units.
3 ..	5 ..	6
2 ..	8 ..	9
	6 ..	7

2d Operation.

	hunds.	tens.	units.	
356 =	2 ..	14 ..	16	
	2 ..	8 ..	9	
		6 ..	7	= 67.

It will be perceived that the principle is the same in this example as in the one above.

4. A farmer raised 1000 bushels of wheat, and sold 679 bu. 3 pks. 6 qts.; how much had he left? Ans. 320 bu. 2 qts.

5. A wine merchant sold from a hogshead of Madeira, 35 galls. 3 qts. 1 pt. 2 gi.; he then put in 28 galls. 1 qt. 3 gi., and afterwards sold 43 galls. 3 gi.; how much had he left?
Ans. 12 galls. 1 qt. 2 gi.

6. A wine merchant drew 4 bls. 1 qt. 1 gi. of wine from a cask containing 3 bls. 31 galls. 3 qts. 1 pt. 3 gi. Required—the quantity left in the cask. Ans. 1 pt. 2 gi.

7. From $1\frac{3}{8}$ hhds. of Port wine was taken $54\frac{3}{4}$ galls. How much was left? Ans. 31 galls. 3 qts. 1 pt.

What is the rule for the subtraction of compound numbers? What is the proof? If any denomination is wanting, how do we proceed? What is the difference between the principles of subtraction of simple and compound numbers? Explain why this is the case.

8. From $2\frac{1}{2}$ bbls. of beer, was taken $34\frac{7}{8}$ galls. How much remained? Ans. 1 bl. 19 galls. 1 pt.

9. From a piece of cloth containing $18\frac{3}{4}$ yds., was made a coat containing 2 yds. 1 qr. 3 na., a vest containing $2\frac{1}{2}$ qrs., and a pair of pants containing $1\frac{7}{8}$ yds.; how much was left?

Ans. 13 yds. 3 qrs. 1 na.

10. A traveled 140 miles: B followed after him 76 M. 7 fur. 35 rds.; how far apart were they? Ans. 63 M. 5 rds.

11. It is 24 miles from Columbus, O. to Delaware; if a man travel $15\frac{2}{3}$ miles, from Columbus, how much farther must he go to get to Delaware? Ans. 8 M. 2 fur. 26 rds. 11 ft.

12. A man owned a farm containing 134 A. $3\frac{3}{4}$ R.; he sold at one time 43 A. 2 R. 27 sq. rds., and at another time $32\frac{5}{8}$ A.; how much had he left? Ans. 58 A. 2 R. 23 sq. rds.

13. A certain pile of wood contains 17 cord, 112 s. feet. 1438 s. in.; another pile contains 14 C. 123 s. ft. 1649 s. in.; how much more in one pile than in the other?

Ans. 2 C. 116 s. ft. 1517 s. in.

14. A silversmith had a piece of silver weighing 4 lbs. 8 oz. $16\frac{5}{6}$ pwts. from this he made a set of spoons weighing 2 lbs. $6\frac{4}{5}$ oz., and thimbles enough to weigh $1\frac{3}{8}$ lbs.; how much was left?

Ans. 9 oz. 10 pwts. 20 grs.

15. A box of goods weighed 62 lbs. 12 oz.; the box alone weighed 18 lbs. $14\frac{3}{4}$ oz.; how much did the goods weigh?

Ans. 43 lbs. 13 oz. 4 drs.

16. If a waggon loaded with hay weighs 28 cwt. $3\frac{4}{5}$ qrs., and the waggon weighs 7 cwt. 3 qrs. 20 lbs., what is the weight of the hay?

Ans. 21 cwt. = 1 T. 1 cwt.

17. A merchant having $12\frac{2}{5}$ cwt. of sugar, sold at one time 3 cwt. 3 qrs. 21 lbs., at another time 2 cwt. 1 qr. 15 lbs., and at another time $6\frac{1}{25}$ cwt.; how much had he left? Ans. None.

18. From a package of medicine containing 4 ℔. 10℥. 6 ʒ. 1℈. 15 grs., was taken 2 ℔. 11 ℥. 7 ʒ. 2 ℈. 18 grs.; how much was left?

Ans. 1 ℔. 10 ℥. 6 ʒ. 1 ℈. 17 grs.

19. A. worked at a certain place 2 yrs. 186 da. 12 hrs.; B. worked at the same place 1 yr. 310 da. 14 hrs.; how much longer did A. work than B.? Ans. $241\frac{1}{6}$ da.

20. Henry's age is 17 yrs. 6 mo. 23 d.; George's age is 10 yrs., 7 mo. 21 d.; how much older is Henry than George?

Ans. 6 yrs. 11 mo. 2 d.

27. Washington was born Feb. 2d, 1732; he died Dec. 14th. 1799; what was his age at the time of his death?

Ans. 67 yrs. 9 mo. 22 d.

Operation.		
yrs.	mo.	d.
1799 ..	12 ..	14
1732 ..	2 ..	22
Ans. 67 ..	9 ..	22

We set down the year, the number of the month, (calling Jan. 1, Feb. 2, &c.) and the day of the month, and subtract as usual, reckoning 30 days to the month, and 12 months to the year. In this example, Dec. is the 12th month, Feb. the 2nd., &c.—Hence—

Obs. 1. *The period of time between any two dates may be found by subtracting the earlier date from the latter, the months being reckoned 30 days.* This rule is much used in calculating interest. (Sect. XIV, Art. 5.)

21. A note was given Jan. 24th, 1841, and paid Aug. 12th 1843; what time did it run? Ans. 2 yrs. 6 mo. 18 d.

Obs. 2. *The time which elapses from the giving of a note until the payment of it is called the time it has to run.*

22. A man left home Sept. 28th, 1837, and returned June 18th, 1840; how long was he absent? Ans. 2 yrs. 8 mo. 20 d.

23. How long has a note to run, which is given Nov. 16th, 1828, and paid Jan. 3d. 1847? Ans. 18 yrs. 1 mo. 17 d.

24. London is 51° 32′, and Gibralter 36° 6′ 30″ north latitude, what is the difference of latitude between the two places? Ans. 15° 25′ 30″.

25. From an area of 180°, take 57° 48′ 39″. Ans. 122° 11′ 21″.

26. The longitude of Boston is 71° 3′, and that of the Sandwich Islands 155° west from Greenwich; what is their difference in longitude? Ans. 83° 57′.

27. A man having 6 dozen dozen eggs, broke a half a dozen dozen; how many had he left? Ans. 792.

28. Take 149 lbs. from a barrel of flour, and how much remains? Ans. 47 lbs.

29. From a barrel of pork, take 97 lbs. Ans. 103 lbs.

30. From $\frac{5}{8}$ of a bushel, take $\frac{3}{4}$ of a peck. Ans. 1 pk. 6 qts.

31. From $\frac{5}{6}$ of a hogshead, wine measure, take $\frac{3}{8}$ of a gallon. Ans. 52 galls. 1 pt.

32. From $\frac{1}{2}$ a mile take $\frac{7}{10}$ of a furlong. Ans. 3 fur. 12 rds.

33. From $\frac{4}{5}$ of a cwt., take $\frac{7}{20}$ of a quarter. Ans. 2 qrs. 21 lbs. 4 oz.

34. From $\frac{2}{3}$ of a year, take $\frac{1}{6}$ of a day. Ans. 243 days 8 hrs.

35. From $\frac{7}{16}$ of a circle, take $\frac{7}{8}$ of a degree. Ans. 156° 37′ 30″.

How is the period of time between two dates found? What is meant by the time a note has to run?

Article 6. Multiplication and Division of Compound Numbers.

Ex. 1. A farmer has 6 bags of wheat each containing 2 bu. 3 pks. 6 qts. how much wheat in all the bags?

Operation.

bu.	pks.	qts.
2 --	3 --	6
		6
Ans. 17 --	2 --	4

We write our numbers as usual, placing our multiplier under the lowest denomination of the multiplicand, (qts.) Then, $6 \times 6 = 36$ qts.; 36 qts. = 4 pks. 4 qts.; we set down the 4 qts. and carry the 4 pks., $3 \times 6 = 18$ pks. and 4 pks. to carry make 22 pks.; 22 pks. = 5 bu. 2 pks.; we set down the 2 pks. and carry the 5 bu.; $2 \times 6 = 12$ and 5 to carry = 17 bu.

From this illustration we derive the following

RULE FOR THE MULTICATION OF COMPOUND NUMBERS.

I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Commence at the right hand multiply as in simple numbers and carry as in addition of compound numbers.*

Proof.—*The same as in simple numbers.* (Sect. IV, Art 2, Obs. 7.)

Remark 1. If any denomination is wanting in the multiplicand, supply its place with a cipher.

2. A farmer has 17 bu. 2 pks. 4 qts. of wheat put up in 6 bags, each containing an equal quantity; how much is there in each?

Operation.

bu.	pks.	qts.
6)17 --	2 --	4
Ans. 2 --	3 --	6

$17 \div 6 = 2$, and 5 remainder. This 5 is bushels, because the dividend, 17, is bushels. (Sect. V, Art. 2, Obs. 12, *a.*) 5 bu. = 20 pks.; 20 pks. + 2 pks. = 22 pks.; 22 pks. $\div 6 = 3$ pks. and 4 pks. remainder. 4 pks. = 32 qts. 32 qts. + 4 qts. = 36 qts.; 36 qts. $\div 6 = 6$ qts.

From this illustration we derive the following

RULE FOR THE DIVISION OF COMPOUND NUMBERS.

I. *Write the dividend and divisor as usual.* (Sect. V, Rule.)

II. *Commence with the highest denomination; divide, and place the quotient as in division of simple numbers.* (Sect. V, Rule.)

III. *If a remainder occurs reduce it to the next lower denomination and add to it the number given, (if any) of this denomination; after which divivde as usual.*

Proof.—*The same as in Simple Numbers.* (Sect. V, Art. 2 Obs. 9.

Remark 3. If any denomination

What is the rule for the multiplication of compound numbers? What is the proof? If any denomination is wanting in the multiplicand, how do we proceed? What is the difference between the principles of the multiplication of simple and compound numbers? Explain by the example given, why this is the case? Why do we not reduce in this way in simple numbers? What is the rule for the division of compound numbers? What is the proof?

2. The principles of multiplication of simple and compound numbers are the same as can be shown by the following example.

Multiply 256 by 8.

Operation.

thou.	hunds.	tens.	units.
	2	5	6
			8
Ans. 2	0	4	8

$6 \times 8 = 48 \div 10 = 4$ tens, 8 units. $5 \times 8 = 40$, and 4 to carry $= 44$; $44 \div 10 = 4$ hund. 4 tens. $2 \times 8 = 16$ and 4 to carry $= 20$; $20 \div 10 = 2$ thou. 0 hun.

It will be perceived that there is no difference between this operation, and that of Ex. 1.— But as the number that we set down is always the right hand figure of the partial product, it is evident that the above formal process is unnecessary.

3. A farmer took 7 loads of wheat to market, each load containing 22 bu. 2 pks. 5 qts.; how much was there in all?

5. A wine merchant has 9 casks of wine, each containing 20 galls. 1 pt.; how much had he in all?

7. A gentleman has 8 casks of brandy, each containing 45 galls. 1 pt.; how many gallons has he in all?

9. How many gallons in 5

in the divdend is wanting supply its place with a cipher.

4. The principles of the division of simple and compound numbers are the same, as can be shown by the following example:

Divide 2048 by 8.

Operation.

thou.	hund.	tens.	units.
8)2	0	4	8
Ans. 2	5	6	

2 thou. $\times 10 = 20$ hund.; $20 \div 8 = 2$ hund. and 4 hunds. rem.: $4 \times 10 = 40$ tens, and 4 tens $= 44$ tens. $44 \div 8 = 5$ tens and 4 tens rem.; $4 \times 10 = 40$ units, and 8 units $= 48$ units; $48 \div 8 = 6$ units.

It will be perceived that the operation is similar to that of Ex. 2. But the denomination given only occupies the place of the cipher, when added to the one that is reduced, this process may be dispensed with. (Sect. V, Art. 2, Ex. 9, and Obs. 13.)

4. If 7 equal loads of wheat contain 158 bu. 2 pks. 3 qts., how much does each contain?

6. If 9 casks of wine contain 181 galls. 1 pt., how many gallons does each contain, each containing an equal quantity?

8. If 8 casks of brandy, containing each an equal quantity, hold 361 galls., how many gallons are there in each?

10. If 5 equal sized flasks con-

If any denomination is wanting in the dividend, how do we proceed?— What is the difference between the principles of the division of simple and compound numbers? Explain why this is the case. Why is not this process adopted in simple numbers?

flasks of beer, each containing 6 galls. 2 qts. 1 pt?

11. How many yards of cloth will it take to make 7 coats, each containing 3 yds. 2 qrs. 1 na.?

13. If a man walk 22 M. 6 fur. 29 rds in one day, how far can he walk in 6 days?

15. A man has 4 farms, each containing 54 A. 3 R. 23 sq. rds.; how much in all?

17. There are 6 piles of wood each containing 12 C. 110 s. ft. 1129 s. in.; how many cords in all?

19. A silversmith made 12 spoons, each weighing 2 oz. 3 pwts. 8 grs. Required, the weight of the whole?

21. A farmer sold 11 loads of hay, each weighing 18 cwt. 3 qrs. 21 lbs. Required, the weight of the whole?

23. A merchant bought 10 sacks of coffee, each weighing 156 lbs. 12 oz. 14 drs. Required, the weight of the whole?

25. What is the weight of 9 packages of medicine, each weighing 3 ℔. 9 ℥. 5 ʒ. 1 ℈. 16 grs.?

27. If it takes a man 1 hr. 10 min. 20 sec. to walk a league, how long will it take him to walk 7 leagues?

29. The earth moves daily in her orbit, about 59′ 8″; how much does it move in 9 days?

31. There are 18 sacks of flaxseed, each containing 9 bu. 3 pks. 6 qts.; how much in all?

REMARK 5. When the multiplier is a composite number we may first mul-

tain 33 galls. 1 pt., how much does each contain?

12. If it takes 24 yds., 3 qrs. 3 na. to make 7 coats, how much does it take to make one?

14. If a man walks 137 M. 14 rds. in 6 days, how far does he walk in a day?

16. If 4 equal farms contain 219 A. 2 R. 12 sq. rds., how many acres does each contain?

18. If 6 equal piles of wood contain 77 C. 23 s. ft., 1590 s. in., how much is there in each pile?

20. If 12 spoons weigh 2 lbs. 2 oz., what is the weight of each, each weighing the same?

22. If 11 equal loads of hay weigh 10 T. 8 cwt. 2 qrs. 6 lbs., what is the weight of each load?

24. If 10 sacks of coffee weigh 15 cwt. 2 qrs., 18 lbs. 12 drs., what is the weight of each sack, all being equal?

26. If 9 equal packages of medicine weighs 34 lb. 3 ℥. 2 ʒ. 1 ℈. 4 grs., what is the weight of each?

28. If it takes a man 8 hrs. 12 min. 20 sec. to walk 7 leagues, how long will it take him to walk one league?

30. If the earth moves in her orbit 8° 52′ 12″ in 9 days, how far does she move per day?

32. If 18 sacks contain 178 bu. 3 pks. 4 qts. of flaxseed, how much does each contain, all being of the same size?

REMARK 6. When the divisor is a composite number, we may first divide

When the multiplier or divisor is a composite number, how may we proceed?

tiply by one factor and then by the other. (Sect. IV, Art. 4, Obs. 4.)

Operation of the last example.

bu. pks. qts.
9 -- 3 -- 6 $18 = 6 \times 3$
6

59 -- 2 -- 4 = quantity in
3   [6 sacks.

Ans. 178 -- 3 -- 4 = quantity in
[$6 \times 3 = 18$ sacks.

33. In 36 casks of wine each containing 28 galls. 3 qts. 1 pt. 3 gi.; how many gallons?

35. If it takes 2 yds. 1 qr. 3 na. of cloth to make a pair of pantaloons, how much will it take to make 24 pair?

37. A man took 49 loads of wood to the city, each containing 1 C. 15 s. ft., 124 s. in.; how many cords in all?

39. A merchant bought 63 kegs of butter, each weighing 32 lbs. 14 oz. 12 drs. Required, the weight of the whole?

41. If a man walk 25 M. 6 fur. 37 rds. in a day, how far can he walk in 127 days?

Remark 7. When the multiplier exceeds 12, and is not a composite number, we multiply each denomination, and reduce it on a part of the slate separate from the multiplier and multiplicand, and merely set down the result as when the multiplier is less than 12.

first by one factor, and then by the other. (Sect. V, Art. 4, Obs. 3.)

Operation of the last example.

bu. pks. qts. $18 = 3 \times 6$
3)178 -- 3 -- 4
[of 18, or 6 sks.
6) 59 -- 2 -- 4 = quantity in $\frac{1}{3}$

Ans. 9 -- 3 -- 6 = quantity in $\frac{1}{6}$
[of 6, or 1 sk.

34. If 36 equal casks of wine contain 16 hhds. 34 galls. 3 qts. 1 pt.; how much does each contain?

36. If it takes $58\frac{1}{2}$ yds. of cloth to make 24 pair of pantaloons, how much does it take to make one pair?

38. If 49 equal loads of wood contain 54 C. 98 s. feet. 892 s. in., how much is that per load?

40 If 63 kegs of butter weigh 1 T. 2 qrs. 24 lbs. 1 oz. 4 drs., what is the weight of each, all weighing alike?

42. If a man travel 3284 M. 7 fur. 19 rds. in 127 days. traveling the same distance each day how far does he travel per day?

Remark 8. When the divisor is greater than 12, and is not a composite number, perform the operation by long division.

When the multiplier exceeds 12, and is not a composite number, how do we proceed? By what other method? What is the third method? When the divisor exceeds 12, and is not a composite number, how do we proceed?

Operation of the last example.

```
          M.   fur.  rds.
          25 .. 6 .. 37
                    127
      ---------------------
Ans.  3284 .. 7 .. 19

                rds.
   37×127 = 4699
          4|0)469|9
          ------
      fur. 117–19 rds. rem.
    6×127 = 762 fur. add.
            -----
         8)879 fur.
           -----
         M. 109–7 fur. rem.
  25×127 = 3175 M. add.
           ------
           3284  M.
```

Operation of the last example.

```
        M.    fur.  rds.       [Ans.
127)3284 .. 7 .. 19(25 .. 637
    254
    ---
    744
    635
    ---
    109 M. remainder.
      8
    ---
    872 furlongs.
      7 fur. add.
    ---
127)879(6 fur.
    762
    ---
    117 fur. remainder.
     40
    ----
    4680 rods.
      19 rds. add.
    ----
127)4699(37 rds.
    381
    ---
    889
    889
    ---
    000
```

Or, if the student prefers, he may first multiply by 10, and the product thence arising by 10, which will give the product by 100: then multiply the product by 100, by the number of hundreds, the product by 10, by the number of tens; and the first multiplicand by the number of units; and add these last three results together. Thus—

It will be perceived that the only difference between this operation, and that of Ex. 2, is that in this operation the entire work is set down, whilst in Ex. 2 it is not.

Operation 2.

$127 = 100 + 20 + 7$, or 1 hund., 2 tens, 7 units.

M.	fur.	rds.		M.	fur.	rds.
25 ..	6 ..	37	× 7 =	181 ..	0 ..	19
		10				
258 ..	5 ..	10	× 2 =	517 ..	2 ..	20
		10				

Add	2586 ..	4 ..	20	= product by 100.
	517 ..	2 ..	20	= product by 2 tens, or 20.
	181 ..	0 ..	19	= product by 7 units, or 7.
Ans.	3284 ..	7 ..	19	= product by $100 + 20 + 7 = 127$.

Operation 3.

25 M. 6 fur. 37 rds. = 8277 rds.

```
  8277
   127
 -----
 57939
16554
8277
-------
1051179 rds.
```

1051179 rds. = 3284 M. 7 fur. 19 rds. Ans.

Obs. 1. It may be asked by some pupils, why we cannot multiply compound numbers, when the multiplier exceeds 12, in the same manner as simple numbers; that is, by multipiying by each figure of the multiplier separately, and writing the first figure of each product under the figure by which we multiply? We answer, that we can, if the multiplicand is first reduced to but one denomination, as in Operation 3. But when the multiplicand is not thus reduced, *each denomination must be multiplied separately by the entire multiplier, on account of the inequality of their increase.* Did all the denominations increase by the same ratio, we could then multiply in compound as in simple numbers.

What is the difference between the operations of Ex. 41 and 42? When can we multiply compound numbers by multipliers exceeding 12, in the same manner as in simple numbers? Why cannot we proceed in this way when the multiplicand is not thus reduced? What would be necessary, in order to have the same rule apply to compound and simple numbers?

43. How much wine in 29 casks, each containing 47 galls. 3 qts. 2 pts. 3 gi.?

44. If 29 casks of wine, each containing the same quantity, hold 22 hhds. 8 galls. 2 qts. 1 pt. 3 gi., how much is that for each?

45. How much wheat can be put in 23 casks, each containing 4 bu. 3 pks. 6 qts.?

46. If 23 casks hold 113 bu. 2 pks. 2 qts. of wheat, how much is that for each, they all being of the same size?

47. If it takes 4 yds. 3 qrs. 3 na. of cloth to make a suit of clothes, how much will it take to make 349 suits?

48. If it takes 1723 yds. 3 na: of cloth to make 349 suits of clothes, how much does it take to make one suit?

49. How much sugar in 53 hhds., each containing 12 cwt. 2 qrs. 23 lbs.?

50. If 53 hhds. of sugar contain 33 T. 14 cwt. 2 qrs. 19 lbs,, how much is that for each?

To find the difference in the time of two places :

Obs. 2. The earth in revolving on its axis, performs one entire revolution in 24 hours, and as the circumference of the earth is 360°, she makes $360° \div 24 = 15°$ of motion in 1 hour of time, and $60 \div 15 = 4$, *i.e.* 1° of motion in 4 minutes of time. Therefore, *there is a difference of 1 hour in the time between two places for every 15° of longitude between them, and of 4 minutes for every degree.* Hence—

To find the difference in the time of two places:

Obs. 3. *Multiply the difference in the longitude of the two places, (in degrees and minutes,) by 4: the result will be the difference in time, in minutes and seconds.*

REMARK.—The student will bear in mind, that as the earth revolves from west to east, *places easterly have the time earlier than places westerly.** Hence—

Obs. 4. *To find the time of places easterly* ADD *the difference in the time of the two places to the given time, and to find the time of places westerly* SUBTRACT *the difference in the time of the two places, from the given time.*

51. The difference in the longitude between Boston and Wash-

How often does the earth perform one revolution on its axis? How many degrees of motion does it make in one hour of time? Why is this the case? How long does it take it to make 1° of motion? What is the inference deduced from these considerations? How then do we find the difference in the time of two places? Which has the time the earliest, places easterly, or places westerly? Why so?

* The teacher can best explain this by means of an apple or other round substance.

ington City is 6° 40′; now when it is 4 o'clock, P. M. at Washington, what is the time at Boston?

Operation.

6°	40′
	4
26 min.	40 sec.

We find the difference in the time of the two places to be 26 min. 40 sec.; and Boston being east of Washington, the time is earlier, and consequently it must be 26 min. 40 sec. past 4 o'clock at Boston, when it is 4 o'clock at Washington.

52. Washington, D. C., being 77° 1′ 30″, and Columbus. O., 83° 3′ W. longitude from Greenwich, I wish to know what time it is at Columbus, when it is 9, A. M., at Washington?

Ans. 35 min. 54 sec. past 8 o'clock, A. M.

53. Reading, Eng., is about 1°, and Buffalo, N. Y., is about 78° 55′ W. longitude from Greenwich. Required—the difference in the time of the two places.

Ans. 5 hrs. 11 min. 40 sec.

54. If it is 9 o'clock, 30 min. P. M. at Buffalo, what is the time at Reading?

Ans. 41 min. 40 sec. past 2 o'clock, A. M. (on the next morning.)

55. If Buffalo is 78° 55′ W. longitude, and Tunis, in Africa about 10° E. longitude from Greenwich, when it is 5 min. 20 sec. past 6 o'clock, P. M. at Buffalo; what is the time at Tunis?

Ans. 1 min. past 12 o'clock, midnight.

Remark 1. When one place is East, and the other place West longitude, we must add their longitude together to find the distance between them.

56. In the last example, when it is 26 min. 10 sec. past 1 o'clock A. M. at Tunis, what is the time at Buffalo?

Ans. 30 min. 30 sec. past 7 o'clock P. M. (*i.e.* the day before.)

Remark 2. The learner will observe in these examples, that we count our time by *half days* or 12 hours; and therefore, when we obtain more than 12 hours, it is some hour less than 12, of the preceding or succeeding A. M. or P. M. part of the day. Consequently, if our given time is less than the difference of time between the two places, and we wish to subtract the difference of time, we must add 12 hours to our given time, and then subtract as usual.

57. If a certain place is 160° E. longitude, and another place is 120° W. longitude, and it is 5 o'clock, P. M., at the latter place, what time is it at the former?

Ans. 40 min. past 11 o'clock, A. M. (*i. e.* before.)

How do we find the time of places easterly? Of places westerly? When one place is east, and another place is west longitude, how do we find the distance between them? In these examples, how do we count our time? What then is the result when we have more than 12 hours? If our given time is less than the difference between the time of the two places, and we wish to subtract, how do we proceed?

58. In the last example, when it is 10 o'clock, A. M. at the former place, what time is it at the latter?

Ans. 20 min. past 3 o'clock, P. M. (afterwards.)

59. What is the difference of time between the places last mentioned? Ans. 5 hrs. 20 min.

60. Suppose a meteor to appear so high in the heavens as to be visible at the same moment, at Boston 71° 4′ 9″, at the city of Washington 77° 1′ 30,″ and at the Sandwich Islands 155° W. longitude, and that its appearance at Washington, be at 45 min. 45 sec. past 11 o'clock in the evening, what would be the time of its appearance at Boston, and at the Sandwich Islands.

Ans. { At Boston, 9 min. 34 sec. past 12 o'clock, midnight (after.) At the Sandwich Islands, 33 min. 51 sec. past 6 o'clock, P. M. (before.) }

SECTION X.

EXCHANGE.

Obs. 1. *The method of finding the value of the currency of one country in that of another, is called* Exchange.

Obs. 2. Currency signifies the circulating medium of trade, and is called Money.

Obs. 3. Money is said to have two values: an *intrinsic,* and a *commercial value.*

Obs. 4. The *intrinsic value,* is the value of the money of one country, as compared with that of another, with respect both to *weight,* and the *purity of the metal of which it is made.*

Obs. 5. The *commercial value* is the compative value of the money of different countries, with respect to their *weight, fineness,* and *market price.*

Obs. 6. The *relative value* of foreign coins, (that is; the value they bear to each other,) is determined by the laws of the country.

The value of a few of the most common foreign coins in the

What is Exchange? What does currency signify? How many values has money? What is the intrinsic value? The commercial value? How is the relative value of foreign coins determined?

United States as established by Act of Congress, 1842, is shown in the following

TABLE:

The value of a	Pound Sterling, or Sovereign,	of England,	is	$4.846.
The "	a Guinea,	"	is	5.075.
The "	a Franc.	of France	is	0.185.
The "	a Five Franc piece,	"	is	0.93.
The "	a Dollar, of Mexico, Peru, and Chili,		is	1.00.
The "	a Doubloon, of Spain, Mexico, &c.,		is	15.535.
The "	a Ducat of Russia,		is	2.297.

Case 1. *To change Foreign Coins to Federal Money.*

Ex. 1. What is the value of 20 Pounds Sterling, in Federal Money?

Operation.

One Pound Sterling contains $4.846, therefore 20 Pounds Sterling contains 20 times $4.846.

$$
\begin{array}{r}
\$4.846 \\
20 \\
\hline
\text{Ans. } \$96.920
\end{array}
$$

Remark.—This character (£.) usually stands for the Pound or Sovereign.

The other coins are reduced to Federal Money in the same manner. Hence—

To change foreign coins to Federal Money:

Obs. 7. *Multiply the given number of coins, by the number of dollars in one coin as taken from the table.*

2. Reduce 50 Sovereigns to Federal Money. Ans. $242.30.
3. Change 75 Guineas to Federal Money. Ans. $380.62½.
4. Change 90 Guineas to Federal Money. Ans. $456.75.
5. Change 55 Francs to Federal Money. Ans. $10.175.
6. Change 180 Francs to Federal Money. Ans. $33.30.
7. In 84 Five Franc pieces, how many dollars? Ans. $78.12.
8. What is the value of 125 Five Franc pieces in Federal Money? Ans. $116.25.
9. Change 35 Doubloons to Federal Money. Ans. $543.725.
10. Change 18 Doubloons to Federal Money. Ans. $279.63.
11. Change 38 Ducats to Federal Money. Ans. $87.286.
12. Change 22 Ducats to Federal Money. Ans. $50.534.

Give the value of some of the foreign coins as shown in the table? How do we change foreign coins to Federal Money?

CASE 2. *To change Federal Money to other currencies.*

REMARK—This case is the opposite of the preceding one, and each proves the other.

Ex. 1. Change $72.69 to Pounds Sterling. Ans. 15.

Operation.

As 1 Pound contains $4.846. there will evidently be as many Pounds in $72.69, as $4.846 is contained in $72.69.

```
4.846)72.690(15 Ans.
      4846
      -----
      24230
      24230
      -----
      00000
```

Federal Money is changed to other currencies in the same manner. Hence—

To change Federal Money to other currencies;

Obs. 8. Divide the given sum by the number of dollars it takes to make one of the coin to which it is to be reduced, and point off as in Division of Decimals. (Sect. VIII, Art. 12. Rule.)

REMARK.—4 farthings make 1 penny; 12 pence make 1 shilling; and 20 shillings make 1 Pound or Sovereign, Hence—if there is a remainder in reducing Federal Money to Pounds Sterling, we can reduce it lower as in Division of Compound Numbers. (Sect. IX, Art. 6. Rule for the division of Compound Numbers). Farthings are marked, far.; pence, d. and shillings, s.; 1, 2, and 3 farthings are generally written as $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of a penny.

2. Change $121.15 to Sovereigns. Ans. 25.
3. Change $253.75 to Guineas. Ans. 50.
4. In $69.75 how many Five Franc pieces? Ans. 75.
5. In $217.49 how many Doubloons? Ans. 14.
6. In $68.91 how many Ducats? Ans. 30.
7. Change $32.83165 to pounds, shillings, and pence. Ans. £6, 15s; 6d.
8. Change $21.807 to Sterling Money. Ans. £4, 10s.
9. Change $52.3368 to Sterling Money. Ans. £10, 16s.
10. Change $27.591 to Sterling Money. Ans. £5, 13s. 5d. 3 far.+

How do we change Federal Money to other currencies? How is the Pound divided? If there is a remainder in reducing Federal Money to Pounds Sterling, how may we proceed?

SECTION XI.

ANALYSIS.

Article 1. Definitions and Mental Exercises.

Obs. 1. *Analysis signifies the separation of a body into parts, and is used in mathematics for the developement and illustration of principles, and may also be applied to the solution of questions with great practical advantage.* In the preceding sections we have analyzed *Principles;* in this section we design to analyze *practical questions,* or *Problems.*

Note.—We often hear people tell of "doing sums in their head." They in in reality, solve their questions by *Analysis,* or the *common sense rule.*

Obs. 2. This method is of extensive application, and great utility. *It strengthens the mental faculties, and accustoms the pupil to habits of close thinking,* and with a little practice he can become very expert at it.

No particular direction can be given for solving every question in analysis. The following may perhaps answer for general directions, but the learner must try and think for himself and not depend too much on others.

Obs. 3. *Examine the question attentively; reason according to the nature of the sum, and work as common sense directs.*

Remark.—The operation of solving questions by analysis is called an *Analytic Solution.*

Obs. 4. *In solving questions analytically, we generally reason from the given number to* 1, *and then from* 1 *to the required number.*

MENTAL EXERCISES.

1. If 5 tons of hay cost $30, what will 8 tons cost?

Ans. 48.

Solution.—If 5 tons cost $30, 1 ton will cost $\frac{1}{5}$ of $30, or $6, and 8 tons will cost 8 times $6 or $48.

2. If 3 firkins of butter cost $15, what cost 4 firkins? 6; 8; 10; 12?

What does analysis signify? To what is it used in mathematics? To what may it be applied? How do people who "do sums in their head" in reality solve their questions? What advantages are derived from this method? What directions are given for analyzing? What is the operation of solving a question by analysis called? How do we generally reason in solving questions analytically?

3. If a man can earn \$14 in 7 days, how much can he earn in 3 days? 4; 5; 6; 8; 10; 12; 15?

4. If a man spend \$18 in 6 days, how much does he spend in 4 days? 3; 5; 9; 10; 12?

5. If a boy walk 72 miles in 6 days, how far does he walk in 5 days? 8; 10; 12?

6. If 4 men can do a piece of work in 3 days, how long will it take 6 men to do the same work? Ans. 2 days.

Solution.—If it takes 4 men 3 days to do the work, it will take 1 man $3\times4=12$ days to do it. Again if it takes 1 man 12 days to do it, 6 men can do it in $\frac{1}{6}$ of the time, or 2 days.

7. If it takes 3 men 8 days to do a piece of work, how long will it take 2 men to do it? 4; 5; 6; 8; 12; 24?

8. If 4 horses consume a stack of hay in 12 weeks, how long would it keep 2 horses? 6; 8; 12; 24?

9. If a certain pasture keeps 6 cows 6 weeks, how long will it keep 4 cows? 8; 9; 12; 18; 36;

10. Two boys were counting their money: one said, "I have 50 cents;" the other said, "I have $\frac{4}{5}$ as much as you." How much had the latter boy?

Solution.—$\frac{1}{5}$ of 50 cents is 10 cents, and $\frac{4}{5}$ is 4 times as much as $\frac{1}{5}$, and 4 times 10 are 40. Ans. 40 cents

11. A. and B. were talking of their ages: A.'s age was 63 years, and B.'s age was $\frac{5}{7}$ of A.'s age. What was the age of B.?

12. John and Thomas were playing marbles: John had 24, and Thomas had $\frac{3}{8}$ as many as John. How many had Thomas?

13. A. has 96 sheep; B. has $\frac{7}{12}$ as many as A. How many has B.

14. One man has 132 hogs, another man has $\frac{9}{11}$ as many; how many has the latter man?

15. Thomas has 120 bushel of oats; $\frac{9}{10}$ of Thomas' is equal to $\frac{6}{7}$ of James'; how many bushels has James? Ans. 126.

Solution.—$\frac{1}{10}$ of 120 is 12, and $\frac{9}{10}$ of 120 is 9 times 12 or 108; then 108 is $\frac{6}{7}$ of James'; if 108 is $\frac{6}{7}$, $\frac{1}{7}$ is $\frac{1}{6}$ of 108 or 18, and $\frac{7}{7}$ or the whole quantity 7 times 18 or 126.

16. Two boys were talking of their money; one says, "I have 12 cents;" the other says, "$\frac{3}{4}$ of your money is equal to $\frac{1}{2}$ of mine;" how much had he?

17. Two boys talking of their ages, one said "I am 15 years old;" the other said, $\frac{4}{5}$ of your age is equal to $\frac{6}{7}$ of mine". How old was he?

18. A. has 14 sheep; $\frac{5}{7}$ of his number is equal to $\frac{2}{5}$ of B.'s; how many has B.?

19. Charles has $36; $\frac{4}{9}$ of his is equal to $\frac{8}{25}$ of Francis'; how much has Francis?

20. A man bought a horse and paid $30 down, which was $\frac{5}{9}$ of the price of him. What was the price of the horse?

Ans. $54.

Solution.—If 30 is $\frac{5}{9}$ of a number, $\frac{1}{9}$ will be $\frac{1}{5}$ of 30 or 6, and $\frac{9}{9}$, or the number itself will be 9 times 6, or 54.

21. A note has run 45 days, which is $\frac{5}{8}$ of the time it has to run; how long has it to run?

22. A man bought a waggon, paying $36 down, which was $\frac{3}{8}$ of the price of it. Required, its price?

23. James bought some iron, paying $48 in trade; this was $\frac{3}{5}$ of the cost of it; what did it cost?

24. A lady purchasing some goods, paid $72 down; this was $\frac{9}{10}$ of her purchases; how much did she trade?

25. A man in trading, paid down $84; this was $\frac{7}{12}$ of his bill; required the amount of his bill, and how much he had yet to pay?

26. Frank and Henry were speaking of their money. Henry says "I have 84 cents;" says Frank "I have $\frac{6}{7}$ as much as you, and am going to buy lemons with it, at 4 cents apiece;" how many can he buy?

Ans. 18.

Solution.—$\frac{6}{7}$ of 84 is 72, the number of cents Frank had. Then as the lemons are 4 cents apiece, he can buy $72 \div 4 = 18$ lemons for 72 cents.

27. A man has $\frac{7}{12}$ of $144; how many yards of cloth can he buy at $6 per yard?

28. Henry has $\frac{6}{11}$ of $132; how many steers can he buy with it at $9 apiece?

29. A man having $\frac{9}{10}$ of $120, bought 9 coats with it; how much were they apiece?

30. Charles had a note for $96; he obtained $\frac{5}{8}$ of it, and spent it for sheep at $3 apiece; how many did he buy?

31. William spent $14 for books, which was $\frac{2}{7}$ of all he had; he spent the remainder for 7 yards of cloth; what did the cloth cost him per yard?

Ans. $5.

Solution.—If $14 was $\frac{2}{7}$ of all he had, $\frac{1}{7}$ would be $\frac{1}{2}$ of $14, or $7, and $\frac{7}{7}$ would be 7 times $7, or $49. Then as he spent $14, he would have $49–$14=$35 remaining; with this he bought 7 yards of cloth, which must have cost him $35÷7=$5 a yard.

32. A farmer paid $30 for goods, which was $\frac{3}{4}$ of his bill; he paid the balance in wheat at $$1\frac{1}{4}$ per bushel; how many bushels did it take?

33. A man spent $150 for sheep, which was $\frac{3}{11}$ of all he had: he spent the remainder for land at $10 an acre; how many acres did he buy?

34. A man bought a horse paying \$36 down, which was $\frac{4}{9}$ of what he was to pay; he paid the remainder in flour at \$5 a barrel; how many barrels did it take?

35. Lewis spent \$72 for cloth, which was $\frac{6}{11}$ of all he had; he spent the remainder for books at \$4 apiece; how many books did he buy?

36. Silas spent 24 cents for a slate, which was $\frac{3}{8}$ of all he had; he bought 10 lead pencils with what was left; how much were the pencils apiece?

37. $\frac{5}{6}$ of 24 is how many times 10? Ans. 2.

Solution.—$\frac{5}{6}$ of 24 is 20; $20 \div 10 = 2$; hence, 20 is twice 10.

38. $\frac{7}{8}$ of 48 is how many times 6?
39. $\frac{5}{9}$ of 54 is how many times 3?
40. $\frac{6}{7}$ of 56 is how many times 4?
41. 48 is $\frac{8}{9}$ of how many times 6? Ans. 9.

Solution.—We first inquire, 48 is $\frac{8}{9}$ of what number? If 48 is $\frac{8}{9}$, one ninth is $\frac{1}{8}$ of 48, or 6; and $\frac{9}{9}$ or the number itself is 9 times 6, or 54. Then 54 is how many times 6? $54 \div 6 = 9$, that is 54 is 9 times 6?

42. 63 is $\frac{7}{10}$ of how many times 15?
43. 108 is $\frac{9}{10}$ of how many times 30?
44. 6 times 6 is $\frac{4}{9}$ of what number? Ans. 81.
45. 8 times 9 is $\frac{6}{7}$ of how many times 3 times 4? Ans. 7.
46. 3 times 30 is $\frac{9}{10}$ of how many times 4 times 5? Ans. 5.
47. $\frac{5}{6}$ of 36 is $\frac{3}{10}$ of what number? Ans. 100.

Solution.—$\frac{5}{6}$ of 36 is 30; 30 is $\frac{3}{10}$ of 100.

48. $\frac{6}{7}$ of 42 is $\frac{4}{9}$ of what number?
49. $\frac{3}{4}$ of 48 is $\frac{6}{7}$ of how many times 3? Ans. 14.
50. $\frac{5}{8}$ of 72 is $\frac{9}{10}$ of how many times 5 times 5?

Ans. 2.

Article 2. Exercises for the Slate.

1. If 25 yds. of cloth cost \$100, what will 40 yds cost?

Ans. \$160.

Solution.—If 25 yds. cost \$100, 1 yd. will cost $\$100 \div 25 = \$4$. Then 40 yds. will cost 40 times as much as 1 yd., and $\$4 \times 40 = \$160$, Ans.

Remark.—In solving such questions, the learner can often shorten the operation, by merely expressing the work and canceling. Thus—

How can the operation in analysis often be shortened?

$\cancel{25}$	$\cancel{100}$—4
	40
	——
	$160 Ans.

When the learner performs the operation by cancelation, (or any other way) he should be required to give his reason for every step he takes. It is a matter of unimportance, how or where he sets his figures, if he thoroughly understands his subject

NOTE.—Questions of this kind are usually worked by Simple Proportion, or the Rule of Three, which is explained in the next section. Business men, however, generally solve them by analysis.

2. What cost 48 cows, if 20 cows cost $240? Ans. $576.
3. What cost 48 lbs. of coffee, if 60 lbs. cost $6.60? Ans. $5.28.
4. If I pay $31.20 for the use of $520 a certain time, how much must I pay for the use of $720 the same time? Ans. $43.20.
5. If a stage travel 90 miles in 15 hours, how many miles will it travel in 36 hours? Ans. 216.
6. What cost 850 bushels of barley if 600 bushels cost $300? Ans. $425.
7. If 25 horses eat $112\frac{1}{2}$ bushels of oats in a week, how many bushels will 14 horses eat? Ans. 63.
8. If 416 lbs. of wool cost $104, what will 599 lbs. cost? Ans. $149.75.
9. If 48 tons of hay cost $720, what will 36 tons cost? Ans. $540.
10. If a railroad car travels 154 miles in 11 hours, how far will it travel in 18 hours? Ans. 252 miles.
11. A man can do a piece of work in 8 days, laboring 12 hours per day: how long will it take him when he labors but 9 hours per day? Ans. $10\frac{2}{3}$ days.
12. If 4 lbs. of coffee cost $0.50, what will 40 yds. of calico cost, if $7\frac{1}{2}$ lbs. of coffee are worth 5 yds. of calico? Ans. $7.50.
13. If 12 men can do a piece of work in 24 days, how long would it take 30 men to do the same work?

Suggestion.—If 12 men do the work in 24 days, it will take 1 man 12 times as long to do it, and 30 men will do it in $\frac{1}{30}$ of the time in which one man could perform it. Ans. $9\frac{3}{5}$ days.

14. If 18 men eat a barrel of flour in 8 days how long will it last 45 men? Ans $3\frac{1}{5}$ days.
15. If a certain quantity of oats last 16 horses 18 days, how long will it last 28 horses? Ans. $10\frac{2}{7}$ days.

Is it a matter of importance where or how the figures are set if the learner understands his subject?

16. If a barrel of flour lasts 8 men 30 days, how long will it last 18 men? Ans. $13\frac{1}{3}$ days.

17. If 16 men can do a piece of work in 12 days, how long will it take 24 men to do it? Ans. 8 days.

18. If 6 stacks of hay keep 40 cattle 90 days, how long will they keep 50 cattle? Ans. 72 days.

The learner will perceive that the number of stacks is not to be regarded in solving this question.

19. If $\frac{1}{2}$ a bushel of wheat cost \$0.50, what will $\frac{3}{4}$ of a bushel cost? Ans. \$0.75.

Suggestion.—If $\frac{1}{2}$ a bushel cost \$0.50, a bushel will cost twice as much, and $\frac{3}{4}$ of a bushel will cost $\frac{3}{4}$ as much as a bushel.

20. If $\frac{4}{5}$ of a yard of cloth cost \$3.20, what costs $\frac{7}{8}$ of a yard? Ans. \$3.50.

21. If $\frac{6}{7}$ of an acre of land costs \$18, what costs $\frac{2}{3}$ of an acre? Ans. 14.

22. If $\frac{9}{10}$ of a ton of hay costs \$10.80, what will $\frac{5}{6}$ of a ton cost? Ans. 10.

23. If $\frac{7}{9}$ of a ton of coal costs \$4.90 what will $\frac{9}{10}$ of a ton cost? Ans. \$5.67.

24. A merchant bought a load of wheat, $\frac{4}{5}$ of which cost \$24, and afterwards sold $\frac{3}{10}$ of his load at cost; how much did he get for what he sold? Ans. \$9.

25. A has 420 sheep; $\frac{2}{5}$ of A's is equal to $\frac{4}{7}$ of B's; how many has B? Ans. 294.

26. A man bought 16 yds. of cloth for \$60; if he should sell $\frac{5}{8}$ of it at cost, how much would he get for it? Ans. \$37.50.

27. If a man pay \$33.50 for $26\frac{4}{5}$ days work, how much must he pay for $67\frac{2}{10}$ days work? Ans. \$84.

28. If a man ride 645 miles in $18\frac{3}{7}$ days, how far can he ride in $29\frac{3}{5}$ days? Ans. 1036 miles.

29. If 7 lbs. of tea cost $\$5\frac{1}{4}$, how much will 12 lbs. cost? Ans. \$9.

Suggestion.—$\$5\frac{1}{4} = \$\frac{21}{4}$. If 7 lbs. cost $\$\frac{21}{4}$, 1 lb will cost $\frac{1}{7}$ of $\$\frac{21}{4}$, and 12 lbs will cost 12 times as much as 1 lb.

30. If 15 acres of land cost $\$131\frac{1}{4}$, what will 43 acres cost? Ans. \$376.25.

31. If it cost $\$2\frac{7}{8}$ to build 112 rds. of fence, how much will it cost to build 216 rds.? Ans. $\$5.54\frac{13}{28}$.

32. If 15 pair of side combs cost $\$0.93\frac{3}{4}$, what cost 27 pair? Ans. $\$1.68\frac{3}{4}$.

33 If $\frac{5}{6}$ of a bushel of wheat costs $\frac{10}{12}$ of a dollar, what costs $\frac{3}{4}$ of a bushel?

Solution.—If $\frac{5}{6}$ of a bushel costs $\frac{10}{12}$, $\frac{1}{6}$ will cost $\frac{1}{5}$ as much, or $\frac{2}{12}$ $= \frac{1}{6}$ of a dollar, and a bushel will cost 6 times as much, or \$1.

Again, if a bushel costs $1, $\frac{3}{4}$ of a bushel will cost $\frac{3}{4}$ of a dollar, or $0.75.

34. If $\frac{7}{8}$ of a yard of cloth costs $\frac{7}{10}$ of a dollar, what costs $\frac{13}{20}$ of a yard? Ans. $0.52.

35. If $\frac{7}{12}$ of a farm costs $2100, what costs $\frac{5}{9}$ of the same farm? Ans. $2000.

36. If $\frac{8}{13}$ of a ship costs $48000, what is $\frac{28}{45}$ of her worth? Ans. $56000.

37. Suppose an army of 1500 men have 120000 lbs. of bread; how long will it last them, allowing each soldier $1\frac{1}{4}$ lbs per day? Ans. 64 days.

38. Suppose the above named army should lose 120 barrels of bread, each containing 250 lbs.; how much must be taken from each soldier's allowance per day, in order that the remainder may last the same time, and how much will be the allowance of each soldier per day after the reduction is made?

Ans. Each soldier loses 5 oz. per day, and has 15 oz. remaining.

39. Suppose an army of 1500 men, having lost $\frac{1}{4}$ of their bread, were obliged to subsist upon 15 oz. per day for 64 days; had none of their bread been lost, they would have had $1\frac{1}{4}$ lbs. per day for the same time; how much bread had they at first, and how much was lost?

Ans. { They had at first 120000 lbs.
They lost 30000 lbs.

40. Suppose the allowance of an army of 1500 men was shortened 5 oz. per day for 64 days in consequence of a part of their bread being spoiled, and the amount spoiled to be $\frac{1}{4}$ of the whole; what was the whole amount both good and bad, the amount spoiled and the allowance of each soldier per day, both before and after their bread was spoiled?

Ans. { Total amount 120000 lbs.
Amount spoiled 30000 lbs.
Daily allowance of each soldier at first $1\frac{1}{4}$ lbs.
Daily " " " after a part was spoiled 15 oz.

Note.—Questions similar to the four preceding, and the 21 following ones, are generally solved by reasoning from one statement to another, without reference to any particular rules.

41. A man owning $\frac{7}{8}$ of a farm, sold $\frac{2}{3}$ of his share for $2800; required the worth of the farm? Ans. $4800.

Solution.—As he sold $\frac{2}{3}$ of his share, he sold $\frac{2}{3}$ of $\frac{7}{8}$ of the entire farm. $\frac{2}{3}$ of $\frac{7}{8}=\frac{7}{12}$. (Sect. VIII, Art. 3, Obs. 10 or 11.) Then $2800 is $\frac{7}{12}$ of the worth of the farm, and $\frac{1}{12}$ must be $\frac{1}{7}$ of $2800, or $400, and the entire farm must be worth 12 times as much as $\frac{1}{12}$ or $\$400\times12=\$4800$.

42. A merchant owning $\frac{3}{4}$ of a store, sold $\frac{4}{5}$ of his share for \$15-000; what was the worth of the store? Ans. \$25000

43. A farmer owning $\frac{5}{6}$ of a field of wheat, sold $\frac{4}{9}$ of his share for \$100; what was the field worth at this rate? Ans. \$270.

44. A cistern holding 400 gallons is to be filled with water. By one pipe 30 gallons run in in an hour, whilst by another pipe 10 gallons run out in an hour; if both are left open, how long will it take the cistern to fill?

Solution.—As 30 galls. run in in an hour, and 10 galls. run out in the same time, it fills $30-10=20$ galls. per hour. Then as it holds 400 galls. it will take it $400 \div 20 = 20$ hours to fill.

Ans. 20 hours.

45. A boat traveling up stream is driven by steam at the rate of 14 miles per hour, whilst she is retarded by the current $4\frac{1}{2}$ miles per hour; how long will it take her to travel 418 miles?

Ans. 44 hours.

46. A vessel sprung a leak at sea, and it was not discovered until she had 215 gallons of water in the hold; the pumps empty 5 gallons per minute, whilst the leak lets in $2\frac{1}{2}$ gallons per minute; how long at this rate would it take to empty the ship?

Ans. 86 min.=1 hr. 26 min.

47. A man earns \$9 per week, and spends \$3.50 per week; how much can he save in 66 weeks? Ans. \$363.

48. A cistern can be filled by one pipe in 10 hours, and by another pipe in 12 hours. If both are left open how long will it take the cistern to fill? Ans. $5\frac{5}{11}$ hours.

Solution.—If a pipe fills it in 10 hours, it will fill $\frac{1}{10}$ of it in 1 hour. Again, if a pipe fills it in 12 hours it will fill $\frac{1}{12}$ of it in 1 hour; therefore both will fill $\frac{1}{10}+\frac{1}{12}=\frac{11}{60}$ of it in an hour. Then to fill the whole cistern, or $\frac{60}{60}$, it will take $60 \div 11 = 5\frac{5}{11}$ hours.

49. A cistern has a pipe which will fill it in 8 hours, and another that will empty it in 10 hours. If both are left open, how long will it take the cistern to fill? Ans. 40 hours.

Suggestion.—It fills $\frac{1}{8}$ full in 1 hour, and emties $\frac{1}{10}$ in the same time; hence, it gains $\frac{1}{8}-\frac{1}{10}=\frac{1}{40}$ of its contents in an hour.

50. A. can do a piece of work in 6 days, whilst it will take B. 8 days to do the same work. If both work together, in what time will they do it? Ans. $3\frac{3}{7}$ days.

51. A. can do a piece of work in 9 days; B. in 12 days, and C. in 15 days; how long would it take them all working together, to do it? Ans. $3\frac{39}{47}$.

52. A. can do a certain piece of work in 12 days; B. in 16 days; C. in 20 days; and D. in 24 days; in what time will they all, working together, do it? Ans. $4\frac{12}{57}$ days.

53. A. and B. together can plow a certain piece of ground in 10 days. A. can do it alone in 16 days; in what time can B do it?

Ans. $26\frac{2}{3}$ days.

54. A man and his wife eat the flour of a bushel of wheat in 12 days; the woman would eat it alone in 27 days; in what time would the man eat it? Ans. $21\frac{3}{5}$ days.

55. A cistern holding 600 gallons, has two pipes which together will let in 140 gallons of water in 3 hours, it has also one pipe which will let out 158 gallons in 5 hours; If all are left open, how long will it take the cistern to fill? Ans $39\frac{93}{113}$ hours.

56. A. B. and C. can do a piece of work in 15 days; A. and B. can do it in 24 days; in what time can C. do it alone?

Ans. 40 days.

57. James and Henry wish to divide 25 cents between them so that Henry may have 5 cents more than James; how much must each have?

Solution.—If Henry takes the 5 cents, he is to have more than James, it is evident that the remainder should be divided equally between them. Then $25-5=20$; $20\div2=10$ cts., James' share, and $10+5=15$ cts.; Henry's share. (Sect. VI, Art 1, Obs. 14.) Proof, $15+10=25$.

58. Divide 36 marbles between two boys, giving one 10 more than the other? Ans. 13 and 23.

59. A man left $4500 to be divided between two children, giving one $800 more than the other; what was the share of each?

Ans. One's share $1850; the other's $2650.

60. A man left his property amounting to $7500 to be divided between his wife, son, and daughter, giving the wife $1200 more than to the son, and to the son $800 more than to the daughter.—What was the share of each?

Ans. { Wife's share, 3566.66\frac{2}{3}$; Son's 2366.66\frac{2}{3}$; Daughter's 1566.66\frac{2}{3}$.

61. Divide $1200 between four person, giving A. $200 more than B; B $150 more than C; and C $100 more than D.

Ans. A's share $550; B's $350: C's $200; D's $100.

62. What cost 40 yds. of cloth, at $0.25 per yard?

Ans. $10.

Solution.—Had the cloth been $1 per yard, it would evidently have cost $40. Now $0.25 is $\frac{1}{4}$ of $1, therefore it will cost $\frac{1}{4}$ as many dollars as there are yards; and $\frac{1}{4}$ of 40 is 10.

Obs. 1. *The solution of questions by taking parts*, as in this example, *is called* PRACTICE.

Obs. 2. The chief advantage derived from this method of operating, is, *that it generally contracts the operation, thus often ennabling the learner to solve questione mentally which would otherwise require the use of the slate.*

Obs. 3. Any number that forms an exact part of another number, is called an *aliquot part* of that number. Thus, 4 is an aliquot part of 12; 50 cts. of \$1.00; &c.

REMARK.—It will be perceived that the terms *aliquot parts, component parts*, and *factors*, are synonymous; that is, they express the same meaning.

To obtain the cost of any number of articles, when the price of 1 is an aliquot part of a dollar:

Obs. 4. *Divide the given number of articles by that part of \$1 at which a single article is priced: the result will be the answer, in dollars.*

REMARK,—If there is a remainder after dividing, the quotient may be extended to cents, and mills, by annexing ciphers.

Obs. 5. *Practice is chiefly used in computing the price of various articles of trade;* as groceries, cloth, land, grain, &c.

The aliquot parts of \$1 are shown in the following

TABLE.

$12\frac{1}{2}$	cents	is $\frac{1}{8}$ of	\$1.
$16\frac{2}{3}$	"	is $\frac{1}{6}$ of	\$1.
20	"	is $\frac{1}{5}$ of	\$1.
25	"	is $\frac{1}{4}$ of	\$1.
$33\frac{1}{3}$	"	is $\frac{1}{3}$ of	\$1.
50	"	is $\frac{1}{2}$ of	\$1.

ALSO:

$12\frac{1}{2}$	cents	is $\frac{1}{2}$ of	\$0.25.
$12\frac{1}{2}$	"	is $\frac{1}{4}$ of	\$0.50.
25	"	is $\frac{1}{2}$ of	\$0.50.
25	"	is $\frac{1}{3}$ of	\$0.75.
$37\frac{1}{2}$	"	is $\frac{1}{2}$ of	\$0.75.
$37\frac{1}{2}$	"	is $\frac{3}{8}=\frac{1}{4}+\frac{1}{8}$ of	\$1.
$62\frac{1}{2}$	"	is $\frac{5}{8}=\frac{1}{2}+\frac{1}{8}$ of	\$1.
75	"	is $\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ of	\$1.
$66\frac{2}{3}$	"	is $\frac{2}{3}=\frac{1}{3}+\frac{1}{3}$ of	\$1.
$87\frac{1}{2}$	"	is $\frac{7}{8}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, or $1-\frac{1}{8}$ of	\$1.

What is practice? What is the advantage derived from this method of operating? What is an aliquot part of a number? Give examples. What is the difference in the meaning of the terms aliquot parts, component parts, and factors? How do we obtain the cost of any number of articles when the price of 1 is an aliquot part of a dollar? If there is a remainder how do we proceed? For what is practice chiefly used? Give examples. Give some of the aliquot parts of \$1.

63. What cost 72 lbs. of coffee, at $12\frac{1}{2}$ cents a pound?
Ans. $9.

64. What cost 68 yds. of muslin, at $12\frac{1}{2}$ cents a yard?
Ans. $8.50.

65. What cost 75 slates, at $16\frac{2}{3}$ cents apiece?
Ans. $12.50.

66. What cost 110 books, at 20 cents apiece? Ans. $22.

67. What cost 50 palm leaf hats at 25 cents apiece?
Ans. $12.50.

68. What cost 45 bushels of potatoes, at $33\frac{1}{3}$ cents per bushel?
Ans. $15.

69. What cost 1200 bushels of barley, at $37\frac{1}{2}$ cents per bushel?
Ans. $450.

Operation.

4	$1200 = cost at $1.
2	$300 = cost at 25 cts.
	$150 = cost at $12\frac{1}{2}$ cts.
Ans.	$450 = cost at $37\frac{1}{2}$ cts

The cost at $1 per bushel, would be $1200, the cost at 25 cts. per bushel would be $\frac{1}{4}$ of $1200 = $300; the cost at $12\frac{1}{2}$ cts. per bushel would be $\frac{1}{2}$ as much as at 25 cts., or $\frac{1}{2}$ of $300 = $150; then the cost at $37\frac{1}{2}$ cts. per bushel would be equal to the cost at 25 cts. + the cost at $12\frac{1}{2}$ cts., or $300 + $150 = $450, because $37\frac{1}{2}$ cts. = 25 cts. + $12\frac{1}{2}$ cts.

70. What cost 400 pair of socks, at $37\frac{1}{2}$ cts. a pair?
Ans. $150.

71. What cost 247 yds. of cloth, at 50 cts. a yard?
Ans. $123.50.

72. What cost 471 bushels of rye, at $62\frac{1}{2}$ cts a bushel?
Ans. $\$294.37\frac{1}{2}$.

73. What cost 600 bls. of cider, at $66\frac{2}{3}$ cts. per barrel?
Ans. $400.

74. What cost 300 lbs. of tea, at 75 cts. per pound?
Ans. $225.

75. What cost 400 yds. of satinet, at $87\frac{1}{2}$ cts. per yard?
Ans. $350.

76. What cost 150 acres of land, at $8.25 per acre?
Ans. $1237.50.

Suggestion.—$\$8.25 = \$8\frac{1}{4}$; therefore multiplying 150 by $8\frac{1}{4}$, it s evident we shall have our answer in dollars. Hence—

When the price of an article is in dollars and cents,

Obs. 6. *Multiply the number of articles, by the number of dollars, take aliquot parts for the cents, and add the several results together.*

Explain the solution of Ex. 69. When the price of an article is in dollars and cents how do we proceed?

77. What cost 250 bushels of wheat, at $\$1.12\frac{1}{2}$ per bushel?
Ans. $281.25.

78. What cost 360 yds. of cloth, at $\$2.16\frac{2}{3}$ per yard?
Ans. $780.

79. What cost 480 silver pencils, at $2.25 apiece?
Ans. $1080.

80. What cost 260 gallons of wine, at $\$2.33\frac{1}{3}$ per gallon?
Ans. $\$606.66\frac{2}{3}$.

81. What cost 80 yds of cloth, at $\$4.37\frac{1}{2}$ per yard?
Ans. $350.

82. What cost 160 acres of land, at $12.50 per acre?
Ans $2000.

83. What cost 40 sacks of coffee, at $\$12.62\frac{1}{2}$ per sack?
Ans. $505.

84. What cost 72 bls. of flour, at $\$7.66\frac{2}{3}$ per barrel?
Ans. $552.

85. What cost 120 acres of land, at $5.75 per acre?
Ans. $690.

86. What cost 24 stoves, at $\$15.87\frac{1}{2}$ apiece? Ans. $381.

REMARK.—The preceding examples in practice, are confined to Federal Money; we now design to show its application to compound numbers.

87. What cost 12 acres, 40 sq. rds. of land at $20 per acre?
Ans. $245.

40 sq. rds. $= \frac{1}{4}$ of an acre. Then $\$20 \times 12\frac{1}{4} = \$225$. Hence—

Operation.

```
4 | $20
  |  12
  | ----
  | $240=cost of 12 acres.
  |     5=cost of 40 sq. rds.
  | ----
Ans. $245=cost of the whole.
```

To find the value of a quantity consisting of several denominations:

Obs. 7. *Multiply the price by the number of the denomination at which the price is rated, and take aliquot parts, to find the value of the lower denominations. The sum of the several values will be the value of the entire quantity.*

A few of the principal aliquot parts in compound numbers, are shown in the following

---

How do we find the value of a quantity consisting of several denominations?

TABLE.

LONG MEASURE.			SQUARE MEASURE.		
2 furlongs	are	$\frac{1}{4}$ of a mile.	40 sq. rds.	are	$\frac{1}{4}$ of an acre.
4 "	"	$\frac{1}{2}$ " "	80 "	"	$\frac{1}{2}$ " "
6 "	"	$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ "	120 "	"	$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ "
40 rds.	"	$\frac{1}{8}$ of a mile.	AVOIRDUPOIS WEIGHT.		
80 "	"	$\frac{1}{4}$ " "	5 cwt.	is	$\frac{1}{4}$ of a ton.
160 rds.	"	$\frac{1}{2}$ " "	10 "	is	$\frac{1}{2}$ " "
240 "	"	$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ "	15 "	is	$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ ton.
20 "	"	$\frac{1}{2}$ a furlong.	4 oz.	is	$\frac{1}{4}$ of a pound.
1 ft.	is	$\frac{1}{3}$ of a yard.	8 oz.	is	$\frac{1}{2}$ " "
6 inches	"	$\frac{1}{2}$ a foot.	12 oz.	is	$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$ "

These are the parts that occur the most frequently in common business. Others might be given, but it is thought unnecessary, as the pupil can easily discover them by the following directions:

Obs. 8. *Make the given number the numerator of a fraction, and that number which it takes of this denomination to make 1 of the next higher, the denominator.*

Thus, if it were required to find what part of an hour were 30 minutes, we make 30 the numerator, and 60 (minutes in an hour) the denominator; thus, $\frac{30}{60}=\frac{1}{2}$.

88. How much would 12 bu. 2 pks. 4 qts. of wheat cost, at \$1.25 per bushel? Ans. \$15.78$\frac{1}{8}$.

Operation.

2	\$ 1.25	
	12	
	\$15.00	= cost of 12 bu.
4	.62$\frac{1}{2}$	= cost of 2 pks.
	.15$\frac{5}{8}$	= cost of 4 qts:
	\$15.78$\frac{1}{8}$	= cost of the whole.

2 pks. = $\frac{1}{2}$ a bushel. Having found the cost of 12 bushels, we find the cost of 2 pks., or $\frac{1}{2}$ a bushel, by taking $\frac{1}{2}$ the price of 1 bushel, (\$1.25.) 4 qts. = $\frac{1}{2}$ a peck, or $\frac{1}{4}$ of 2 pks. Therefore, we take $\frac{1}{4}$ of the cost of 2 pks., (\$0.62$\frac{1}{2}$) as the cost of 4 qts., and the sum of all these is equal to the whole.—(Sect. IV. Art. 4. Obs 4. Rem. 2.)

89. How much will 15 bu. 3 pks. 6 qts. of wheat cost, at \$1.44 per bushel? Ans. \$22.95.

The learner will perceive that *as many decimals are pointed off in the result, as there are in the given price of the single article.*

Name some of the aliquot parts of Long Measure? Square Measure? Avoirdupois weight? How do we find what part one number is of another? In finding the quantity of several denominations, how many decimals do we point off in the result?

90. If a man walk 24 miles, 80 rods per day, how far can he walk in 48 days? Ans. 1164 miles.

91. How much would 15 acres, 2 R. 20 sq. rds. of land cost, at \$8.50 per acre? Ans. \$132.81$\frac{1}{4}$.

92. How much would 18 A. 1 R. 10 sq. rds. of land cost, at \$10 an acre? Ans. \$183.12$\frac{1}{2}$.

93. How much would 32 A. 3 R. 30 sq. rds. of land cost, at \$15 an acre? Ans. \$494.06$\frac{1}{4}$.

94. How much would 6 T. 10 cwt. 20 lbs. of hay cost. at \$8 a ton? Ans. \$52.08.

95. How much would 15 cwt. 2 qrs. 12 lbs. 8 oz. of cheese cost, at \$12.50 per cwt.? Ans. \$195.31$\frac{1}{4}$.

96. If a man work 12 hrs. 30 min. per day, how much time will he work in 36 days? Ans. 18 d. 18 hrs.

97. If a man earn \$1.50 per day, how much will he earn in 24 days, 18 hrs. 45 min.? Ans. \$37.17$\frac{3}{16}$.

98. How much would 15768 lbs. of hay cost, at \$8.40 per ton? Ans. \$66.25.

Solution.—A ton is 2000 lbs. Therefore, the hay costs \$8.40 ÷ 2 = \$4.20 per thousand pounds, and its cost is found according to Sect. VIII. Art. 11. Obs. 1. to be \$66.225. Hence—

To find the cost of articles by the ton, or 2000 lbs.:

Obs. 9. *Multiply half the cost of a ton by the number of pounds, and point off three additional decimals from the right hand.*

99. How much will be the storage on 27480 lbs. of goods, at \$3.60 per ton? Ans. \$49.464.

100. How much would 7240 lbs. of coal cost, at \$6.90 per ton? Ans. \$24.978.

101. How much would be the transportation of 12415 lbs. of goods at \$11.80 per ton? Ans. \$73.248.

102. At \$7 per ton, how much would 1694 lbs. of hay cost? Ans. \$5.929.

103. A farmer sold 50 bushels of wheat, at \$1.25 per bushel, and took his pay in cloth, at \$0.75 per yard. How many yards did it take? Ans. 83$\frac{1}{3}$.

Solution.—We first find the cost of the wheat to be \$62.50. (Obs. 6.) Next we divide this by .75, because the coth is \$0.75 per yard, and he can evidently buy as many yards as .75 is contained in \$62.50. (Sect. VI. Art. 1. Obs. 24. *a.*)

How do we find the cost of articles by the ton of 2000 lbs.? Demonstrate this rule.

Questions of this kind are sometimes solved by a rule called BARTER.

Barter signifies an exchange of different commodities, at prices agreed upon by the parties.

NOTE.—It is not necessary to give an additional Rule for such questions. They can all be solved by *Analysis*, and the process is so simple that any scholar of moderate capacity can easily understand it.

104. How many bushels of oats, at \$0.31$\frac{1}{4}$ a bushel, must be given in exchange for 48 yards of muslin, at 12$\frac{1}{2}$ cts. a yard? Ans. 19$\frac{1}{5}$.

105. How many cords of wood, at \$2.75 a cord, must be given for 75 lbs of sugar, at 10$\frac{1}{4}$ cts. per pound? Ans. 2$\frac{35}{44}$.

106. How many pounds of butter, at \$0.16 per pound, must be given for 48 yards of calico, at \$0.25 per yard? Ans. 75.

107. If I give 45 acres of land, worth \$25 an acre, for 15 horses, how much do the horses cost me apiece? Ans. \$75.

108. Bought a hogshead of molasses, at \$0.45 per gallon, and paid for it in butter, at \$0.18 per pound. How many pounds did it take? Ans. 157$\frac{1}{2}$.

109. How many sheep, at \$1.75 apiece, must I give for 35 cows, at \$10 apiece? Ans. 200.

110. How much tobacco, at 12$\frac{1}{2}$ cents a pound, must I give in exchange for 300 pounds of saleratus, at 5 cts. a pound? Ans. 120 lbs.

111. How many pair of shoes, at \$2.25 a pair, will it take to pay for 144 yards of satinet, at \$1.25 a yard? Ans. 80.

112. Bought, 720 bushels of apples, at \$0.75 a bushel, and paid for them with 1620 bushels of potatoes. How much were the potatoes per bushel? Ans. \$0.33$\frac{1}{3}$.

113. A man mixed 5 lbs. of tea, worth \$0.50 per pound, with 6 lbs. worth \$0.75 per pound, and 8 lbs worth \$0.80 per pound. How much was a pound of the mixture worth? Ans. \$0.70$\frac{10}{19}$.

Operation.

\$0.50 × 5 = \$2.50 = the price of 5 lbs. at \$0.50 per pound.
0.75 × 6 = 4.50 = " " 6 lbs. at \$0.75 "
0.80 × 8 = 6.40 = " " 8 lbs. at \$8.80 "

19) 13.40 (.70$\frac{10}{19}$ cts. Ans.
13.3

10

The whole number of pounds mixed are 5 + 6 + 8 = 19 lbs.

What does barter sifinify?

The cost of the whole mixture is $2.50 + $4.50 + $6.40 = $13.40. Then one pound must be worth $13.40 ÷ 19 = 0.70\frac{10}{19}$. Hence—

To find the cost of a pound, bushel, &c., of a mixture :

Obs. 10. *Divide the whole cost of the mixture by the whole number of simples.*

Obs 11. *The mixing of several simples of different qualities together*, as in the last example, *to form a compound of a mean or middle quality, is called* ALLIGATION.

Alligation is of two kinds—MEDIAL and ALTERNATE.

Obs. 12. ALLIGATION MEDIAL *is used when the prices and quantities of the simples are given, to find the price of the mixture compounded of them, as in the last example.*

REMARK.—*Alligation Alternate* is not much used in common business, and is not treated of in this work. Its chief object is to find the proportional quantity to be taken of each simple, when the price of the mixture, and of the several simples, is given.

114. A farmer made a mixture, containing 12 bushels of oats, worth $0·25 per bushel ; 15 bushels of corn, at 0.37\frac{1}{2}$ per bushel, and 8 bushels of peas at 0.62\frac{1}{2}$ cents per bushel. What is a bushel of the mixture worth? Ans. 0.38\frac{13}{14}$.

115. A grocer mixed 30 gallons of water with 60 gallons of whiskey, worth $0.20 per gallon. How much is a gallon of the mixture worth? Ans. 0.13\frac{1}{3}$.

116. A grocer mixed 18 lbs. of sugar, at 8 cts. per pound, with 22 lbs., at 10 cts ; 24 lbs., at 12 cts. ; and 30 lbs.. at 14 cts. per pound. How much is a pound of the mixture worth?

Ans. 0.11\frac{19}{47}$.

117. A man mixed 40 gallons of wine, worth $2.25 per gallon, with 30 gallons of brandy, worth $3.50 per gallon, and 20 gallons of water. How much was a gallon of the mixture worth?

Ans. 2.16\frac{2}{3}$.

118. A goldsmith mixed 8 oz. of gold, 16 carats fine ; 12 oz., 18 carats fine ; 16 oz., 20 carats fine ; 20 oz., 22 carats fine ; and 24 oz., 24 carats fine. Required—the fineness of the mixture.

Ans. 21 carats.

119. James and William wish to divide 24 cents between them, in such a manner that James may have 3 as many times as William. How many must each one have?

Ans. James 18, and William 6.

How do we find the cost of a pound, bushel, &c., of a mixture? What is Alligation? How is it divided? When is Alligation Medial used?

Solution.—Whilst James takes 3 cents, William takes 1; and both take 4 cents. Then James must receive as many times 3 cents, and William as many times 1 cent, as 4 is contained in 24. $24 \div 4 = 6$. Therefore, $3 \times 6 = 18$ cents for James' share, and $1 \times 6 = 6$ cents for William's share.

PROOF.—$18 + 6 = 24$. $18 = 6 \times 3$.

Obs. 13. *The process of dividing a number into two or more parts that shall bear a certain relation to each other, as in the last example, is called* PROPORTIONAL DIVISION.

To divide a number into proportional parts:

Obs. 14. *Divide the number to be divided by the sum of the parts: the result will be one part, from which the other parts can be found.*

PROOF —*Add the several results together; if their sum is equal to the number divided, the work is correct.*

REMARK 1.—The reason of the proof depends upon the self-evident principle, that *the whole is equal to the sum of all its parts.*

120. Suppose two men start, one from Columbus, and the other from Wooster, Ohio, and walk towards each other, the former at the rate of 3 miles per hour, and the latter at the rate of 4 miles per hour. How far would each one walk before they would meet, the distance being 85 miles?

Ans. The former walked $36\frac{3}{7}$ miles, the latter $47\frac{4}{7}$ miles.

121. Divide 126 into three parts, that shall be to each other as the numbers 2, 3, and 4. Ans. 28, 42, 56.

122. Divide 144 into 4 parts, the numbers of which shall be to each other as the number 3, 4, 5, and 6.

Ans. 24, 32, 40, 48.

123. Divide 120 into four parts, that shall be to each other as the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$. Ans. 48, 32, 24, 16.

REMARK 2.—By multiplying each fraction by the least common multiple of their denominators, we find they are to each other as the numbers 6, 4, 3, and 2.

124. The standard for gold and silver coins in the United States is 9 parts pure metal to 1 part alloy. How much of each is there in an eagle, which weighs 10 pwts. 18 grs.?

Ans. Gold, 9 pwts. $16\frac{1}{5}$ grs. Alloy, 1 pwt. $1\frac{4}{5}$ grs.

125. Gunpowder is composed of 76 parts of nitre, 14 of char-

What is Proportional Division? How do we divide a number into proportional parts? What is the method of proof? Upon what does the reason of this method of proof depend? How do we find the relation which fractions bear to each other?

coal, and 10 of sulphur. How much of each is there in 200 lbs.?

Ans. Nitre, 152 lbs. Charcoal, 28 lbs. Sulphur, 20 lbs.

126. If a man ride 288 miles in 8 days, riding 9 hours per day, how far, at the same rate, can he ride in 15 days, riding 12 hours per day?

Solution.—If he rides 288 miles in 8 days, he will ride $288 \div 8 = 36$ miles per day; and if he rides but 9 hours per day, he will ride $36 \div 9 = 4$ miles per hour. Again—if he rides 4 miles per hour, he will ride $4 \times 12 = 48$ miles in a day of 12 hours length; and in 15 such days he would ride $48 \times 15 = 720$ miles.

Ans. 720 miles.

Or, the operation may be performed by cancelation. Thus:

$$\begin{array}{c|l} \cancel{8} & \cancel{288}\text{--}\cancel{36}\text{--}4 \\ \cancel{9} & 15 \\ & 12 \\ \hline \end{array}$$

The learner will perceive that we multiply and divide the same as in the above solution.

Ans. 720 miles.

NOTE.—Questions of this kind are usually solved by Compound Proportion, which is explained in the next section. But the solution by Analysis is preferable to any other. The learner will observe that the canceling part is not the *solution*, but merely the *operation*. The operation is the *mechanical*, and the solution the *mental*, or *reasoning* part, of the work, and a pupil ought never to accustom himself to the use of the former, without the aid of the latter, as a *thorough knowledge of all the reasons why he performs his operations as he does, is absolutely necessary*, if he ever wishes to become a good arithmetician.

127. If a man plough 12 ~~72~~ acres in 10 days, how long will it take 18 men to plough 230 acres? Ans. $10\frac{35}{54}$ days.

128. If 5 men earn \$75 in 12 days, how long will it take 8 men to earn \$200? Ans. 20 days.

129. If 15 men dig 360 rods of ditch in 4 days, how many rods will 24 men dig in 12 days? Ans. 1728.

130. If 12 horses eat 126 bushels of oats in 14 days, how many bushels will 16 horses eat in 24 days? Ans. 288.

131. If a family of 5 persons spend \$87.50 in 7 months, how much will they spend in 16 months, if 6 more persons are added to the family? Ans. \$440.

132. A farmer has $\frac{1}{2}$ of his hogs in one field, $\frac{1}{3}$ in another, and 24 in a third field. How many has he in all? Ans. 144.

Solution.—$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} =$ what he has in two fields; then the third field must contain $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$ of the whole. But the third

---

Explain the difference between the operation and the solution of a question in mathematics? Should the former ever be used without the latter? Why not?

field contains 24; therefore, 24 is $\frac{1}{6}$ of the whole, and $24 \times 6 = 144$, the whole lot.

NOTE.—Questions of this kind are sometimes solved by a rule called *Position*, or *Trial and Error*. Analytic solutions, however, are preferable.

REMARK 3.—Position might, with propriety, be called the *Guess Work Rule*. In Arithmetic, all questions should be solved upon *true principles*,† without any guess work about it; and as all examples in *Position* can be solved by *Analysis*, to give an extra rule for their solution would be unnecessary.

133. In a certain school $\frac{2}{7}$ of the pupils study Arithmetic, $\frac{3}{7}$ study Philosophy, 12 study Algebra, and the remainder, who are $\frac{1}{7}$ of the school, study various branches. How many scholars are there in the school? Ans. 84.

134. Of a certain army, $\frac{1}{6}$ were killed, $\frac{1}{4}$ were wounded, $\frac{1}{3}$ were taken prisoners, and 3000 fled. How many were in the army at first? Ans. 12000.

135. A drover has sheep in 6 lots: In the first he has $\frac{1}{4}$ of his flock; in the second, $\frac{1}{5}$; in the third, $\frac{1}{6}$; in the fourth, $\frac{1}{8}$; in the fifth, $\frac{1}{12}$; and in the sixth he has 84. How many sheep has he? Ans, 480.

136. If a pole is $\frac{1}{4}$ in the mud, $\frac{1}{3}$ in the water, and 20 feet out of the water, what is its length? Ans. 48 feet.

137. A schoolmaster being asked how many children he had, answered: "If I had as many more as I now have, $\frac{3}{5}$ as many, $\frac{1}{3}$ as many, $\frac{1}{5}$ as many, and $\frac{7}{15}$ as many, I should then have 270." How many had he? Ans. 75.

138. What number is that, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of which is 36? Ans. 48.

139. What number is that, $\frac{1}{2}$ of which exceeds $\frac{1}{3}$ of it by 10? Ans. 60.

140. What number is that, from which if you take $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ of itself, the remainder will be 44? Ans. 96.

141. What number is that, which, if you increase it by $\frac{1}{3}$, $\frac{1}{10}$, and $\frac{1}{12}$ of itself, the result will be 157? Ans. 120.

142. A.'s age is 3 times B.'s, and twice A.'s age equals C.'s; the sum of all their ages is 80 years. How old is each? Ans. A. 24, B. 8. and C. 48 years.

This example might be performed by Proportional Division, B.'s part being 1, A.'s part 3, and C.'s $3 \times 2 = 6$; then $6 + 3 + 1 = 10$, the sum of all the parts.

What might Position with propriety be called? How should all questions in arithmetic be solved? What do you mean by true principles? How can all questions in Position be solved?

†By *true principles*, are meant principles capable of demonstration, or proof.

143. A., B., and C. talk of their ages. A. said his age was $\frac{15}{7}$ of B.'s, and C. said his age was $\frac{7}{4}$ of the sum of the ages of A. and B. both; the sum of all their ages was 121 years. Required—the age of each.

Ans. A.'s age, 30 years. B.'s, 14 years. C.'s 77 years.

144. A m n bought a sheep, cow, and horse; the cow cost 5 times as much as the sheep, and the horse cost 5 times as much as the cow and sheep both. They all cost $72. Required—the cost of each?

Ans. The sheep, $2; the cow, $10; and the horse, $60.

145. A. and B. have the same annual income. A. saves $\frac{1}{6}$ of his, but B., by spending $\frac{1}{2}$ as much again as A., loses $75 in the course of the year. Required—the amount of the annual income of each. Ans. $300.

146. A. and B. have the same annual income: A. saves $\frac{1}{10}$ of his, but B., by spending $50 per year more than A., at the end of 5 years finds himself $75 in debt. Required—the annual income of each. Ans. $350.

Solution.—B. runs in debt $75 in 5 years. This is $75 $\div$ 5 = $15 a year more than his income; and as he spends $50 per year more than A., A. must save $50 — $15 = $35 a year; and as A. saves $\frac{1}{10}$ of his income, their income must be $35 $\times$ 10 = $350.

147. A men hired 90 days on these conditions: for every day he worked, he was to receive $0.75; and for every day he was idle, he was to pay $0.25 for his board. At the end of his time he received $40. How many days did he work, and how many days was he idle? Ans. He worked $62\frac{1}{2}$ days, and was idle $27\frac{1}{2}$.

148. A. and B. commence trading with equal sums of money: A. gained a sum equal to $\frac{1}{4}$ of his stock, and B. lost $200. B. had then $\frac{2}{3}$ as much money as A. How much had each at first?

Ans. $1200.

149. A man left his property to his 3 children, giving A. $\frac{1}{3}$ wanting $200; to B. $\frac{2}{5}$, and to C. the rest, which was $200 less than the share of B. What was the value of the estate, and what was each one's share?

Ans. Estate, $3000; A.'s share, $800; B.'s share, $1200; and C.'s [share, $1000.

150. A person being asked the time, replied: "The time past noon is equal to $\frac{5}{11}$ of the time till midnight." What time was it?

Ans. 45 min. past 3 o'clock.

151. A certain pole is composed of three pieces: The top piece is 15 feet long, the middle piece is as long as the top piece, and $\frac{2}{5}$ of the length of the lower piece; and the lower piece is as

both the oth r pieces. How long is the pole and how long is each piece?

Ans. Length of the p le, 100 feet; length of pieces, 15, 35, 50 ft.

152. A man has a gold and a silver watch, and a chain worth \$20. If he put the chain on the gold watch, its value will be 6 times that of the silver watch; but f he put the chain on the silver watch, its value will be but half that of the gold watch. Required—the value of the two watches.

Ans. The gold one \$70; the silver one \$15.

Solution.—As the chain, when put on the gold watch, makes its value 6 times that of the silver watch, the silver watch must be worth \$20 $\div$ 6 $=$ \$$3\frac{1}{3}$ $+$ $\frac{1}{6}$ of the value of the gold watch. Therefore when the chain is put on the silver watch its value must be \$20 $+$ \$$3\frac{1}{3}$ $+$ $\frac{1}{6}$ of the value of the gold watch. But by the question this is $\frac{1}{2}$ the value of the gold watch; therefore, twice this, or \$40 $+$ \$$6\frac{2}{3}$ $+$ $\frac{1}{3}$ of the value of the gold watch must be the value of the gold watch. Now \$40 $+$ \$$6\frac{2}{3}$ $=$ \$$46\frac{2}{3}$; and from the above reasoning, this must be $\frac{2}{3}$ of the value of the gold watch. If \$$46\frac{2}{3}$ is $\frac{2}{3}$, \$$46\frac{2}{3} \div 2 =$ \$$23\frac{1}{3}$, is $\frac{1}{3}$ and \$$23\frac{1}{3} \times 3 =$ \$70, the value of the gold watch.

153. A man has two horses, and a saddle worth \$25. If he puts the saddle on the first horse, his value will be $\frac{17}{9}$ of that of the second horse; but if he puts the saddle on the second horse, his value will be $\frac{7}{6}$ of that of the first horse. Required—the value of each horse. Ans. The first \$60; the second \$45.

154. The hou and minute hand of a clock are together at 12 o'clock. At what time are they next together?

Ans. 5 min. $27\frac{3}{11}$ sec. past 1.

Solution.—The minute hand moves over 12 spaces, whilst the hour hand moves over but 1, and therefore gains 11 spaces in an hour. If the minute hand overtook the hour hand precisely at 1 o'clock, it would gain 12 spaces in an hour. But it gains only 11 spaces in an hour; consequently to overtake the hour hand it must ga n $\frac{12}{11}$ of the distance it has to run. $\frac{12}{11}$ of 1 hour, or 60 minutes is 1 hour, 5 minutes, $27\frac{3}{11}$ seconds. (Sect. IX. Art 3. Case 2.)

155. At what time between 3 and 4 are the hands of a clock together? Ans. 16 min. $21\frac{9}{11}$ sec. past 3 o'clock.

156. At what time between 10 and 11 are the hands of a clock together? Ans. 54 min. $32\frac{8}{11}$ sec. past 10 o'clock.

157. At what time between 9 and 10 are the hands of a clock exactly opposite each other? Ans. $16\frac{4}{11}$ min. past 9 o'clock.

Suggestion.—The learner will perceive that to be opposite each other, the hands must be 30 min ites apart. Now at 9 o'clock the

minute hand is 15 minutes ahead of the hour hand; and therefore, it has but 15 minutes more to gain to be opposite the hour hand.

158. At what time between 2 and 3 are the hands of a clock exactly opposite each other? Ans. $43\frac{7}{11}$ min. past 2 o'clock.

159. Four men trade together: A. put in $\frac{7}{11}$ as much as B.; C. put in $\frac{13}{15}$ of $\frac{35}{39}$ of what A. and B. both put in; and D. put in $\frac{7}{12}$ of $\frac{18}{35}$ of what B. and C. put in. They all put in \$9875. What was each one's share?

Ans. A.'s \$1750; B.'s \$2750; C.'s \$3500; and D.'s \$1875.

160. A., B. and C. trade in company: A. put in \$4500; $\frac{8}{9}$ of what A. put in was equal to $\frac{4}{5}$ of what B. put in; and $\frac{18}{25}$ of what B. put in, added to $\frac{8}{15}$ of what A. put in, was $\frac{12}{11}$ of what C. put in. They gained a certain sum, of which A.'s share was equal to $\frac{2}{7}$ of what B. and C. put in; B.'s share was equal to $\frac{1}{3}$ of what A. and C put in; and C.'s share was equal to $\frac{11}{45}$ of what they all put in. What was the whole amount invested? How much did B. and C. put in; what did they gain, and what was each one's share of the gain?

Ans. { Whole stock, \$15000; B. put in \$5000; C. put in \$5500. Whole gain, \$10000; A.'s share, \$3000; B.'s share, \$3333.33$\frac{1}{3}$; C.'s share. \$3666.66$\frac{2}{3}$.

SECTION XII.

RATIO AND PROPORTION.

Article 1. Ratio.

Obs. 1. Ratio *means relation, and signifies that relation which one number has to another of the same kind. It is expressed by the quotient of one divided by the other.*

Thus, if we inquire, *What is the ratio of* 4 *to* 2? if we make the first number (4,) the standard of comparison, the question resolves itself into this: *What part of* 4 *is* 2? and the answer is, 2 is $\frac{1}{2}$ of 4.

Again: if we make a second number (2,) the standard of comparison, the question is, *What part of* 2 *is* 4? and the answer is, 4 is twice 2.

The first of these methods is called the *French method*, by which the ratio of 4 to 2 is $2 \div 4 = \frac{1}{2}$.

What does ratio mean? What does it signify? How is it expressed? How do we find the ratio between two numbers?

The second method is called the *English method*, by which the ratio of 4 to 2 is, $4 \div 2 = 2$.

Each method has advantages peculiar to itself, but we shall follow the *French* method, as it is generally adopted in this country. Hence—

To find the ratio of one number to another:

Obs. 2. *Divide the second number by the first.*

MENTAL EXERCISES.

1. What is the ratio of 9 to 3? of 6 to 12? Ans. $\frac{1}{3}$, 2.

2. What is the ratio of 16 to 2? Of 21 to 7? Of 48 to 12? Of 63 to 9? Of 84 to 7?

3. What is the ratio of 30 to 6? Of 36 to 4? Of 120 to 10? Of 132 to 11?

4. What is the ratio of 3 to 4? Of 5 to 8? Of 7 to 9? Of 8 to 15? Of 23 to 37?

5. What is the ratio of 15 to 25? Of 18 to 36? Of 36 to 84? Of 90 to 100?

6. What is the ratio of 75 to 25? Of 144 to 48? Of 132 to 1728?

7. What is the ratio of 288 to 576? Of 132 to 96? Of 196 to 210?

Obs. 3. The two numbers thus compared, when spoken of together, as in the preceding examples, are termed a COUPLET; but when spoken of separately, they are called the *terms* of the couplet.

Obs. 4. The *first* term is called the ANTECEDENT, *because it comes before the other;* the *second* term is called the CONSEQUENT, *because it follows after the other.*

8. Of what numbers is $\frac{5}{6}$ the ratio? Or, what number, divided by another, will produce $\frac{5}{6}$?

It is evident that the least numbers that will do this are 5 and 6; or, the ratio of 5 to 6 is $\frac{5}{6}$. Hence—

To find two numbers of which a given fraction is the ratio:

Obs. 5. *Make the numerator the second number, and the denominator the first term of the couplet.*

9. Of what two numbers are the following fractions the ratio? $\frac{2}{3}$; $\frac{7}{8}$; $\frac{6}{7}$; $\frac{12}{11}$; $\frac{15}{9}$; $\frac{16}{4}$. $\frac{20}{25}$; $\frac{288}{144}$; $\frac{79}{63}$; $\frac{100}{48}$; $\frac{1728}{4376}$; $\frac{2131}{576}$?

10. Of what two numbers is 7 the ratio?

When two numbers compared are spoken of together, what are they called? When spoken of separately, what are they called? What is the first term called? Why? What is the second term called? Why? How do we find two numbers, of which a given fraction is the ratio?

It is evident that we wish to find two numbers, one of which multiplied by 7 will produce the other. Then assuming 1 as one number, the other must be $1 \times 7 = 7$; or, assuming 2 as one number, the other is $2 \times 7 = 14$; also, $3 \times 7 = 21$, &c. That is, 7 is the ratio of 1 to 7; of 2 to 14; of 3 to 21, &c. Hence—

To find two numbers of which a given integer is the ratio:

Obs. 6. *Take any number as the first term of the couplet, and multiply it by the ratio to obtain the second term of the couplet.* The same rule will apply when the ratio is a fraction, and other terms are desired than the numerator and denominator.

11. Of what two numbers is 6 the ratio? 9; 4; 8; 12; 10; 15; 26; 38; 18?

12. Of what two numbers is 3 the ratio? 5; 11; 13; 23; 47; 108; 150?

Remark 1.—As the ratio is found by dividing one number by another, it can always be expressed *fractionally*. Thus, the ratio of 6 to 3 is $\frac{3}{6}$; and the ratio of 8 to 16 is $\frac{16}{8}$.

2.—Or, the ratio may be expressed by *a couplet of dots* (:) placed between the terms of the couplet. Thus the ratio of 10 to 15 may be expressed thus: 10 : 15. Hence—

Obs. 7. *Ratio may be expressed in two ways, and it is immaterial which we use.*

Obs. 8. As ratio expresses the *relation* of numbers, *one number cannot have a ratio to another number of a different kind.* Because it would be absurd to compare bushels with hours, or feet with pounds, as a bushel cannot be said to be part of an hour, or a foot longer or shorter than a pound. But we can compare bushels with bushels, or feet with feet, or pounds with pounds, because they express things of the same kind.

Remark.—From this we perceive that ratio refers only to the *relative*, and not to the *absolute* magnitude of the two quantities. For 1 lb. has the same relation to 2 lbs. as 1 ton to 2 tons, or 1 ounce to 2 ounces.

13. What is the ratio of 2 pecks to 1 peck?

Ans. $\frac{1}{2}$, or 1 peck is $\frac{1}{2}$ of 2 pecks.

How do we find two numbers of which a given integer is the ratio? Will the same rule apply when the ratio is a fraction? How can ratio always be expressed? How may it otherwise be expressed? What is the inference deduced from this? Can ratio express the relation between numbers of different kinds? Can we compare bushels with hours? Why not? Can we compare feet with pounds? Why not? To what can we compare bushels? With what can we compare feet? With what can we compare pounds? Why? Does ratio refer to the relative or absolute magnitude of two quantities? Give an example.

14. What is the ratio of 2 bushels to 3 pecks?

Note.—First reduce both to the same denomination. 2 bu. = 8 pks.

Ans. $\frac{3}{8}$.

Obs. 9. Hence—*When the terms of the couplet are of the same kind, but of different denominations, they must both be reduced to the same denomination before their ratio can be found.*

15. What is the ratio of 2 feet to 4 yards?

16. What is the ratio of 1 ton to 16 cwt.? Of 3 seconds to 4 minutes?

17. What is the ratio of 120 sq. rds. to 2 acres? Of 7 acres to 21 acres? Of 320 acres to 1 sq. mile? Of 6 miles to 3 miles? Of 8 miles to 4 furlongs?

18. What is the ratio of 2 miles to 180 rods? Of 1 bushel to 6 qts.? Of 1 gallon to 2 qts.? Of 1 ton to 16 lbs.?

Remark.—As the ratio of any two numbers can always be expressed fractionally, it is obvious, that as far as the *value* is concerned, the same general principles are equally applicable to both ratio and fractions. Hence—

Obs. 10. *If we multiply the consequent, or divide the antecedent by any number, we multiply the ratio by that number.* (Sect. VIII. Art. 2. Obs. 1 and 5.)

Thus,		The ratio of 2 : 4 is 2.
Multiplying the consequent	by 2,	the ratio of 2 : 8 is 4, or 2×2.
Dividing the antecedent	by 2,	" " 1 : 4 is 4, or 2×2.

Remark.—This is evident from the fact that *we either multiply the numerator or divide the denominator*, either of which has the same effect upon the quotient. (Sect. VIII. Art. 2. Obs. 7.)

Obs. 11. *If we divide the consequent, or multiply the antecedent by any number, we divide the ratio by that number.* (Sect. VIII. Art. 2. Obs. 2 and 4.)

Thus,		The ratio of 4 : 16 is 4.
Dividing the consequent	by 2,	" " 4 : 8 is 2, or 4÷2.
Multiplying the antecedent	by 2,	" " 8 : 16 is 2, or 4÷2.

Remark.—This is evident from the fact, that *we either divide the numerator or multiply the denominator*, either of which has the same effect upon the quotient. (Sect. VIII. Art. 2. Obs. 7.)

¶ When the terms of the couplet are of the same kind, but of different denominations, how do we proceed? The ratio of any two numbers can always be expressed fractionally; what is the inference deduced from this fact? What effect does it have upon the ratio, to multiply the consequent, or divide the antecedent by any number? Show why this is correct. What effect does it have upon the ratio to divide the consequent, or multiply the antecedent, by any number? Show why this is correct?

Obs. 12. *If we multiply and divide both the antecedent and the consequent by the same number, we do not alter the ratio.* (Sect. VII. Art. 2. Obs. 8.)

Thus,		The ratio of 12 : 6 is $\frac{1}{2}$.	
Multiplying both terms of the couplet by 3,	"	" 36 : 18 is $\frac{1}{2}$.	
Dividing both terms of the couplet by 3,	"	" 4 : 2 is $\frac{1}{2}$.	

Remark.—This is evident from the fact, that *we multiply or divide both divisor and dividend by the same number,* which does not alter the quotient. (Sect. VI. Art. 1. Obs. 23.)

Obs. 13. *If the antecedent and consequent are equal, the ratio is 1, and it is called a ratio of equality* (Sect. VIII. Art. 2. Obs. 10. *a.*)

Thus, the ratio of 3 : 3 is 1, or 3 is equal to 3.

Remark.—This is evident from the fact, that *the divisor and dividend are equal.* (Sect. VI. Art. 1. Obs. 26. *d.*)

Obs. 14. *If the antecedent is greater than the consequent, the ratio is less than 1, and it is called a ratio of inequality.* (Sect. VIII. Art. 2. Obs. 10.)

Remark.—This is evident from the fact, that *the divisor is greater than the dividend.* (Sect. VI. Art. 1. Obs. 26. *d.*)

Obs. 15. *If the antecedent is less than the consequent, the ratio is greater than 1, and it is called a ratio of greater inequality.* (Sect. VIII. Art. 2. Obs. 10. *b.*)

Thus, the ratio of 5 : 15, or 3, is a ratio of greater inequality.

Remark.—This is evident from the fact, that *the dividend is greater than the divisor.* (Sect. VI. Art. 1. Obs. 26. *d.*)

19. Of what kind is the ratio of 5 to 7?

Ans. A ratio of greater inequality.

20. Of what kind are the ratios of the following couplets: 6 : 4; 8 : 8; 7 : 14; 36 : 24; 15 : 15? 20 : 40; 60 : 30; 15 : 24; 23 : 17?

Obs. 16. *Ratio is of two kinds*—Simple *and* Compound.

What effect does it have upon the ratio to multiply or divide both the antecedent and consequent by the same number? Show why this is correct. If the antecedent and consequent are equal, what is the ratio? What is it called? Why is this correct? If the antecedent is greater than the consequent, what is the ratio? What is it called? Why is this correct? If the antecedent is less than the consequent, what is the ratio? What is it called? Why is this correct? How is ratio divided?

a. A Simple ratio *is the ratio of a single expression, or couplet.*

b. A Compound ratio *is the ratio of the products of the corresponding terms of two or more couplets.*

Thus, the ratio of 6 : 2, or 12 : 24 is a simple ratio.
Again the simple ratio of 3 : 6 is 2.
And the simple ratio of 2 : 8 is 4.
And the ratio compounded of these is the same as the simple ratio of 6 : 48, which is 8.

Note.—Compound ratio is not different from any other ratio. It is only used in some cases to denote the origin of the ratio.

21. What is the simple ratio of 4 : 2? Of 8 : 16? Of 48 : 96; Of 56 : 28?

22. What is the compound ratio of the following ratios:

$\left.\begin{array}{r} 6 : 12 \\ 4 : 24 \end{array}\right\}$; $\left.\begin{array}{r} 8 : \\ 9 : 2 \end{array}\right\}$; $\left.\begin{array}{r} 2 : 8 \\ 6 : 24 \\ 4 : 12 \end{array}\right\}$; $\left.\begin{array}{r} 12 : 6 \\ 24 : 8 \\ 16 : 9 \end{array}\right\}$; $\left.\begin{array}{r} 18 : 9 \\ 12 : 36 \\ 4 : 16 \end{array}\right\}$

Article 2. Proportion.

Obs. 1. A Proportion *consists of an equality of ratios.*

Thus, the ratio 3 : 9 is equal to the ratio of 4 : 12; that is, the ratio of both is 3. Hence, the two couplets 3 : 9 and 4 : 12 form a proportion.

Obs. 2. *The terms of the two couplets which compose the proportion are called* Proportionals.

Obs. 3. *Proportion is usually expressed by four dots* (: :) *placed between the two couplets.*

Thus: 4 : 8 : : 6 : 12 is a proportion, and is read, 4 is to 8 as 6 is to 12; or, the ratio of 4 to 8 is the same as that of 6 to 12: i. e. 2.

a. Proportion may also be expressed by placing the sign of equality between the two ratios.

Thus: 2 : 8 = 4 : 16 is a proportion, and is read, the ratio of 2 to 8 is equal to the ratio of 4 to 16.

What is a Simple ratio? A Compound ratio? Is there any difference between Compound ratio, and any other ratio? Why then is it used? Of what does a proportion consist? Give an example. What are the terms of the couplets which compose the proportion called? How is proportion usually expressed? How read, when expressed in this manner? How may it otherwise be expressed?

Obs. 4. From these illustrations, we conclude that *four numbers are in proportion, when the first has the same ratio to the second, that the third has to the fourth.*

Obs. 5. The learner will perceive from what has been said, *that Ratio and Proportion are different:*

1st. *Because a ratio consists of but two terms, or one couplet; whilst a proportion cannot exist without four terms, or two couplets.*

2nd. *Because one ratio may be greater or less than another;* thus, the ratio of 2 : 8 is greater than that of 4 : 12, and less than that of 3 : 15; *but in a proportion the ratios must be equal, and consequently one cannot be greater or less than another.*

Obs. 6. Although it takes *four terms* to form a proportion, still it can be formed with *three numbers,* as one of the numbers may be repeated.

Thus: 3 : 6 : : 6 : 12 is a proportion, of which 6 is the consequent of the first couplet, and the antecedent of the second couplet.

Obs. 7. When a number is repeated, as in this example, it is called a *mean proportional* between the other two numbers; and the last term is called a *third proportional* to the other two numbers.

Thus, in the above example, 6 is a mean proportional between 3 and 12, and 12 is a third proportional to 3 and 6.

Obs. 8. *The first and last terms of a proportion are called the* EXTREMES; *the two middle terms are called the* MEANS.

Thus: in the proportion 15 : 3 : : 20 : 4, 15 and 4 are the extremes, and 3 and 20 the means.

Obs. 9. *In every proportion the product of the extremes is equal to the product of the means; otherwise, they are not proportional.*

Thus: 4 : 2 : : 6 : 3 is proportional, because $4 \times 3 = 2 \times 6$.

But 6 : 2 : : 12 : 3 is not proportional, because $6 \times 3 = 18$, and $12 \times 2 = 24$, which is greater.

How read, when expressed in this manner? When are four numbers in proportion? Are ratio and proportion alike? Why not? Of how many different numbers can proportion consist? How can this be done? What term in each couplet is the number repeated? What is the number that is repeated called? What is the last term called? In the proportion 3 : 6 : : 6 : 12, which is the mean proportional, and which the third proportional? Which are the extremes in a proportion? Which the means? In the proportion 15 : 3 : : 20 : 4, which are the extremes, and which the means? What principle is true with regard to every proportion? If this is not the case, what is the conclusion? Is 4 : 2 : : 6 : 3 proportional? Why? Is 6 : 2 : : 12 : 3 proportional? Why not?

Illustration 1st.—In the proportion 4 : 2 : : 6 : 3, the ratio of 4 : 2 is $\frac{1}{2}$; also, that of 6 : 3 is $\frac{1}{2}$: hence, the ratios of the two couplets being equal, the numbers are proportional, according to Obs. 1.

2nd.—From the fact that proportion is an equality of ratios, it is evident that the product, either of the extremes or means, must contain, as factors, *the antecedents of both couplets and the ratio.* But the *consequent* of each couplet is equal to the *antecedent of this couplet multiplied by the ratio.* (Art. 1. Obs. 6.) Therefore, *the products of the antecedent of each couplet, multiplied by the consequent of the other couplet, are equal.*

Obs. 10. *As in Ratio, so in Proportion, the two terms of each couplet must be of the same denomination.* (Art. 1. Obs. 8.)

Remark.—The learner will observe that it is not necessary for *all four* terms of the proportion to be of the same denomination, but only those of each couplet. For \$6 has the same ratio to \$3, that 8 days has to 4 days; that is, the ratio of each is 2. Therefore, \$6 : \$3 : : 8 days : 4 days: is a correct proportion.

Obs. 11. By attentively examining Obs. 9, we notice two considerations:

a. 1st. *If we divide the product of the means by either extreme, the quotient will be the other extreme.*

b. 2nd. *If we divide the product of the extremes by either mean, the quotient will be the other mean.*

Thus, in the proportion, 5 : 10 : : 6 : 12;

If we have given the two means, and either mean, } $12\times5=60$; $60\div10=6$, one mean.

Or, ... $12\times5=60$; $60\div6=10$, the other mean.

If we have given the two extremes, and either extreme, } $10\times6=60$; $60\div5=12$ one extreme.

Or, ... $10\times6=60$; $60\div12=5$, the other extreme.

Obs. 12. Hence—*If any three terms of a proportion are given, the other may be found by dividing the product of the means by the given extreme, or the product of the extremes by the given mean.*

Explain this point by the illustration given? Proportion is an equality of ratios; what inference is deduced from this? To what is the consequent of each couplet equal? What do we conclude from this fact? What is necessary in both ratio and proportion? Is it necessary for all four of the terms to be of the same denomination? Which then? Give an example illustrating this point. What is the first consideration we notice from examining Obs. 9? The second? If we have given three terms of a proportion, how do we find the fourth?

REMARK.—This is evident from the fact, that *in all such cases we have the product of two factors, and one factor given to find the other*, which is always found by dividing the product by the given factor. (Sect. VI. Art. 1. Obs. 16.)

MENTAL EXERCISES.

1. If the extremes are 4 and 6 and one mean is 3, what is the other mean?
2. If the extremes are 8 and 15, and one mean is 5, what is the other mean?
3. If the means are 6 and 12, and one extreme is 8, what is the other extreme?
4. If the means are 7 and 8, and one extreme is 4, what is the other extreme?
5. If the extremes are 4 and 12, and one mean is 8, what is the other mean?
6. If the means are 4 and 9, and one extreme is 3, what is the other extreme?
7. If the extremes are 8 and 9, and one mean is 12, what is the other mean?
8. If the means are 4 and 10, and one extreme is 20, what is the other extreme?
9. Is 3 : 4 : : 9 : 12 proportional? Is 7 : 8 : : 9 : 10 proportional? Is 12 : 6 : : 20 : 8 proportional?
10. Is 21 : 7 : : 24 : 8 proportional? Is 10 : 12 : : 15 : 18 proportional?
11. Is 36 : 9 : : 42 : 7 proportional? Is 72 : 8 : : 108 : 9 proportional?
12. Is 40 : 50 : : 70 : 90 proportional? Is 40 : 60 : : 60 : 90 proportional?

NOTE.—The learner should be required to give his reasons in the above examples, why they are proportional or not, as the case may be.

Obs. 13. *Proportion is of two kinds*—SIMPLE *and* COMPOUND.

Case 1.—SIMPLE PROPORTION.

Obs. 14. SIMPLE PROPORTION *is an equality between two simple ratios.*

Thus : 6 : 2 : : 9 : 3 is a simple proportion.

REMARK 1.—The chief use of Simple Proportion is *to find the fourth term of a proportion, when the first terms are given.* It is in this manner that it is chiefly applied to practical business.

2.—Simple Proportion is often called the *Single Rule of Three*, because three

---

Show why this is correct. How is Proportion divided? What is Simple Proportion? For what is it chiefly used? What is Simple Proportion often called? Why?

terms are given to find a fourth. *Proportion*, however, we consider to be the most appropriate name, because we have one couplet, and the antecedent of another couplet given, to find such a consequent of the latter couplet, that both shall have the same ratio, and thus constitute a proportion. (Obs. 1.)

Ex. 1. If 4 yards of cloth cost \$6, how much would 7 yds. cost?
Ans. \$10.50.

Solution.—As in every couplet the terms must be of the same kind, it is evident that the numbers 4 and 7 form the terms of one couplet, because they are both yards, and \$6 is one term of another couplet, and the third term of the proportion, and as this term is dollars, the fourth term must also be dollars, in order that both may be of the same kind. Hence—

Obs. 15. *In every proportion the third term must be of the same kind as the answer.*

We now wish to ascertain which of the other numbers is the first, and which the second term. If we make 7 the first term, and 4 the second, the proportion stands, 7 yds. : 4 yds. : : \$6 : the fourth term, or answer.

Here we have the two means and one extreme of a proportion given, to find the other extreme. Then $6\times4=24$; $24\div7=3\frac{3}{7}$, (Obs. 12,) and the proportion, when completed, is 7 yds. : 4 yds. : : \$6 : \$$3\frac{3}{7}$. Thus, we make 7 yds. cost less than 4 yds., which is absurd. Therefore, we will make 4 the first term, and 7 the second term, and the proportion stands thus: 4 yds. : 7 yds. : : \$6 : the fourth term, which we find, as above, to be \$10.50.—Hence—

Obs. 16. *If the answer ought to be greater than the third term, make the greater of the remaining numbers the second term, and the less number the first term of the proportion.*

Remark.—This is evident from the fact, that if the answer should be greater than the third term, *the ratio of each couplet is greater than unity*; (Art. 1. Obs. 15.) and therefore *each consequent is greater than its antecedent.*

The following form is usually adopted in simple proportion:

We multiply the 6 and 7 together, and divide their product by 4, which gives \$10.50, as the fourth term.

$$\begin{array}{ccccccc} \text{yds.} & & \text{yds.} & & & & \$ \\ 4 & : & 7 & : & : & & 6 \\ & & 6 & & & & \\ \hline & 4) & 42 & & & & \end{array}$$

Hence— Ans. $10\frac{1}{2}=\$10.50$.

What is the most appropriate name? Why? What must the answer be in

When three terms of a proportion are given, to find the fourth:

Obs. 17. *Multiply together the second and third terms, and divide the product by the first term.* (Obs. 12.)

Note.—The learner must bear in mind that we cannot multiply two concrete numbers together; (Sect. IV. Art. 2. Obs. 6. Rem. 1. and Sect. VI. Art. 1. Obs 15.) thus in the above example we multiply 6 and 7 together, and divide the product by 4, because the ratio of \$6 to the answer, must be the same as 4 yds. to. 7 yds. In this way we make the terms all abstract.

Proof.—$10.50\times4=42$; $7\times6=42$. (Obs. 9.) Hence—

To prove Simple Proportion:

Obs. 18. *Multiply together the first term and the answer, or fourth term; and also the second and third terms; if the two products are equal, the work is correct.* (Obs. 9.)

Questions in Simple Proportion can be worked by cancelation:

The above example is worked thus: We place the 7 and 6 at the right for the same reason that we make them the second and third terms in the proportion, and we place the 4 at the left for the same reason that we make it the first term. Again, the second and third terms are multipliers, and the first term is a divisor. (Obs. 17., and Sect. VII. Art. 1. Obs. 3.)

2..$\cancel{4}$	7
	$\cancel{6}$..3
2	$21=10\frac{1}{2}$.

Analytic Solution.—If 4 yds. cost \$6, 1 yard will cost $6\div4=$ \$1.50, and 7 yds. will cost $\$1.50\times7=\$10.50$.

2. If 8 lbs. of tea cost \$7, what will 5 lbs. cost?

Ans. $\$4.37\frac{1}{2}$.

---

every proportion? Why? If the answer should be greater than the third term, which of the remaining numbers do we make the first, and which the second term of the proportion? Show why this is correct. When three terms of a proportion are given, how do we find the fourth? Can we multiply concrete numbers together? How do we avoid this in Proportion? How do we prove Simple Proportion? Why is this correct? By what other method can questions in Proportion be performed? How are the numbers arranged for canceling? Why arranged in this manner? If the answer ought to be less than the third term, which of the remaining numbers do we make the first, and which the second term of the proportion? Show why this is correct.

Operation.

lbs.	*lbs.*	$
8 :	5 ::	: 7

$$\begin{array}{r} 7 \\ \hline 8)35 \\ \hline \end{array}$$

We arrange the terms as in the first example, excepting that we make 5 the second term, because 5 lbs. will evidently cost less than 8 lbs. We then proceed according to Obs. 17.

Hence— Ans. $4\frac{3}{8}$=\$4.37$\frac{1}{2}$.

Obs. 19. *If the answer ought to be less than the third term, make the least of the remaining numbers the second term, and the greater number the first term of the proportion.*

REMARK.—This is evident from the fact, that if the answer should be less than the third term, *the ratio of each couplet is less than unity;* (Art. 1. Obs. 14.) *and therefore, each consequent should be less than its antecedent.*

Analytic Solution.—If 8 lbs. cost \$7, a lb. will cost \$7÷8=\$$\frac{7}{8}$, and 5 lbs. will cost \$$\frac{7}{8}$×5=\$$\frac{35}{8}$=\$4.37$\frac{1}{2}$, as before.

There is also another method by which questions in simple proportion may be performed, which may be thus explained:

A proportion must consist of two couplets. (Obs. 5, *a*) and these couplets must both have the same ratio. (Obs. 1., and Obs. 5. 6.) Now, in examples in simple proportion we have given one couplet, and the antecedent of another couplet, to find the consequent of this latter couplet. (Obs. 4. Rem. 1.) But the consequent of any couplet is equal to its antecedent, multiplied by the ratio; (Art. 1. Obs 6.) therefore, as the first term must have the same ratio to the second, as the third has to the fourth, (Obs. 4.)

Obs. 20. *The fourth term of a proportion may be found by multiplying the third term by the ratio of the first to the second.*

Thus, in the first example, we have the proportion 4 : 7 : : 6 : the fourth term. Here the ratio of 4 : 7 is $\frac{7}{4}$, (Art. 1. Obs. 2.) and \$6×$\frac{7}{4}$=\$10.50, as before.

In the second example, we have the proportion 8 : 5 : : 7 : the fourth term. Here the ratio of 8 : 5 is $\frac{5}{8}$, (Art. 1. Obs. 2.) and \$7×$\frac{5}{8}$=\$4.37$\frac{1}{2}$, as before.

3. If 3 bu. 3 pks. 6 qts. of wheat must be given for 2 galls. 3 qts. 1 pt. of wine, how much wine must be given for 6 bu. 2 pks. of wheat? Ans. 4 galls. 2 qts. $1\frac{61}{63}$ pts.

Of how many couplets must a proportion consist? What relation must the ratios of these couplets have to each other? What have we given in examples in Simple Proportion? What have we to find? To what is the consequent of any couplet equal? What relation have the four terms of a proportion to each other? By what other method then can the fourth term of a proportion be found?

Operation.

$$
\begin{array}{lll}
\text{bu. pks. qts.} & \text{bu. pks.} & \text{galls. qts. pt.} \\
3 \;..\; 3 \;..\; 6 \;: & 6 \;..\; 2 \;: & :\; 2 \;..\; 3 \;..\; 1 \\
4 & 4 & 4 \\
\hline
15 \text{ pks.} & 26 \text{ pks.} & 11 \text{ qts.} \\
8 & 8 & 2 \\
\hline
126 \text{ qts.} & 208 \text{ qts.} \;: & :\; 23 \text{ pts.}
\end{array}
$$

$$
\begin{array}{r}
208 \\
23 \\
\hline
624 \\
416 \\
\hline
126)4784(37\tfrac{61}{63} \text{ pts.} = 4 \text{ galls. } 2 \text{ qts. } 1\tfrac{61}{63} \text{ pts. Ans.} \\
378 \\
\hline
1004 \\
882 \\
\hline
122\tfrac{122}{126} = \tfrac{61}{63}.
\end{array}
$$

In this example, it is first necessary to reduce the first and second terms to one, and the same denomination, and also to reduce the third term to the lowest denomination mentioned in it, before we can proceed further. We then work as usual, and the fourth term of course is pints, which is the denomination to which the third term was reduced. (Obs. 10.) Hence—

Obs. 20. *If the first and second terms contain different denominations, reduce both to one and the same denomination; and if the third term is a compound number, reduce it to the lowest denomination mentioned in it; after this, proceed according to* Obs. 17; *the result will be of the denomination as the third term, when reduced.*

Remark.—The arrangement of the terms, as in the preceding examples, is called *stating the question.*

Analytic Solution.—If 126 qts. pay for 23 pts., 1 qt. will pay for $\frac{1}{126}$ of 23 pts., or $\frac{23}{126}$ pts. Again, if 1 qt. pays for $\frac{23}{126}$ pts. 208 qts. will pay for 208 times as much, and $\frac{23}{126} \times 208 = \frac{2392}{63}$ pts. = 4 galls. 2 qts. $1\frac{61}{63}$ pts., as before.

From the preceding examples, we perceive that all questions in

---

If the first and second terms contain different denominations, how do we proceed? If the third term is a compound number, how do we proceed? Of what denomination is the result?

Simple Proportion are solved on precisely the same principles. Hence—

Obs. 22. *All questions in Simple Proportion can be solved by Analysis.*

From the preceding remarks and illustrations, we derive the following

GENERAL RULE FOR SIMPLE PROPORTION.

I. *Make that number the third term which is of the same kind as the answer required.* (Obs. 15.)

II. *Consider from the nature of the question whether the answer should be greater or less than this term.*

III. *If the answer ought to be greater than the third term, make the greater of the two remaining numbers the second term, and the less number the first term.* (Obs. 16.)

IV. *If the answer ought to be less than the third term, make the least of the two remaining numbers the second term, and the greater number the first term.* (Obs. 19.)

V. *Then multiply the second and third terms together, and divide the product by the first term; the quotient will be the fourth term, or answer,* (Obs. 17) *which is always of the same denomination as the third term.* (Obs. 21.)

Proof.—*Multiply the first and fourth terms together, and also the second and third terms; if the two products are equal, the work is correct.* (Obs. 18.)

Remark 1.—If either, or all the terms, are compound numbers, the first and second terms must be reduced to the *same denominations*, and the third term must be reduced to the *lowest denomination mentioned in it*, before the multiplication or division can be performed (Obs. 21.)

2.—If the learner choose, *he may reduce the lower denominations to fractions or decimals of a higher*, instead of reducing the *higher* to the *lower* denominations. (Sect. IX. Art. 3. Cases 2, 3, and 4. Rules.)

3.—We can often shorten the operation by multiplying the third term by the ratio of the first term to the second. (Obs. 20.) When the ratio is an *integer*, or *small fraction*, this method is preferable.

4.—If there is a remainder after the division has been performed, reduce it to the next lower denomination, and divide as before.

5.—This rule is equally applicable, whether the numbers are *Integral, Fractional, or Decimal.*

What is the arrangement of the terms as in the preceding examples called? How can all questions in Simple Proportion be solved? What is the rule in Simple Proportion? What is the proof? If either or all the terms are compound numbers, how do we proceed? By what other method can we proceed with compound numbers? How can we often shorten the operation? When is this method preferable? If there is a remainder after the division has been performed, how do we proceed?

Rule—By Cancelation: *Consider from the nature of the question which are the different terms; then after placing the second and third terms at the right, and the first term at the left, cancel as usual.*

Note.—The above Remarks, (the third excepted,) are equally applicable to Cancelation or Proportion.

EXERCISES FOR THE SLATE.

1. If 8 bushels of apples cost $4, how much will 14 bushels cost? Ans. $7.

2. If 9 bushels of peaches cost $6.75, how much will 12 bushels cost? Ans. $9.

3. If 14 yds. of cloth cost $21, how much will 11 yds. cost? Ans. $16.50.

4. If 13 pencils cost $22.75, how much will 9 pencils cost? Ans. $15.75.

5. If $9 buy 12 bushels of apples, how many bushels will $15 buy? Ans. 20.

6. If $24.50 will pay for 14 saws, how many saws can be bought for $14? Ans. 8.

7. If 6 men can do a piece of work in 18 days, in what time can 9 men do it? Ans. 12 days.

8. If 12 men can do a piece of work in 21 days, in what time can 7 men do it? Ans. 36 days.

9. If 10 lbs of sugar cost $1.25, how much will 1 cwt. cost? Ans. $12.50.

10. If a man can travel 75 miles in 3 days, how far will he travel in 3 weeks? Ans. 450 miles.

Note.—The learner will observe that he travels 6 days in the week.

11. If 2 cwt. 3 qrs. 15 lbs. of coffee cost $30.45, how much will 2 qrs. 22 lbs cost? Ans. $7.56.

12. If 2 yds. 3 qrs. of cloth cost $5.50, how much will 3 qrs. 2 na. cost? Ans. $1.75.

13. If the moon moves $105° 24' 40''$ in 8 days, how long will it take it to perform one revolution, or 360°? Ans. 27 d. 7 h. 43 m.+.

14. If a pole 5 feet high cast a shadow 3 ft. 6 in. in length, how high must a pole be to cast a shadow 130 feet? Ans. $185\frac{5}{7}$ feet.

15. If 7 yards of cloth cost $10.50, how much will 15 bushels of apples cost, if 9 bushels of apples are worth 3 yards of cloth? Ans. $7.50.

Is this rule applicable to either integral, fractional, or decimal numbers? What is the rule by cancelation?

16. If from a staff that's three feet long,
A shadow five is made,
What is a steeple's heighth in yards,
That's ninety feet in shade?

Ans. 18 yards.

17. If I give \$15 for the use of \$200 a certain time, how much must I give for the use of \$700 for the same time? Ans. \$52.50.

18. If a family of 6 persons consume 50 lbs. of flour in a certain time, how much will they consume in the same time, if 5 persons more are added to the same family? Ans. $91\frac{2}{3}$ lbs.

19. If a man can perform a piece of work in 16 days, working 10 hours per day, how long will it take him to perform it if he works 12 hours per day? Ans. $13\frac{1}{3}$ days.

20. If 84 bushels of oats last 30 horses 7 days, how long will 215 bushels last them? Ans. $17\frac{11}{12}$ days.

Upon carefully examining the question, it will be perceived that the number of horses is not one of the terms of the proportion.

21. If the interest of \$600 for 12 months is \$36, what is the interest of the same sum for 9 months? Ans. \$27.

Fractions.—22. If $\frac{2}{3}$ of a yard of cloth cost $\frac{7}{8}$ of a dollar, how much will $\frac{11}{12}$ of a yard cost?

$\frac{2}{3}$ yd. : $\frac{11}{12}$ yd. : : \$$\frac{7}{8}$: \$$\frac{77}{64}$ = \$$1\frac{13}{64}$. Ans.

23. If $\frac{11}{7}$ of a barrel of flour cost \$8, what will $\frac{15}{16}$ of a barrel cost? \$$4\frac{17}{22}$.

24. If $\frac{4}{5}$ of a pound of tea cost \$$\frac{7}{10}$, how much will $\frac{6}{7}$ of a pound cost? Ans. \$0.75.

25. If $\frac{2}{3}$ of $\frac{5}{6}$ of a gallon of wine cost $\frac{9}{10}$ of a dollar, how much would $\frac{4}{5}$ of $\frac{5}{7}$ of a gallon cost? Ans. \$$0.92\frac{4}{7}$.

26. If \$$18\frac{7}{8}$ buy $15\frac{1}{10}$ bushels of wheat, how many bushels can be bought for \$$28\frac{11}{16}$? Ans. $22\frac{19}{20}$.

27. If $4\frac{3}{4}$ yds. of cloth cost \$$5\frac{1}{2}$, how much will $7\frac{1}{8}$ qrs. cost? Ans. $2\frac{1}{16}$.

28. If 215 bushels of oats last 30 horses $17\frac{11}{12}$ days, how long will 84 bushels last them? Ans. 7 days.

29. If $\frac{3}{4}$ of an acre of land cost \$$\frac{27}{4}$, how much would $\frac{15}{16}$ of an acre cost? Ans. \$$8\frac{7}{16}$.

30. If a railroad car goes 162.75 miles in 7.75 hours, how far, at this rate, would it travel in 2.5 days? Ans. 1260 miles.

31. If $22\frac{1}{7}$ lbs. of butter cost \$4.74, how much would $1\frac{7}{8}$ cwt. cost? Ans. \$$39.37\frac{1}{2}$.

32. If 5.25 bushels of wheat cost \$$4.72\frac{1}{2}$, how much would 8.37 bushels cost? Ans. \$7.533.

33. If 12.45 pounds of tea cost \$9.3375, how much would 9.875 lbs. cost? Ans. \$$7.40\frac{5}{8}$.

34. If 20.125 acres of land cost \$322, how much would 15.5 acres cost? Ans. \$248.

35. If 12 horses eat 62.75 bushels of oats in 7 days, how much will they eat in 15 days? Ans. 134.46+ bushels.

36. A field was measured and found to be 148 rods in length; but afterwards the line with which it was measured was found to be but $31\frac{3}{4}$ feet long instead of 33 feet as was supposed. Required—the true length of the field? Ans: $142\frac{13}{33}$ rds,

Case 2.—Compound Proportion:

Obs. 23. A Compound Proportion *is an equality between a compound and a simple ratio.*

Thus: $\left.\begin{matrix}3:12\\4:20\end{matrix}\right\} :: 5:100$ is a compound proportion, and is read—"3 into 4 is to 12 into 20 as 5 is to 100;" or, "the ratio of $3\times4:12\times20$ is equal to the ratio of $5:100$."

Remark 1.—The learner will perceive that it is not the ratio of 3 : 12, or that of 4 : 20 alone, which is equal to that of 5 : 100; but *it is the ratio compounded of these* which is equal to that of 5 : 100. Thus:

$3\times4 : 12\times20 :: 5 : 100$; because $3\times4\times100=12\times20\times5$.—(Obs. 9.)

2.—*Compound Proportion is chiefly used when more than one statement is necessary in Simple Proportion.* It is often called *The Double Rule of Three*, but we consider the term *Compound Proportion* preferable, as it expresses what it really is.

Ex. 1. If a man ride 246 miles in 6 days, riding 8 hours per day, how far can he ride in 14 days, riding 12 hours per day? Ans. 861 miles.

Solution.—In this example two things are to be considered:

1st. *The number of days he rode.* From this we deduce this question: *If a man rides 246 miles in 6 days, how far will he ride in 14 days.* And the proportion and result stand thus:

6 : 14 days : : 246 miles : 574 miles.

2nd. *The number of hours he rode per day.* From this and the last proportion we obtain this question: *If a man rides 574 miles in a certain number of days, riding 8 hours per day, how far will he ride in the same time, riding 12 hours per day?* And the proportion and result stand thus:

8 hrs. : 12 hrs. : : 574 miles . 861 miles.

What is Compound Proportion? How do you read the expression given? Is it the ratio of 3 : 12, or of 4 : 20 alone which is equal to that of 5 : 100? What ratio then is equal to that of 6 : 100? Why? When is Compound Proportion chiefly used? What is it often called? What is a more appropriate name? Why?

All questions in Compound Proportion involve the same principles. Hence—

Obs. 24. *All questions in Compound Proportion can be performed by two or more statements in Simple Proportion.*

This example, however, can be worked by one statement, thus:

Operation.

$$\left.\begin{array}{lcl} 6 \text{ days} & : & 14 \text{ days} \\ 8 \text{ hours} & . & 12 \text{ hours} \end{array}\right\} : : 246 \text{ miles} : \text{the answer.}$$

```
48        : 168          : : 246 miles : the answer.
            246
           ----
           1008
           672
          336
         ------
   68)41328(861 miles. Answer.
      384
      ---
      292
      288
      ---
       48
       48
       --
       00
```

As we desire miles for our answer, we make 246 miles the third term. (Obs. 15.) Then, if he rides 246 miles in 6 days, he will evidently ride farther in 14 days; therefore, we make 14 the second term, and 6 the first term. (Obs. 16.) Again, if he rides 12 hours per day, he will evidently ride farther than if he rides but 8; therefore we make 12 another second term, and 8 the first. (Obs. 16.) But we perceive this to be a compound proportion, (Obs. 23.) and is equivalent to the simple proportion 48 (6×8) : 168 (14×12) : : 246 : the answer. (Obs. 23. Rem. 1.) which is found according to Obs. 17. Hence—

To perform operations by one statement in Compound Proportion.

Obs. 25. *Arrange the third term, and each couplet as in Simple*

How can all questions in Compound Proportion be performed? When we make but one statement, how do we arrange the terms?

Proportion. (Obs. 15, 16, and 17.) *Then multiply the continued product of all the second terms by the third term, and divide the result by the contined product of all the first terms.*

The above example may be proved thus: $6\times8\times861=14\times12\times246$. (Obs. 9.) Hence—

To prove Compound Proportion:

Obs. 26. *Multiply the continued products of the first terms by the fourth term; also, the continued product of the second terms by the third term; if the two results are alike, the work is correct.* (Obs. 9.)

REMARK.—In all questions in Compound Proportion, there is *one* number of a different kind from any of the rest, and *this number is always made the third term.*

The fourth term of the above proportion may also be found thus: The ratio compounded of two couplets 6 : 14 and 8 : 12 is $3\frac{1}{2}$. (Art. 1. Obs. 16. *b.*) Now the third term (246) must have the same ratio to the fourth term, or answer, (Obs. 23, Rem, 1.) and $246\times3\frac{1}{2}=861$. (Obs. 20.)

Hence—we can obtain the fourth term of a Compound Proportion:

Obs. 27. *By multiplying the third term by the ratio compounded of the first and second terms.*

Analytic Solution.—If he rides 246 miles in 6 days, he will ride $\frac{1}{6}$ of 246 miles, or 41 miles, in 1 day; then if he rides but 8 hours per day, he will ride $\frac{1}{8}$ of 41 miles, or $\frac{41}{8}$ miles per hour; and in 12 hours he will ride 12 times as far, or $\frac{123}{2}$ miles; and in 14 days, of 12 hours each, he will ride 14 times $\frac{123}{2}$ miles, which is 861 miles, as before.

All questions in Compound Proportion are solved on the same principles. Hence—

Obs. 28. *All questions in Compound Proportion can be solved by Analysis.*

Operations in Compound Proportion can frequently be shortened by canceling factors common to the first, and the second or third terms, as in the following

---

How do we next proceed? How do we prove the operation? What occurs in all questions in Compound Proportion? What term is this number made? By what other method can the fourth term of a compound proportion be found? How may all questions in Compound Proportion be solved? How can we often shorten the operation?

Operation.

$$\begin{array}{r|l} \cancel{6} & \cancel{14}\ldots 7 \\ \cancel{2}\ldots\cancel{4}\ldots 8 & \cancel{12}\ldots 2 \\ & \cancel{246}\ldots 123 \end{array}$$

$123 \times 7 = 861$

Our first terms are all factors of some number which is a divisor, and our second and third terms are all factors of some number which is a dividend; (Obs. 25.) therefore we place all our first terms at the left of the line, and all the other numbers at the right of the line, (Sect. VII. Art. 1. Obs. 3.) and cancel as usual.

REMARK 1.—We have presented these different methods of obtaining the result, both in Simple and Compound Proportion, for the purpose of showing the relation which numbers bear to each other, and also to show how the same result may be obtained by different processes, thus proving clearly the correctness of the principles by which we operate. And this we consider to be one of the principal beauties of mathematics—*that we are obliged to take no principle, or believe no theory, unless it is capable of demonstration, or proof.* And one indubitable proof of the correctness of mathematical principles, is, that *we always obtain the same result to a question or problem, no matter what principles we apply, or what process we use, provided we make no mistake in our reasoning, or mode of application.* The case would be otherwise, were our principles incorrect, because then the application of different principles to the same question, would often produce different results; but as this never happens, we conclude that they are *absolutely true.*

2.—Of the different methods of obtaining the result in proportion, that by *Analysis* is decidedly preferable, as *it is better adapted to the discipline of the mind, and strengthens the mental faculties.* It is recommended to the pupil to solve all the questions, both in Simple and Compound Proportion, by Analysis, after he has obtained the result by Proportion. He will by this means gain sufficient mental culture to amply compensate him for his trouble.

From the foregoing remarks and illustrations, we derive the fol-following

GENERAL RULE FOR COMPOUND PROPORTION.

I. *Make that number the third term which is of the same kind as the answer.* (Obs. 15.)

II. *Take two of the remaining numbers of the same kind, and arrange them in the same manner as in Simple Proportion.* (Obs. 16 and 17.) *Proceed in the same manner with all the other numbers.*

III. *Multiply the continued product of the second terms by the third term, and divide the result by the continued product of the first terms.* (Obs. 25.)

How do we arrange the terms for canceling? Why do we arrange them in this manner? What do these different methods in obtaining the result in Simple and Compound Proportion show? What does this prove? What is one of the principal beauties in mathematics? What is one indubitable proof of the correctness of mathematical principles? Would this be so were our principles incorrect? Why not? As this never happens, what do we conclude? Of the different methods of obtaining the result in Proportion, which is preferable? Why so? What is the rule in Compound Proportion?

PROOF.—*Multiply the fourth term by the continued product of the first terms; also, multiply the third term by the continued product of the second terms; and if the two results are equal, the work is correct.* (Obs. 26.)

NOTE.—The remarks under the rule in Simple Proportion are equally applicable to Compound Proportion, except the third, of which it will be observed in Compound Proportion that we multiply the third term by the ratio compounded of the first and second terms, instead of the simple ratio of either couplet. (Obs. 27.)

RULE—By Cancelation.—*Consider which are the different terms: then place the third term together with all the second terms at the right, and all the first terms at the left.*

EXERCISES FOR THE SLATE.

1. If 15 men earn \$225 in 12 days, how much will 9 men earn in 24 days, at the same rate? Ans. \$270.

2. If 8 men earn \$84 in 7 days, working 10 hours per day, how long will it take 12 men to earn \$300, working 12 hours per day? Ans. $13\frac{8}{9}$ days.

3. If 7 men cut 49 acres of grass in 4 days, how many days will it take 11 men to cut 112 acres of grass? Ans. $5\frac{9}{11}$.

4. If the interest on \$200 for 2 years is \$24, what is the interest on \$150 for 1 year and 9 months? Ans. \$15.75.

5. If 12 students spend \$100 in 3 weeks, how much will 7 students spend in 5 weeks? Ans. $\$97.22\frac{2}{9}$.

6. If 10 men dig a trench 240 yards long, 6 feet wide, and 5 feet deep, in 8 days, how long will it take 18 men to dig a trench 400 yards long, 8 feet wide, and 4 feet deep? Ans. $7\frac{73}{81}$ days.

7. If 6 men, working $12\frac{3}{10}$ hours per day, dig a cellar $22\frac{1}{2}$ ft. long, $17\frac{3}{10}$ ft. wide, and $4\frac{3}{4}$ ft. deep, in $2\frac{1}{2}$ days, how long will it take 9 men to dig a cellar 45 ft. long, $35\frac{3}{5}$ ft. wide, and $5\frac{7}{10}$ ft. deep, working $8\frac{1}{5}$ hours per day? Ans. 12 days.

8. If 80 men dig 20 cellars, each 45 ft. long, 28 ft. wide, and 3 ft. deep, in 10 days, working 12 hours per day, how many men will it require to dig 30 cellars, each 24 ft. long, 21 ft. wide, and $4\frac{1}{2}$ ft. deep, in 18 days, working but 10 hours per day, supposing the strength of the men in the former case to be $1\frac{5}{9}$ that of those in the latter, and the hardness of the ground in the latter case to be only $\frac{3}{4}$ of that in the former? Ans. 56.

What is said respecting the remarks under the rule in Simple Proportion? What must be observed with respect to remark third? What is the rule by cancelation?

Article 3. Partnership.

Obs. 1. Partnership is the associating together of two or more persons in joint trade, with a mutual agreement to share the respective gains or losses in proportion to the capital each one has invested, and the time it has been employed ; this association is called a Company, or Firm, and each individual is called a Partner.

Obs. 2. Stock is the name given to the money, or value of the articles employed in trade. It is sometimes called Capital.

The gain or loss divided is called the Dividend.

There are two cases of Partnership.

Case 1.—*This case is used when no consideration is made with regard to time.*

Ex. 1. A. and B. enter into partnership. A. furnished $300, and B. $500; they gained $250. What was each man's share of the gain?

Solution.—A. put in $300, and B. $500 ; hence, $300+$500=$800, the whole stock invested. Now it is evident that each one's share, of the gain must have the same relation to the whole gain, as each one's share has to the whole stock. Hence, the following

Operation.

		Whole stock.		A.'s stock.		Whole gain.		A.'s share of gain.
$300 A.'s stock.								
$500 B.'s stock.	Then,-	$800	:	$300	: :	$250	:	$93.75.
				B.'s stock.				B.'s share of g'n
$800 Whole stock.	And,	$800	:	$500	: :	$250	:	$156.25.

Proof:
$ 93.75 A.'s share of the gain.
$156.25 B.'s share of the gain.
$250.00 Whole gain.

Solution by Analysis.—As A. put in $300, he owns $\frac{300}{800}=\frac{3}{8}$ of the stock ; therefore he must have $\frac{3}{8}$ of the gain, and the $\frac{3}{8}$ of $250 is $93.75, A.'s gain.

Also, as B. put in $500, he owns $\frac{500}{800}$ of the stock, and therefore must have $\frac{5}{8}$ of the gain ; $\frac{5}{8}$ of $250 is $156.25, B.'s gain

2. Suppose, in the above case, they had lost $200 ; what would have been each one's share of the loss?

What is Partnership? What is the association called? What is each individual called? What is stock? What is it sometimes called? What is the gain or loss to be divided called? Of how many cases does Partnership consist? When is Case 1 used?

Operation.

$300
$500 Then, $800 : $300 : : $200 : $ 75, A.'s loss.
—— And $800 : $500 : : $200 : $125. B.'s loss.
$800

PROOF: $\begin{cases} \$\ 75, \text{ A.'s loss.} \\ \$125, \text{ B.'s loss.} \\ \text{——} \\ \$200 \text{ whole loss.} \end{cases}$

Analytic Solution.—A. put in $\frac{3}{8}$ of the stock, and therefore must sustain $\frac{3}{8}$ of the loss : $\frac{3}{8}$ of $200 is $75, A.'s share of the loss. B. put in $\frac{5}{8}$ of the stock, and therefore must sustain $\frac{5}{8}$ of the loss : $\frac{5}{8}$ of 200 is $125, B.'s share of the loss.

From these illustrations we derive the following

RULE FOR PARTNERSHIP.

As the whole stock is to each man's share of the stock, so is the whole gain or loss to each man's share of the gain or loss.

PROOF.—*Add together the several shares of the gain or loss; if the sum is equal to the whole gain or loss, the work is correct.*

REMARK.—This case is often called *Single Fellowship;* but as a partnership cannot exist without at least *two* individuals, the propriety of calling it single is somewhat doubtful.

3. A., B., and C. trade together: A. puts in $500, B. $800, and C. $1000; they gain $690. What is each one's share.
Ans. A.'s, $150; B.'s, $240; and C.'s, $300.

4. A., B., and C. speculate together. A. furnishes $3000, B. $5000, and C. $4000; they lose $3000. How much was each one's loss? Ans. A's $750; B's $1250; and C's $1000.

5. P., Y., and R. own a vessel worth $10000. P's share is worth $3500; Y.'s share is worth $4250, and R. owns the rest. In a certain trip they gain $1250. How much is each one's share of the gain?
Ans. P.'s $437.50; Y.'s $531.25; and R.'s $281.25.

6. U. V. and W. own a store. U. owns $\frac{2}{7}$, V. owns $\frac{5}{14}$, and W. owns the rest; they lose $480. How much is each one's share of the loss?
Ans. U.'s 137.142\frac{6}{7}$; V. and W., each 171.428\frac{4}{7}$.

7. A., B. and C. loaded a ship with flour. A. put on 400 bar.

What is the method of proof? What is this case often called? Is this term appropriate? Why not?

rels, B. 350, and C. 250; in a gale the Captain threw overboard 200 barrels. What loss did each one sustain?

A.'s loss 80 bbls.; B.'s 70 bbls; and C.'s 50 bbls.

8. Four persons trade together. A. put in \$200; B. \$400; C. \$350; and D. \$550; they gain \$600. What is the share of each?

Ans. A.'s, \$80; B.'s, \$160; C.'s \$140; and D.'s \$220.

Obs. 4. BANKRUPTCY.—*A Bankrupt is a person who is unable to pay his debts.*

Questions in Bankruptcy are usually performed by the rule of Partnership.

9. A bankrupt owes \$4000; his property is worth but \$3000. How much can he pay on the dollar? Ans. \$0.75.

10. A man's property is worth \$5000. He owes A. \$1000, B. \$3500, C. \$850, D. \$1500, and E. \$2150. How much can he pay each in proportion to their debts?

Ans. A. \$555.555$\frac{5}{9}$; B. \$1944.444$\frac{4}{9}$; C. \$472.222$\frac{2}{9}$; D. \$833.333$\frac{1}{3}$; and E. \$1194.444$\frac{4}{9}$.

11. A man died, leaving property worth \$10000. He owed F. \$2500; G. \$3050; H. \$4500; and I. \$2450. How much can each creditor receive, and how much can the estate pay on the dollar?

Ans. F. receives \$2000; G. \$2440; H. \$3600; and I. \$1960. And the estate pays \$0.80 cents on the dollar.

Obs. 5. GENERAL AVERAGE.—When the master of a vessel, in consequence of a storm, or other casualties at sea, is obliged to throw overboard a part of the cargo, in order to save the ship and crew, the law requires that the loss shall be sustained by the owners of the vessel and cargo, in proportion to the value of each individual's property at stake. The property sacrificed is called the *Jettison.*

The process of finding each one's loss in such cases is called GENERAL AVERAGE. The rule is the same as in Partnership.

12. A vessel being in distress, the Captain threw overboard goods to the value of \$15000 The cargo was owned by three persons, L., M. and N. L. owned \$20000; M. \$35000; and N. \$25000: the vessel was worth \$20000. How much was each man's loss?

Ans. L.'s \$3000; M.'s \$5250; N.'s \$3750; and the owner of the vessel \$3000.

What is a bankrupt? How are questions in bankruptcy usually performed? When the master of a vessel is obliged to throw overboard a part of his cargo, in order to save his ship or crew, how is the loss sustained? What is the process of finding each one's loss in such cases called? What is the rule?

13. In consequence of a storm at sea, a vessel sustained the following loss:

For a part of her cargo thrown overboard, valued at	\$4850
For repairing damages of the ship, &c.,	450
Other expenses,	300

The loss was sustained by a general average, the value of the ship and cargo being as follows:

Value of the vessel,	\$25000.
Part of the cargo owned by Wm. Trader, & Co. valued at	48000.
Part " " " Thos. Dealer, & Co., "	15000.
Part " " " James Lovegain, "	27000.
Part " " " Captain of the vessel, "	10000.

What share of the loss must each sustain?

Ans. The owner of the vessel \$1120; Tra'er & Co. \$2150.40; Dealer & Co. \$672; James Lovegain \$1209.60; Captain of the vessel \$448.

A number can also be divided into proportional parts by the rule of Partnership.

14. Divide the number 576 into three parts that shall be to each other as the numbers 7, 8, and 9.

$7+8+9=24$; $24:7::576:168$, &c.

Ans. 168, 192, and 216.

15. Divide the number 1728 into 6 parts, that will be to each other as the numbers 3, 4, 5, 6, 8, and 10.

Ans. 144, 192, 240, 288, 384, and 480.

Case 2.—Obs. 6. *This case is used when time is considered.*

Ex. 1. A. and B. hire a pasture for \$10. A. put in two cows 4 months, and B. put in 4 cows 3 months. How much ought each to pay?

Solution.—A. put in 2 cows 4 months; this is the same as 8 cows 1 month. B. put in 4 cows 3 months; this is the same as 12 cows 1 month. Hence, they must pay in the proportion of 8 to 12.

Operation.

$2\times 4 = 8.$
$4\times 3 = 12$ Then $20 : 8 :: \$10 : \$4$ A.'s share.
And $20 : 12 :: \$10 : \$6$ B.'s share.
—
20

Proof. \$ 4 A.'s share.
6 B.'s share.
—
\$10 Whole cost.

When is Case 2 used?

Analytic Solution.—A. has in the same as 8 cows 1 month, and B. has in the same as 12 cows 1 month; and both have in the same as $8 + 12 = 20$ cows 1 month. Then A. must pay $\frac{8}{20} = \frac{2}{5}$, and B. $\frac{12}{20} = \frac{3}{5}$ of the cost. $\frac{2}{5}$ of \$10 is \$4, A.'s share, and $\frac{3}{5}$ of \$10 is \$6, B.'s share, as before. Hence—

To solve questions in Partnership, when time is considered:

Obs. 7. *Multiply each man's stock by the time he continues it in trade, and use the product for his share.*

2. A., B. and C. trade together. A. put in \$400 for 6 months; B. put in \$500 for 4 months; and C. put in \$350 for 8 months; they gained \$300. How much was each one's share of the gain?
Ans. A.'s \$100; B.'s \$83.333$\frac{1}{3}$; and C.'s \$116.666$\frac{2}{3}$.

3. Suppose in the last example they had lost \$200. How much would have been each one's share of the loss?
Ans. A.'s \$66.666$\frac{2}{3}$; B.'s \$55.555$\frac{5}{9}$; C.'s \$77.777$\frac{7}{9}$.

4. W., X., Y. and Z. trade together for one year. W. put in \$500 for 6 months, and then put in \$700 more; X. put in \$1200 for 4 months, and then took out \$500; Y. put in \$800 for 5 months, and then took out \$200, but put it back again at the end of 9 months, with \$400 more; Z. put in \$700 for 7 months, and then put in \$800 more; but he took out the whole of his stock at the end of 11 months; at the end of the year they have gained \$2490, How much is each one's share?
Ans. W.'s \$612; X.'s \$624; Y.'s \$600; and Z.'s \$654.

SECTION XIII.

PERCENTAGE.

Article 1. Definitions, Mental Exercises, &c.

Obs. 1. The terms *percentage* and *per cent* are derived from two Latin words *per* and *centum*, which signify *by the hundred.*

Therefore, when we speak of a certain per cent of any number, we evidently mean such a *hundredth part* of that number.

Thus, 5 per cent. of any number is $\frac{5}{100}$ of that number; 12 per cent of any number is $\frac{12}{100}$ of that number, &c.

How do we solve questions in Partnership when time is considered? From what are the terms *per cent* and *percentage* derived? What do these words signify? What do we mean when we speak of a certain per cent of any number?

MENTAL EXERCISES.

1. What is 2 per cent of 100?

2 per cent of 100 is $\frac{2}{100}$ of 100. Now $\frac{1}{100}$ of 100 is 1, and $\frac{2}{100}$ of 100 is $1 \times 2 = 2$. Ans. 2.

2. How much is 4 per cent of 100 cents? Ans. 4 cents.

4 per cent is $\frac{4}{100}$. $\frac{4}{100}$ of 100 cents is 4 cents.

3. A man borrowed $100, paying 6 per cent for the use of it. How much did he pay? Ans. $6.

6 per cent is $\frac{6}{100}$. $\frac{6}{100}$ of $100 is $6.

4. A man having 100 sheep, lost 10 per cent of them by disease. How many did he lose?

10 per cent is $\frac{10}{100}$. $\frac{10}{100}$ of 100 is 10. Hence—

Obs. 2. *Per cent implies so many units for each* 100 *units, so many cents for each* 100 *cents, so many dollars for each* 100 *dollars, so many articles for each* 100 *articles, &c.: and the per centage of any number is as many times the per cent as there are hundreds in this number.*

5. How much is 6 per cent of 300? Ans. $6 \times 3 = 18$.
6. How much is 3 per cent of 200? 400; 700; 500; 800; 900; 290?
7. How much is 8 per cent of 100? 200; 500; 700; 900; 400; 800; 1200?
8. How much is 10 per cent of 200? 1200; 1500; 900; 700; 1000; 2000?
9. A man having found $6, the owner gave him 4 per cent for finding it. How much did he receive? Ans. 24 cents.

Solution.—He received 4 cents for each 100 cents; in $6 are 600 cents. Therefore he received $4 \times 6 = 24$ cents.

10. A merchant sold a coat for $12; his gain was 10 per cent. of what he sold it for. What was his gain?
11. What is 10 per cent of $5? of $8; of $10; of $11; of $6; of $9; of $20?
12. A constable collected $8, and received 5 per cent for his services. How much did he receive?
13. How much is 5 per cent of $2? Of $4? Of $10? Of 7? Of 5? Of $12? Of $6?

What is 5 per cent of any number? 12 per cent? 6 per cent? 3 per cent? 9 per cent? 1 per cent? What is 2 per cent of 100? 6 per cent of 100 cents? 9 per cent of 100 cents? 8 per cent of 100? 2 per cent of $100? 12 per cent of $100? What does per cent imply? What is the percentage of any number?

14. A man had 200 sheep; in two years they increased 50 per cent. How many had they increased?

15. An auctioneer sold $1200 worth of goods, and received 4 per cent for selling. How much did he receive?

16. What is 4 per cent of $200? Of $800? Of $600? Of $1000? Of $300?

17. A man borrowed $460, and gave 6 per cent for the use of it? How much did it cost him?

18. How much is 6 per cent of $200? Of $800? Of $300? Of $500? Of 1200?

19. A man bought a farm for $1000, and sold it so as to gain 12 per cent? How much did he gain?

20. What is 12 per cent of $800? Of $1200? Of $700? Of $1000? Of $500? Of $900? Of $300? Of 600? Of $200? Of $1500? Of $2000?

Obs. 3. *Since* 1 *per cent signifies* $\frac{1}{100}$, 2 *per cent* $\frac{2}{100}$, *&c., it is evident that it may be expressed decimally.* Thus:

1 per cent may be written	....	.01.
2 per cent " "	....	.02.
3 per cent " "	....	.03.
4 per cent " "	....	.04.
7 per cent " "	....	.07.
9 per cent " "	....	.09.

a. When the given per cent is 10 or more, it is merely *written with the decimal point before it.* Thus:

10 per cent is written	....	.10.
12 per cent "	....	.12.
15 per cent "	....	.15.
25 per cent "	....	.25.
50 per cent "	....	.50.
75 per cent "	....	.75.
99 per cent "	....	.99.

b. When the given per cent is 100 or more, it is evidently a *mixed number*, and must be written accordingly. Thus:

100 per cent is written	....	1.00, or 1.
104 per cent "	....	1.04.
125 per cent "	....	1.25.
274 per cent "	....	2.74.

How can any per cent be expressed? Show why this is correct? When the given per cent is 10 or more, how is it written? When the given per cent is 100 or more, what kind of an expression is it? How written?

c. When the given per cent is less than 1, it is either written *fractionally*, or *it occupies three or more decimal places.* Thus:

$\frac{1}{2}$ per cent, that is $\frac{1}{2}$ of 1 per cent is written -- .00$\frac{1}{2}$, or .005.
$\frac{1}{4}$ per cent, that is $\frac{1}{4}$ of 1 per cent is written -- .00$\frac{1}{4}$, or .0025.
$\frac{3}{4}$ per cent, that is $\frac{3}{4}$ of 1 per cent is written -- .00$\frac{3}{4}$, or .0075.
2$\frac{1}{2}$ per cent, is written -- .02$\frac{1}{2}$, or .025&c

21. Write 5 per cent; 6 per cent; 8 per cent; 11 per cent; 14 per cent; 18 per cent; 22 per cent; 39 per cent; 50 per cent; and 18 per cent.

22. Write 120 per cent; 150 per cent; 130 per cent; $\frac{1}{5}$ per cent and $\frac{1}{10}$ per cent in decimals.

23. Write $\frac{3}{5}$ per cent; $\frac{4}{10}$ per cent; $\frac{3}{10}$ per cent; and $\frac{3}{20}$ per cent in decimals.

24. Write 2$\frac{1}{2}$ per cent; 4$\frac{1}{5}$ per cent; 6$\frac{1}{4}$ per cent; 8$\frac{3}{4}$ per cent; and 25$\frac{1}{2}$ per cent in decimals.

25. Write 82$\frac{2}{5}$ per cent; 326$\frac{1}{4}$ per cent; and 512$\frac{3}{4}$ per cent in decimals.

EXERCISES FOR THE SLATE.

1. A constable collected $600, for which he was to receive 4 per cent for collecting. How much did he receive?

Operation.

4 per cent is expressed .04. Therefore, if we multiply $600 by .04, the product will be what the constable must receive. (Obs. 2.) Hence—

$600
.04
———
Ans. $24.00

To find the percentage of any number:

Obs. 4. *Multiply the given number by the given per cent expressed decimally, and point off in the product as in multiplication of decimals.* (Sect. VIII. Art 11. Rule.)

2. What is 6 per cent of $80? Of $100? Of $150? Of $300? Of $1000?

3. What is 5 per cent of $30.26? Ans. $1.513.

4. What is 10 per cent of $75.84? Ans. $7.584.

5. What is 15 per cent of $125.75? Ans. 18.86\frac{1}{4}$.

6. A broker exchanged $1000, for which he was to receive $\frac{1}{2}$ per cent. How much did he receive? Ans. $5.

7. What is $\frac{1}{4}$ per cent of $2000? Of $800? Of $4000? Of $3000? Ans. $5; $2; $10; $7.50.

8. What is $\frac{1}{4}$ per cent of $450.80? Ans. $1.127.

When the given per cent is less than 1, how is it written? How do we find the percentage of any number?

9. What is $\frac{1}{8}$ per cent of $400? Of $800? Of $1200? Of $1500? Ans. $.50; $1; $1.50; 1.87\frac{1}{2}$.

10. What is $\frac{5}{8}$ per cent of $1000? Ans. $6.25.

11. What is $\frac{1}{3}$ per cent of $600? Ans. $2.

REMARK.—When the given per cent is a fraction, *we may first find the per centage at* 1 *per cent, and then take fractional parts of this for the given per cent.* This method is generally preferable.

12. What is $\frac{1}{6}$ per cent of $1200? Of $1500? Of $1800? Ans. $2; $2.50; $3.

13. What is 4$\frac{1}{2}$ per cent of $200? Ans. $9.

14. What is 3$\frac{1}{5}$ per cent of $60? Of $80? Of $100? Of $200? Ans. $1.92; $2.56; $3.20; $6.40.

15. An auctioneer sold $1000 worth of goods, for which he was to receive 12 per cent. How much did he receive? Ans. $120.

NOTE.—The learner will bear in mind that he received 12 per cent on what he *sold*, and not on what was *paid over*, as is often supposed. In the latter case he would only receive $\frac{12}{112}$, instead of $\frac{12}{100}$ of the value of the goods sold. The same remark is applicable to moneys collected by constables, &c.

16. A merchant bought $15000 worth of goods, and sold them so as to gain 15 per cent. How much did he gain? Ans. $2250.

17. A clock dealer shipped 800 clocks for New Orleans, but on the passage 12 per cent of them were washed overboard. How many were lost? Ans. 96.

18. Two men engage in trade with $1500 apiece. One gains 16 per cent, and the other gained 20 per cent. How much did one gain more than the other? Ans. $60.

19. A gentlemen deposited $1200 in a bank, and afterwards drew out 19$\frac{1}{2}$ per cent of it. How much did he take out, and how much had he left?

Ans. He took out $234, and had left $966.

20. What is the difference between 18 per cent of $300, and 24 per cent of $400? Ans. $42.

21. What is the sum of 14 per cent of $200, and 9 per cent of $700? Ans. $91.

22. What is 112 per cent of 200; 800; 1200; 1500; 1824; 4276; 5000?

Ans. in order. 224; 896; 1344; 1680; 2042.88; 4789.12; 5600.

23. What is 103 per cent of 400? Ans. 412.

24. What is 250 per cent of 100? 400; 600; 900; 1400; 1846; 8120?

Ans. in order. 250; 1000; 1500; 2250; 3500; 4615; 20300.

When the given per cent is a fraction, how do we proceed? Is the percentage received by constables, auctioneers, &c., calculated on the value of the property they sell or collect, or on what they pay over? Why so?

25. What is $6\frac{1}{4}$ per cent of \$240? \$273.80; \$320; \$480.40; \$512.96?

Ans. in order. \$15; \$17.11$\frac{1}{4}$; \$20; \$30.02$\frac{1}{2}$; \$32.06.

26. What is $8\frac{1}{3}$ per cent of \$240? \$345; \$462.36; \$512.34; \$680.64?

Ans. in order. \$20; \$28.75; \$38.53; \$42.69$\frac{1}{2}$; \$56.72.

27. What is $37\frac{1}{2}$ per cent of \$250? \$300; \$412.25; \$560.50; \$618.90?

Ans. in order. \$93.75; \$112.50; \$154.593+; \$210.18$\frac{3}{4}$; \$232.08$\frac{3}{4}$.

28. Which is the most—8 per cent of \$500, or 6 per cent of \$700? How much is the difference?

Ans. 6 per cent of \$700 is the greatest by \$2.

29. What is the difference between $4\frac{1}{2}$ per cent and $6\frac{1}{3}$ per cent of \$1500? Ans. \$32.50.

30. What is $\frac{1}{5}$ per cent of \$200? \$240.75; \$300; \$312.25; \$473.95?

Ans. in order. \$0.40; \$0.48+; \$0.60; \$0.624+; \$0.947+.

31. What is $\frac{7}{10}$ per cent of \$120? \$160; \$190; \$210; \$235; \$375.50?

Ans. in order. \$0.84; \$1.12; \$1.33; \$1.47; \$1.645; \$2.628+.

32. What is $\frac{9}{20}$ per cent of \$400? \$800; \$1000; \$1200; \$1500; \$2400?

Ans. in order. \$1.80; \$3.60; \$4.50; \$5.40; \$6.75; \$10.80.

33. What is $\frac{5}{12}$ per cent of \$48? \$72; \$84; \$120; \$288; \$324; \$1728.

Ans. in order. \$0.20; \$0.30; \$0.35; \$0.50; \$1.20; \$1.35; \$7.20.

34. A drover having 300 cattle, sold 40 per cent of them. How many did he sell? Ans. 120.

35. Two men had each \$1500. One gained 20 per cent of his, and the other spent 20 per cent of his. How much then had one more than the other? Ans. \$600.

Obs. 5. Percentage is applied to various calculations in practical business. The most important of these are *Commission, Insurance, Profit and Loss, Stocks, Brokage, Duties, Taxes,* and *Interest.*

Article 2. Commission and Insurance.

Obs. 1. Commission.—*Commission is the per cent, or sum charged by an agent for transacting business for his employer. It is generally applied to buying aad selling goods.*

To what is Percentage applied? Which are the most important of th-
What is Commission? To what is it generally applied?

REMARK 1.—The person who buys or sells goods for another is called a *Commission Merchant, Correspondent*, or *Factor*.

2.—As commission is reckoned at so much per cent on the *money employed* the rule is the same as in percentage. (Art. 1. Obs. 4.)

EXERCISES FOR THE SLATE.

1. A. sold \$200 worth of goods for B., at 6 per cent commission How much did he receive? Ans. \$12.

2. A man sold 100 bushels of wheat at \$1.25 per bushel, on commission, for which he received 5 per cent. How much was his commission? Ans. \$6.25.

3. How much is the commission for selling \$500 worth of goods, at 8 per cent? Ans. \$40.

8. How much is the commision for buying \$1200 worth of goods, at 4 per cent? Ans. \$48.

5. B. sold on commission for C. \$427.80 worth of goods, at $8\frac{1}{3}$ per cent; \$512.49 worth of goods at $10\frac{1}{2}$ per cent; and \$350.50 worth of goods at $12\frac{2}{5}$ per cent. How much did his commission amount to? Ans. \$132.914.

6. D. sold on commission for E. 120 yards of cloth, at \$4.50 per yard, for which he was to receive $4\frac{1}{3}$ per cent. How much was his commission, and how much must he return to E.?

Ans. The commission was \$23.40, and the sum returned was \$516.60.

7. A Southern merchant consigned to a merchant in Boston the following articles, to be sold on commision, at $7\frac{3}{5}$ per cent: 4200 lbs. of cotton, at $7\frac{1}{2}$ cts per pound; 7600 lbs. of sugar, at $8\frac{1}{4}$ cts. per pound; 19 bags of coffee, each containing 90 lbs., at 7 cents per pound; 2100 lbs. of rice, at $4\frac{3}{4}$ cents per pound; and 25 barrels of molasses, at \$15 a barrel. How much was the commission, and what sum must be returned to the owner?

Ans. The commission was \$116.77, and the sum returned was \$1419.68.

INSURANCE.

Obs. 2. *By* INSURANCE *is meant security from loss or damage occasioned by fires, storms, shipwrecks, &c.*

Obs. 3. This security is usually effected with *imdividuals*, or *insurance companies*, by the payment of a stipulated sum, which is generally a certain per cent of the value of the property insured.

What is the person who buys or sells goods for others called? How is commission reckoned? What then is the rule? What is meant by insurance? How is the security effected? When an insurance is effected by a contract with individuals, what is it termed? What is Marine Insurance? How effected?

REMARK 1.—When insurance is effected by a contract with individuals, it is termed *out door insurance.* Insurance at sea is called *Marine Insurance.* It is usually effected for a certain voyage.

2.—The insurers, whether individuals or an incorporated company, are often called *underwriters.*

Obs. 4. *The sum paid for the insurance is called the* PREMIUM; and it is paid when the insurance is effected.

The written instrument that binds the contracting parties is called the POLICY.

The amount of pecuniary responsibility taken by the insurers is called the RISK. It is sometimes equal to the *whole* of the estimated value of the property insured, and sometimes it is equal to only a *part* of the estimated value.

Case 1.—*When the premium is a certain per ecnt of the value of the property insured.*

REMARK.—The very nature of this case shows that the rule is the same as in Percentage. We must add the cost of the policy to the percentage, however, to find the actual cost of the insurance.

EXERCISES FOR THE SLATE.

1. How much must be paid for insuring a house valued at $1200, at $4\frac{1}{2}$ per cent, and the cost of the policy being $1. Ans. $55.

2. How much must be paid for insurance on a store valued at $4000, and on the goods valued at $15000, at $3\frac{2}{5}$ per cent; the policy costing $1.50? Ans. $647.50.

3. How much must be paid for insuring a steamboat valued at $10000, the policy costing $1, and the premium being 5 per cent on $\frac{4}{5}$ of the value of the boat, that being the amount of risk taken by the company? Ans. $401.

4. How much must be paid for insuring a ship valued at $20000, and the cargo valued at $50000, the premium on the ship being $6\frac{1}{4}$ per cent on $\frac{7}{8}$ of its estimated value, and the premium on the cargo being $4\frac{7}{10}$ per cent on $\frac{3}{4}$ of its estimated value, and two policies being required, each costing $1? Ans $2858,25.

REMARK.—In *mutual insurance companies* each one gives a premium note of so much per cent on the property which he wishes to insure, the *rate* being determined by the amount of *Risk.* A certain per cent is paid down on these notes for immediate use, and any losses that occur more than this per cent are

---

What are the insurers often called? What is the Premium? When paid? What is the Policy? The Risk? To what is the Risk equal? What is the rule when the premium is a certain per cent of the value of the property insured?

averaged on the premium notes, and thus each one helps to supply the deficiency in proportion to the amount of property he has insured. The amount of the premium notes forms the capital of the company.

5. How much must be paid for insuring a store valued \$15420, the premium note being 8 per cent, and the assessments 2, $2\frac{1}{2}$, 4, $3\frac{1}{2}$, and $1\frac{1}{2}$ per cent?

Operation.

$$\begin{array}{rl}
\$15420 & \\
.08 & \\
\hline
\$1233.60 & \text{premium note.} \\
.13\frac{1}{2} & \text{sum of assessments.} \\
\hline
61680 & \\
370080 & \\
123360 & \\
\hline
\text{Ans. } \$166.5360 &
\end{array}$$

By taking 8 per cent of \$15420, we find our premium note to be \$1233.60. The sum of the several assessments is $13\frac{1}{2}$. Then $13\frac{1}{2}$ per cent of \$1233.60 gives \$166.536 as our answer.

6. How much must be paid for insuring \$8540 worth of property, at 7 per cent, the assessments being $\frac{1}{2}$, 1, $1\frac{1}{4}$, $\frac{3}{4}$, 2, and $\frac{7}{8}$ per cent? Ans. \$38.109.

7. How much must be paid for insuring \$12500 worth of property, at $12\frac{1}{2}$ per cent, the assessments being 2, $2\frac{1}{2}$, $3\frac{1}{8}$, 4, $\frac{5}{6}$, $3\frac{3}{4}$, and $\frac{5}{8}$ per cent? Ans. \$263.02.

8. A gentleman paid \$50 annually for insurance on his property, at 4 per cent. How much was the value of the property covered by the policy?

Solution.—As the value of the property insured multiplied by the rate per cent of insurance, gives the premium, it is plain that t e premium divided by the per cent will give the value of the property insured. Therefore, $\$50 \div .04 = \$1250$.

Ans. \$1250.

Proof.—$\$1250 \times .04 = \$50$ the premium. Hence—

When we have given the premium and the rate per cent of insurance, to find the value of the property insured:

Obs. 5. *Divide the premium by the rate per cent of insurance*

What is the method pursued by mutual insurance companies? How do we find the value of the property insured, when we have given the premium and he rate per cent of insurance?

expressed decimally, and point off in the quotient as in Division of Decimals. (Sect. VIII. Art. 12. Rule.)

9. If I pay \$270 premium, at 3 per cent, what is the value of the property insured? Ans. \$9000.

10. A merchant paid \$1000 premium on his property, at 5 per cent. What was the value of his property? Ans. \$20000.

11. A gentleman paid \$1500 premium, at 3 per cent, on a ship and cargo. How much were they worth? Ans. \$50000.

12. A man paid \$50 premium on his property, valued at \$1250. What was the rate per cent of insurance? Ans. 4.

Solution.—As the value of the property, multiplied by the rate per cent of insurance, gives the premium, it is evident that the premium divided by the value of the property will give the per cent.

Therefore, $50 \div 1250 = \frac{50}{1250} = .04$. Hence—

When the premium and value of the property are given, to find the per cent of insurance:

Obs. 6. *Make the premium the numerator, and the value of the property the denominator of a common fraction; then reduce it to a decimal.* (Sect. VIII. Art. 9. Obs. 4.)

NOTE.—The pupil will perceive that these three cases of insurance reciprocally prove each other.

13. A gentleman paid \$400 premium on his property, which was valued at \$16000. What was the per cent of insurance? Ans. $2\frac{1}{2}$.

14. A ship captain paid \$1250 premium on his vessel, valued at \$25000. What was the per cent of insurance? Ans. 5.

15. A merchant paid \$1800 premium on his store and goods, valued at \$60000. What was the per cent of insurance? Ans. 3.

Case 2.—*When a person wishes to insure such a sum that in case of a loss, he may receive both the value of his property, and the price for insuring:*

Ex. 1. A gentleman wishes to insure his property, worth \$1140, at 5 per cent, so that if it is destroyed, his policy will cover both the premium and value of the property insured. What sum must his policy cover?

Solution.—The rate of insurance is 5 per cent; therefore on \$100

How do we find the rate per cent of insurance, when we have given the premium and the value of the property insured? What relation have these three cases of insurance to each other?

he receives but $95, as he pays $5 for insuring. Therefore, he receives $\frac{95}{100}$ of the property he insures. Then $1140 is $\frac{95}{100}$ of the sum necessary to be insured, and $1140 is $\frac{95}{100}$ of $1200.

Ans. $1200.

PROOF.—$1200 $\times$ .05 = $60, the premium he would pay. $1200 — $60 = $1140, the value of his property. Hence—

To solve all such questions:

Obs. 7. *Multiply the sum necessary to be insured by* 100, *and divide the product by* 100 *diminished by the rate per cent of insuring.*

2. What sum must I have insured, at 8 per cent, to cover $2760? Ans. $3000.

3. A merchant sent a cargo to Liverpool, valued at $12500. What sum must he have insured at 8 per cent, th t in case of a wreck he may sustain no loss by the operation?

Ans. 13586.95\frac{15}{23}$.

4. What sum must be insured at 10 per cent, in order to cover both the premium, and $10800 worth of property?

Ans. $12000.

ARTICLE 3. PROFIT AND LOSS.

Obs. 1. PROFIT *and* LOSS *in trade signify the sum gained or lost in common business transactions.* They are calculated at a certain per cent on the *purchase price*, or *sum paid* for the articles under consideration.

MENTAL EXERCISES.

1. A man bought a sack of wheat for $5, and sold it at a profit of 10 per cent. For how much did he sell it?

Solution.—As he sold it at 10 per cent profit, he sold it for $5, together with 10 per cent of $5. Now 10 per cent of $5 is $0.50, (Art. 1. Obs. 4.) and $5 + $0.50 = $5.50. Ans. $5.50.

2. Suppose in the last case he had sold the wheat at 10 per cent loss. How much would he have received for it?

Solution.—10 per cent of $5 is $0.50. Then if he sold it at 10 per cent loss he sold for 50 cents less than it cost him, and $5 — $0.50 = $4.50. Ans. $4.50.

3. Bought a stack of hay for $12, and wish to sell it at 8 per cent profit. For how much must I sell it?

What is the rule, when a person wishes to insure such a sum that in case of a loss he may receive both the value of his property, and the price paid for insuring? What do Profit and Loss signify? How calculated?

4. C. bought a piece of cloth for \$10; but it being damaged, he is willing to sell it at 4 per cent loss. For how much must he sell it?

5. A farmer bought some hogs for \$11, and wishes to sell them at 12 per cent profit. For how much must he sell them?

6. A man bought a coat for \$8, and was offered $12\frac{1}{2}$ per cent for his bargain. How much was he offered for it?

7. A merchant bought a barrel of flour for \$5, and sold it at 5 per cent loss. For how much did he sell it?

8. A man bought some tea for \$6, and sold it at 20 per cent profit. For how much did he sell it?

9. A man bought some paper for \$12, and sold it at $8\frac{1}{3}$ per cent profit. For how much did he sell it?

10. A boy bought a cap for \$3, and sold it at $3\frac{1}{3}$ per cent loss. For how much did he sell it?

11. A man bought some land for \$50, and sold it at 6 per cent loss. For how much did he sell it?

12. A drover having 400 sheep, lost 9 per cent of them by disease. How many had he left?

EXERCISES FOR THE SLATE.

Case 1.—*To find at what price to sell an article in order to gain or lose a certain per cent, the purchase price being given.*

1. A man bought a wagon for \$60, and sold it at 5 per cent profit. For how much did he sell it, and how much did he gain?

Operation.

```
 $ 60
   .05
 -----
 $ 3.00 gained.
  60.00
 -----
Ans. $63.00 selling price.
```

By multiplying the purchase price by the per cent, we find his profit to be \$3. Then if he gained \$3, he must have sold it for \$60 + \$3 = \$63.

2. A man bought a farm for \$1200, and sold it at 8 per cent loss. How much did he lose, and for what price did he sell it?

Operation.

```
     $1200          $1200 purchase price.
       .08             96 lost.
     -----          -----
Ans. $96.00         $1104 selling price.
```

8 per cent of \$1200 is \$96. Then as he sold it at 8 per cent loss, he evidently sold it for \$96 less than he gave for it; and \$1200 — \$96 = \$1104. Hence—

To find at what price to sell an article in order to gain or lose a certain per cent, the purchase price being given:

Obs. 2. *First, multiply the purchase price by the per cent; this will give the gain or loss. Then add the gain to, or subtract the loss from, the purchase price, as the case may require; the result in either case will be the selling price.*

REMARK.—As 100 per cent is 1, or unity, it is evident that any per cent profit is equal to 100 increased by this per cent, and any per cent loss is equal to 100 diminished by this per cent. Hence, the selling price may be found thus:

Obs. 3. *Add the per cent gained to, or subtract the per cent lost from* 100, *and multiply the purchase price by the sum or remainder, as the case may be, and point off two decimals; the result in either case will be the selling price.*

NOTE.—The learner will bear in mind that it is only the *selling price* that we obtain by this method, and not the sum gained or lost.

The operations of Ex. 1 and 2, by this method, may be seen, as follows:

Ex. 1.	Ex. 2.
\$60	\$1200
100+5=105	100—8= 92
300	2400
60	10800
\$63.00 selling price.	\$1104.00 selling price.

3. A merchant bought tea for \$0.50 per pound, and sold it at 20 per cent profit. For how much did he sell it per pound?

Ans. \$0.60.

4. A merchant bought a lot of goods for \$450; but getting them damaged, he sold them at 10 per cent loss. How much did he lose, and for how much did he sell them?

Ans. He lost \$45, and sold them for \$405.

4. A drover bought a lot of sheep for \$500, and sold them at 25 per cent profit. How much did he gain, and for how much did he sell them? Ans. He gained \$125, and sold them for \$625.

6. Bought a span of horses for \$250, and sold them at 15 per cent profit. Required—the profit and the selling price.

Ans. Profit, \$37.50; selling price, \$287.50.

How do we find at what price to sell an article in order to gain or lose a certain per cent, the purchase price being given? By what other rule can the selling price be found? Demonstrate this rule.

7. A man having a flock of 200 sheep, lost 40 per cent of them by disease. How many did he lose, and how many had he left?

Ans. He lost 80, and had left 120.

8. A merchant bought cloth for \$6 per yard, but getting it damaged he is willing to sell it at 10 per cent loss. For how much must he sell it per yard? Ans. \$5.40.

9. Suppose he wished to gain 5 per cent on the above cloth. For how much must he sell it per yard? Ans. \$6.30.

10. If he would gain 15 per cent, at what price per yard must he sell it? Ans. \$6.90.

11. A merchant bought 500 bales of cotton, each bale containing 225 lbs., at $9\frac{3}{4}$ cents per pound; but the price declining, he sold it at $4\frac{1}{3}$ per cent loss. How much did he lose, and at what price did he sell it? Ans. Lost \$475.31$\frac{1}{4}$; sold it for \$10493.43.

13. A dealer in grain bought 48 loads of wheat, each containing 44 bushels, at \$1.12$\frac{1}{2}$ per bushel; 30 loads of rye, each containing 52 bushels, at \$0.62$\frac{1}{2}$ per bushel; 115 sacks of oats, each containing 15 bushels. at \$0.20 per bushel; and 230 loads of corn, each containing 35 bushels, at \$0.28 per bushel: he sold the entire quantity so as to gain $16\frac{2}{3}$ per cent on the cost. Required—his gain, and the price at which he sold.

Ans. Gain, \$991.66$\frac{2}{3}$; selling price, \$6941.66$\frac{2}{3}$.

Case 2.—*Two numbers being given, one of which is regarded as a certain per cent of the other, to find the rate per cent.*

1. What per cent of \$20 is \$5?

Solution.—From the nature of the question, \$5 is some per cent of \$20; that is, \$20 multiplied by some number, will produce \$5. Therefore, we have \$5 as the product, and \$20 as one factor, and the other factor must be $\$5 \div \$20 = \frac{5}{20} = .25$. (Sect. VI. Art. 1. Obs. 16.)

Proof.—$\$20 \times .25 = \$5$. Hence—

To find the per cent in such cases:

Obs. 4. *Make the number which is regarded as the percentage the numerator, and the number of which it is so much per cent the denominator of a common fraction, and reduce this to a decimal.* (Sect. VIII. Art. 9. Obs. 4.)

2. What per cent of \$40 is \$11? Ans. $.275 = 27\frac{1}{2}$.
3. What per cent of \$90 is \$30? Ans. $33\frac{1}{3}$.
4. What per cent of \$80 is \$40? Ans. 50.

When two numbers are given, one of which is regarded as a certain per cent of the other, how do we find the per cent?

5. What per cent of 120 is 6? Ans. 5.
6. What per cent of 1 is 2? Ans. 200.
7. What per cent of 60 is 75? Ans. 125.
8. What per cent of \$5 is 5 cents? Ans. 1.
9. What per cent of \$80 is \$120? Ans. 150.
10. What per cent of \$40 is 5 cents? Ans. $.00125 = \frac{1}{8}$.
11. What per cent of \$90 is 6 cents? Ans. $\frac{1}{15}$.
12. What per cent of \$450 is \$0.90? Ans. $\frac{1}{5}$.

The principles of this case are applied to the transactions of business as follows:

13. A merchant bought some cloth at \$5 per yard, and sold it for \$6.25 per yard. What per cent did he gain?

As he bought for \$5, and sold for \$6.25, he evidently gained \$6.25 — \$5 = \$1.25 on each yard. The question then is, *What per cent of* \$5 *is* \$1.25? By Obs. 4 we find this to be 25.

Operation.

\$6.25 selling price.
5.00 purchase price.

\$1.25 gain per yard.
$\frac{125}{500} = 25$ per cent.

PROOF.—$\$5 \times .25 = \$1.25$; $\$5 + \$1.25 = \$6.25$. Hence—

When we have given both the purchase and the selling price, to find the gain or loss per cent on the cost or purchase price:

OBS. 5. *Find the gain or loss on the article by subtracting one price from the other; make this the numerator, and the cost or purchase price the denominator of a common fraction; then reduce this to a decimal.* (Sect. VIII. Art. 9. Obs. 4.)

REMARK.—The learner will perceive that the percentage is calculated on the *cost*, or *sum paid*, for the article, and not on the *selling price*, as is often supposed.

14. Bought a wagon for \$60, and sold it for \$70. Required—the per cent gained. Ans. $16\frac{2}{3}$.
15. A man bought a horse for \$60, and sold him for \$75. Required—the gain per cent. Ans. 25.
16. A tailor bought some cloth for \$120, and sold it for \$135. Required—the gain per cent. Ans. $12\frac{1}{2}$.
17. A man bought a carriage for \$200, and sold it for \$180. Required—the loss per cent. Ans. 10.

When we have given both the purchase and selling price, how do we find the gain or loss per cent? Is the percentage calculated on the buying or selling price?

18. A grocer bought butter for 18 cts. per pound, and sold it for 24 cts. per pound. Required—the per cent profit. Ans. $33\frac{1}{3}$.

19. A gentleman bought a house and lot for \$2500, and sold them for \$2000. What per cent was his loss? Ans. 20.

20. A merchant bought 60 yards of cloth at \$6 per yard, and sold the entire quantity for \$400. What per cent was his profit? Ans. $11\frac{1}{9}$.

21. A speculator laid out \$2000 for land, and afterwards sold his land for \$2800. What per cent did he gain? Ans. 40.

22. Bought cloth for 24 cents per yard, and sold it for 21 cents per yard. What per cent was lost? Ans. $12\frac{1}{2}$.

23. A merchant bought cloth at \$0.50 per yard, and sold it for \$0.60 per yard? Required—the per cent gained. Ans. 20.

24. A man bought a pair of fine boots for \$5, and sold them for \$5.25. What per cent was gained? Ans. 5.

25. A grocer bought 6 sacks of coffee, each containing 200 lbs., at $9\frac{1}{2}$ cts. per lb.; and 3 cwt. of sugar, at 8 cts. per lb.: he sold the whole for \$161. What per cent did he gain? Ans. $16\frac{2}{3}$.

Case 3.—*The price at which an article is sold, and the rate per cent of gain or loss being given, to find the original cost.*

1. A man sold a horse for \$90, and by so doing gained 20 per cent. How much did the horse cost him? Ans. \$75.

Solution.—From the nature of the question, the selling price is 120 per cent of the purchase price. 120 per cent is $\frac{120}{100}$. Then \$90 is $\frac{120}{100}$ of what number?

Proof.—$\$75 \times .20 = \$15$; $\$75 + \$15 = \$90$, the selling price.

2. A man sold cloth for \$4 per yard, and by so doing lost 20 per cent. Required—the original cost. Ans. \$5.

Solution.—The selling price in this example is evidently equal to 80 per cent of the original cost. 80 per cent is $\frac{80}{100}$. Then \$4 is $\frac{80}{100}$ of what number?

Proof.—$\$5 \times .20 = \$1$; $\$5 - \$1 = \$4$, the selling price.

Hence—To find the original cost, when the selling price and the rate per cent of gain or loss are given:

Obs. 6. *Add the gain to, or subtract the loss from* 100; *then multiply the selling price by* 100, *and divide the product by the sum or remainder, as the case may be.*

When the selling price, and the per cent of gain or loss are given, how do we find the original cost?

REMARK.—Pupils sometimes think that if we find the percentage on the *selling price* at the given rate, and add it to, or subtract it from, the selling price, the sum or remainder, as the case may be, will be the original cost. This error may be avoided by recollecting that *the cost or purchase price is always made the basis upon which the gain or loss is calculated.* (Obs. 1.)

3. By selling cloth at \$5 per yard, 25 per cent was gained. Required—the original cost. Ans. \$4.

4. By selling sugar at 11 cents per pound, 10 per cent was gained. How much did it cost? Ans. 10 cts. per lb.

5. A grocer, by selling tea at \$1 per lb., gained $11\frac{1}{9}$ per cent. How much did it cost him? Ans. \$0 . 90 per lb.

6. By selling cloth at \$1 . 50 per yard, 25 per cent was lost. Required—the original cost. Ans. \$2 per yd.

7. A bookseller sold a lot of books for \$400, and by so doing gained $33\frac{1}{3}$ per cent. How much did they cost him?

Ans. \$300.

8. A merchant bought a lot of damaged goods for \$1000, which was $16\frac{2}{3}$ per cent less than cost. Required—the cost.

Ans. \$1200.

9. A man bought two horses, paying the same sum for each. He afterwards sold one of them for \$180, which was 10 per cent loss, and the other at 15 per cent profit. For how much did he sell the latter horse? Ans. \$230.

10. A man sold a piece of land for \$2000, by which means he lost $16\frac{2}{3}$ per cent. He tried at first to sell it at $12\frac{1}{2}$ per cent profit. For how much did he try to sell it? Ans. \$2700.

ARTICLE 4. STOCKS, BROKAGE, DUTIES, AND TAXES.

STOCKS.

Obs. 1. *By* STOCK *is meant the capital of Trading Companies, Banks, Rail-road and Insurance Companies, Manufactories, &c.; also, the funds of governmnnt, state bonds, &c.* It is generally divided into a certain number of parts, called *Shares.* A share is generally valued at \$100, and owners of shares are called *Stock-holders.*

Obs. 2. Stock has two values: a *nominal,* and a *real value.* The *nominal value* is the *original cost,* or *price paid* for a share. The *real value* is the price at which a share can be sold, which varies at different times.

Obs. 3. When stock sells for its nominal value, it is said to be

What error do pupils sometimes fall into with respect to questions of this kind? How may this error be avoided? What is meant by Stock? How is it generally divided? What sum usually constitutes a share? What are the owners of a share called? How many values has stock? What are they? What is the nominal value? The real value?

at *par*. When it sells for more than its nominal value, it is said to be *above par*, or at an *advance;* and when it sells for less than its nominal value, it is said to be *below par*, or at a *discount*. *Par* is a Latin word, signifying *equal*.

REMARK.—Persons who deal in Stock are usually called *Stock-brokers*, or *Stock-jobbers*.

Obs. 4. The variation of the *real value* of Stock is called its *rise* or *fall*. It is reckoned at a certain per cent on its *nominal* or *par value*.

EXERCISES FOR THE SLATE.

1. What is the real value of $1000 of bank stock at 110 per cent; that is, 10 per cent above par? Ans. 1100.

Operation.—$\$1000 \times 1.10 = \$1100$. Or,

$\$1000 \times .10 = \$100$; $\$1000 + \$100 = 1100$, as before.

2. What is the value of $1500 in bank stock, at 94 per cent; that is, 6 per cent below par? Ans. $1410,

Operation.—$\$1500 \times .94 = \$1410$. Or,

$\$1500 \times .06 = \$90$; $\$1500 - 90 = \$1410$, as before.

REMARK.—From these operations we perceive that questions in Stocks are performed in the same manner as in Percentage; (Art. 1. Obs. 4.) or as questions in Profit and Loss, Case 1. (Art. 3. Obs. 2.)

3. Bought 8 shares of railroad stock at 12 per cent advance. How much did it cost? Ans. $896.

NOTE.—When the value of the share is not given in these examples, $100 is understood as its value.

4. A gentleman bought 6 shares of stock in a manufacturing establishment, at 4 per cent discount. How much did it cost?
Ans. $576.

5. A gentleman bought 12 shares of stock in a cotton factory, at 4 per cent discount, and sold it at $6\frac{1}{4}$ per cent advance. How much did he gain, the par value of each share being $75?
Ans. $92.25.

6. A stock-broker bought 20 shares of canal stock, at $2\frac{1}{2}$ per cent discount, and sold it at $3\frac{1}{2}$ per cent advance. Required—his gain, the par value of a share being $100? Ans. 120.

When is stock said to be at par? When above par? When below par? What does par mean? What are persons who deal in stock usually called? What is the variation of the real value of stock called? How is it reckoned? How are questions in stock performed?

7. Bought 6 shares of railroad stock at 94 per cent, and sold it at 112 per cent. Required—the gain? Ans. $108.

8. Bought 9 shares of railroad stock at 106 per cent, and sold it at 96 per cent. How much was lost by the transaction?
Ans. $90.

BROKAGE.

Obs. 5. BROKAGE *is the operation of finding the percentage on Bills of Exchange, Bank notes, &c.* Persons who buy and sell bank notes are called *Brokers*.

Obs. 6. Bank notes have two values: a *nominal*, or *par value*, and a *real value*. The *nominal*, or *par value*, is the sum named on the face of the note. The *real value* is the price at which the note will sell.

The remarks of Obs. 3 apply to bank notes as well as to Stock. The rule is the same as in stocks.

1. What is the value of $1000 in bank notes, at $1\frac{1}{4}$ per cent advance? Ans. $1012.50.

2. What is the value of $1500 in bank notes, at 2 per cent discount? Ans. $1470.

3. A merchant negotiated a bill of exchange of $2500 with a broker, for which he was to give him $4\frac{2}{5}$ per cent. How much did the broker receive, and what sum did he return to the owner?
Ans. The broker received $110, and returned $2390.

4. What is the value of $5000 in bank notes, at $\frac{3}{8}$ per cent premium? Ans. $5018.75.

5. What is the value of $10000 in bank notes at $\frac{5}{8}$ per cent discount? Ans. $9937 . 50.

6. A gentleman in St. Louis exchanged $1500 in specie for Cincinnati bills, at 5 per cent advance; he then started for Cincinnati, and paid for his passage $15; at Cincinnati, he exchanged his bills for notes on a bank in Albany, N. Y., at $2\frac{1}{2}$ per cent discount; he then started for Albany, and paid for his expenses $45; in Albany he sold his notes for Boston bills, at $\frac{1}{2}$ per cent discount: his expenses from Albany to Boston were $10. How much had he when he arrived at Boston; and did he gain or lose by exchanging, and how much, supposing the specie to be every where current?

Ans. { He had when he arrived $1458 62.
Gained by exchanging, $28.62.

DUTIES.

Obs. 7. DUTIES *are sums of money required by government, to*

What is Brokage? What are persons who buy and sell bank notes called? How many values have bank notes? What is the nominal, or par value? The real value? What is the rule in Brokage?

be paid on the importation of goods. They are of two kinds: *Specific* and *Ad Valoram.* The income arising from duties, &c., is called REVENUE; and a table of the duties is called a TARIFF.

Case 1.—SPECIFIC DUTIES.

Obs. 8. *A Specific Duty* is a certain sum imposed upon a ton, pound, yard, gallon, hogshead, &c., without regard to the value of the article.

REMARK 1.—According to law, certain deductions are made, called *tare, draft, leakage,* &c., before specific duties are imposed.

2.—*Tare* is the allowance made for the weight of the *box, cask,* or whatever contains the article on which the duty is paid.

Draft, or Tret, is the allowance made on the weight of the goods for waste or refuse matter.

Leakage is an allowance for waste of liquors in *casks,* &c. It is usually about 2 per cent.

3.—For breakage, 10 per cent is allowed on all *ale, beer,* and *porter,* in *bottles,* and 5 per cent on all other liquors in *bottles;* or the importer may have the duties computed on the actual quantity by tale, if he so chooses at the time of entry.

4.—At the Custom Houses, common sized bottles are estimated to contain $2\frac{2}{3}$ gallons per dozen.

5.— he *entire weight* of any lot of goods, together with the *box,* or whatever contains the goods, is called the *gross weight.* The weight of the goods alone, after all deductions for *tare, draft,* &c., is called the *neat weight.* (Sect. IX. Art. 2. Obs. 21. Rem. 2.)

1. What is the specific duty on 30 hhds. of sugar, at $2\frac{1}{2}$ cents per pound; the gross weight being 9 cwt. 2 qrs. each; the draft 4 lbs. per hhd., and the tare 12 per cent?

Operation.

9 cwt. 2 qrs. = 950 lbs. in each hhd.
30
28500 lbs. in 30 hhds.
30 × 4 = 120 lbs. draft.
28380
28380 × .12 = 34[illegible]6 lbs. tare.
2)24974 lbs. neat weight.
$.02\frac{1}{2}$ duty per lb.
49948
12487
Ans. $624.35

In this operation we notice two considerations:

1st. *The tare is oomputed on the remainder, after the draft is deducted.*

2nd. *The duty is computed on the neat weight, or the remainder after all deductions are made.*

Note.—In casting tare, any remainder which does not exceed half a pound, is not reckoned; but if it exceeds half a pound, it is reckoned a pound. By remainder, in this case, is meant the decimals. Hence—

To find the specific duty on any article of merchandise:

Obs. 9. *First deduct the draft, tare, leakage, &c., from the given article, and then multiply the remainder by the duty per pound, gallon, yard, &c.*

2. What is the specific duty on 10 hogsheads of wine, at 12 cents per gallon, deducting 2 per cent for leakage? Ans. \$74.088.

3. What is the duty on 5 gross of porter, at 15 cents per gallon, 10 per cent being allowed for breakage? Ans. \$300.

4. What is the duty on 30 casks of nails; weight, 140 lbs. per cask, at $4\frac{1}{2}$ cts. per pound, allowing 1 lb. per cask for draft, and 3 per cent for tare? Ans. \$182.02.

5. What is the specific duty on 6 hhds. of sugar, at 3 cents per pound, gross weight as follows: the first, 6 cwt. 3 qrs.; 2nd, 7 cwt. 1 qr.; 3d, 4 cwt.; 4th, 5 cwt. 3 qrs.; 5th, 6 cwt. 1 qr. 20 lbs; 6th, 8 cwt. 15 lbs.: allowance for draft, 4 lbs. per hhd., and for tare 12 per cent? Ans. \$100.62.

Case 2.—Ad Valoram Duties.

Obs. 10. Ad Valoram Duties *consist of a certain per cent of the actual cost of the goods in the country from which they were* brought.

Ad Valoram is a Latin term, signifying *according to value.*

1. What is the ad valoram duty on a quantity of broadcloth, invoiced at \$2000 in London, at 12 per cent? Ans. \$240.

Suggestion.—When the duty is imposed on the actual cost of the goods, of course no deductions are to be made. Then 12 per cent

---

What are Duties? How many kinds of duty are there? What is Revenue? What is Tariff? What are Specific Duties? What deductions are made before Specific duties are imposed? What is Tare? Draft? Leakage? What per cent is allowed on liquors in bottles, for breakage? How much are common sized bottles estimated to contain? What is gross weight? Net weight? Upon what is tare computed? The duty? How do we find the specific duty on an article of merchandise? What are Ad Valoram duties? What does the term ad valoram mean? Are any deductions to be made on ad valoram duties? Why not?

of \$2000, is \$2000 × . 12 = \$240. It will be perceived that the operation is the same as in percentage. (Art. 1. Obs. 4.)

REMARK.—An *Invoice* is a written statement of the goods in question, with the prices of the articles annexed.

2. What is the duty on a quantity of books, invoiced at \$800, at 10 per cent, ad valoram? Ans. \$80.
3. What is the duty on a quantity of wine, invoiced at \$1000, at $16\frac{1}{4}$ per cent, ad valoram? Ans. \$162.50.
4. What is the duty on a quantity of jewelry, invoiced at \$500, at 18 per cent, ad valoram? Ans. \$90.
5. A merchant bought a quantity of silks in Canton for 1200 tales; upon importing them the duty was 15 per cent ad valoram. Required—the duty, the tale being estimated at \$1 . 48. Ans. \$266 . 40.

TAXES.

Obs. 11. TAXES *are sums paid by the people for the support or benefit of the Government, County, District, &c.*

Obs. 12. Taxes are levied either on the person or property of the citizens. When the tax is levied on the person, it is called a *poll tax,* and is usually a specified sum.†

When the tax is levied on the property, it consists of a certain per cent of the value of the entire property possessed by each taxable person.

REMARK.—Property is of two kinds: *Real Estate* and *Personal Property* *Real Estate* denotes property that is fixed: as, lands, houses, &c. *Personal Property* denotes property that is moveable: as, horses, cattle, carriages, notes, tools, &c.

Obs. 13. The first thing necessary to be obtained in levying a tax is an *exact inventory,* or written account, of all the taxable property, both *real* and *personal,* in the State, County, or District &c., where the tax is to be levied, together with the value of the entire property owned by each individual who is to be taxed, and the number of polls.

An *Inventory* is a list of articles.

1. A certain district was taxed \$1850. The district contained 50 polls, which are assessed at \$1 each; and the whole amount of tax-

How do we find the ad valoram duty on an article? What is an invoice? What are taxes? How levied? What is a poll tax? When a tax is levied on property, of what does it consist? Of how many kinds is property? What is Real Estate? Personal Properiy? What is the first thi ng necessary to be obtained in levying a tax?

†In some States there is no poll tax.

able property, both personal and real, in the district, was valued at \$120000. Required—the tax on a dollar, and also the tax of an individual whose real estate is worth \$2500, and whose personal property is worth \$800, and who pays for 1 poll?

Ans. Tax on \$1 is $1\frac{1}{2}$ cts.; on the individual \$50.50.

Operation.

\$1850
  50 poll tax.
―――
\$1800 tax on property.

$120000)1800.00(.01\frac{60000}{120000}=1\frac{1}{2}$ cts. on [the doll.
1200 00
―――
600000

\$2500 real estate.
  800 personal property.
―――
\$3300 taxable property.

\$3300
$.01\frac{1}{2}$
―――
\$3300
1650
―――
\$49.50 + \$1 = \$50.50.

As the poll tax is a specified sum on each individual, it is evident that this must be deducted from the tax to be raised before calculating the tax on the property. By this means we find the tax on the property to be \$1800. Then if \$120000 pays \$1800 tax, \$1 will pay \$1800 ÷ \$120000 = $1\frac{1}{2}$ cts. Also, if a man's real estate is worth \$2500, and his personal property worth \$800, his entire taxable property is worth \$2500 + \$800 = \$3300; and \$3300 × .$1\frac{1}{2}$ = \$49.50, and \$1 for his poll tax makes his entire tax to be \$50.50. Hence—

To assess a State, County, or other tax, we have this

RULE.

I. *Find the tax on all the polls, (if any) at the given rate, and subtract this from the whole tax to be raised. Divide the remainder by the whole amount of taxable property in the State, County, District, &c., and the quotient will be the tax on one dollar.*

II. *Multiply the value of each man's property by the tax on \$1, and the product will be the tax on his property, to which add his poll tax, and the sum will be his total tax.*

Proof.—*Add together the taxes of all the individuals taxed; if the sum is equal to the whole tax assessed, the work is correct.*

2. A town was taxed \$500. It contained 40 polls, valued at \$0.75 each, and the value of the taxable property was \$23500.

What must be done before the tax is levied on the property? Why so? What is the rule for assessing taxes? The proof?

What per cent was the tax, and what is the tax of an individual who pays for 3 polls, and whose property is worth $1575?

Ans. { Tax on $1 is 2 cents. Individual's tax, $33.75.

3. A company was taxed $3135. The value of the taxable property was $125000, and there were 20 polls valued at $0 . 50 each. Required—the tax on $1. Ans. $2\frac{1}{2}$ cts.

Remark.—Assessors generally make a table containing the tax on $1, $2, &c., up to $1000 ; and thus, by knowing the value of any one's property, his tax can easily be found. Thus, in the last example, the tax on $1 being $2\frac{1}{2}$ cts., on $2 it will be twice as much, or 5 cts., &c.

Hence—the following

TABLE.

$1	pays	$0.02½.	$20	pays	$0.50.	$200	pays	$5.00.
2	"	.05.	30	"	.75.	300	"	7.50.
3	"	.07½.	40	"	1.00.	400	"	10.00.
4	"	.10.	50	"	1.25.	500	"	12.50.
5	"	.12½.	60	"	1.50.	600	"	15.00.
6	"	.15.	70	"	1.75.	700	"	17.50.
7	"	.17½.	80	"	2.00.	800	"	20.00.
8	"	.20.	90	"	2.25.	900	"	22.50.
9	"	.22½.	100	"	2.50.	1000	"	25.00.
10	"	.25.						

4. Find, by the above table, the tax of an individual whose property is valued at $1426, and who pays for 2 polls, at $0.50 each. Ans. $36.65.

Operation.

The tax on	$1000	is	$25.00
The "	400	is	10.00
The "	20	is	.50
The "	6	is	.15
And the tax on	$1426	is	$35.65
Two polls at $0.50 each =			1.00
		Total tax,	$36.65

$1426 = 1000 + 400 + 20 + 6$. Therefore, by finding the tax on each of these in the table; and adding them together we find the tax on the whole. (Sect. 4. Art. 4. Obs. 4. Rem. 2.)

5. What would be A's tax in the above company, who pays for 1 poll, and whose property is worth $1594?

Ans. $40.35.

How do assessors proceed in levying a tax? How do we find a tax by the Table?

6. C pays for 3 polls, and his property is worth $2875. Required his tax? Ans. $73.37½.

7. D pays for 1 poll, and his property is worth $882. Required his tax? Ans. $22.55.

8. E pays for 2 polls, and for property worth $27740. Required his tax? Ans. $694.50.

9. F pays for 4 polls, and for property worth $48450. Required his tax? Ans. $1213.25.

10. G pays for 3 polls, and for property worth $30012. Required his tax? Ans. $751.80.

11. H. pays for 4 polls, and for property valued at $12021. Required his tax? Ans. $302.525.

12. Prove by the 8 preceding examples, that the result in example 3 is correct.

Article 5. Interest.

Obs. 1. Interest *is the sum paid for the use of money.*

It is computed by *per centage:* that is, so many dollars are paid for the use of $100 for 1 year; and in the same proportion, for a sum greater or less than $100, or for a longer or shor er time than a year.

Obs. 2. The sum on which the interest is paid is called the Principal.

The interest is generally *a certain per cent. of the principal.* The per cent. on $100 for 1 year is called the Rate.

The sum of the principal and interest is called the Amount.

Illustration.—If I borrow $100 for 1 year, and agree to pay 6 per cent. for the use of it, at the end of the year, I must pay back, not only the $100 which I borrowed, but 6 per cent. of it, or $6 also, making in all $106. In this case $100 is the principal; $6 the interest, 6 per cent. the rate; and $106 the amount.

Obs. 3. *Per annum* signifies by the year. Thus, 6 per cent. per annum, signifies that $6 is paid for the use of $100 for 1 year.

Remark.—When the time is not mentioned with the rate per cent., a year is always understood.

Obs. 4. *Legal interest* is the rate per cent. established by law.

What is interest? How is it computed? What is the principal? The interest? The rate? The amount? In the illustration given, which is the principal? The interest? The rate? The amount? What does per annum signify? Give an example. When the time is not mentioned, with the rate per cent., what is understood? What is legal interest?

It varies in different countries, and also in the different States. Thus, it is

6 *per cent.* in all the New England States, in New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, Arkansas, District of Columbia, and on debts or judgments in favor of the United States.

7 *per cent.* in New York, South Carolina, Michigan, Wisconsin, and Iowa.

8 *per cent.* in Georgia, Alabama, Mississippi, Texas, and Florida.

5 *per cent.* in Louisiana.

REMARK. 1. When the rate per cent. is not mentioned in transacting business, the *legal rate* is always understood.

2. Any rate per cent. higher than the legal rate is called USURY, and the person who exacts it is liable to punishment. If the parties agree, however, any rate *less* than the legal rate may be taken.

Obs. 5. The learner will observe this difference between per centage and interest: *in interest, time is considered; in per centage it is not.*

Interest is of two kinds: SIMPLE and COMPOUND.

SIMPLE INTEREST.

Obs. 6. SIMPLE INTEREST *is the interest on the principal only.*

CASE 1. *To calculate interest for any number of years, at any rate per cent.*

Ex. 1. What is the interest of \$450 for 3 yrs. at 5 per cent? — The amount?

The interest is 5 cents for 100 cents, or \$1 for 1 year. Therefore, the interest of \$450 for 1 year is \$450 × .05 = \$22.50; and for 3 years it is 3 times as much or \$67.50. The sum of the interest (\$67.50) and the principal (\$450) is the amount. (Obs. 2.)

Operation.

```
         $450
          .05
        ------
       $22.50 int. for 1 yr.
            3
        ------
Ans.   $67.50 int. for 3 yrs.
       450.00 principal.
       ------
Ans. $517.50 amount.
```

Hence—to find the interest of any sum of money, for any number of years, at any rate per cent.

What is the legal rate in most of the United States? When the rate per cent. is not mentioned in transacting business, what is understood? What is usury? What is the difference between per centage and interest? How is interest divided? What is simple interest? How do we find the interest of any sum, for any number of years, at any rate per cent?

Obs. 7. *Multiply the principal by the rate per cent. expressed decimally, this will be the interest for 1 year.*

Multiply the interest for 1 year by the number of years, if the time is longer than 1 year.

REMARK. To find the amount; *Add the principal to the interest.*

EXERCISES FOR THE SLATE

1. What is the interest of \$120.50 for two years at 6 per cent? At 8 per cent.? At 7 per cent.?

Ans. in order. \$14.46; \$19.28, \$16.87.

2. What is the interest of \$640.20 for 5 years at 4 per cent.? At 5 per cent.? At $7\frac{1}{2}$ per cent.?

Ans. in order. \$128.04; \$160.05; \$240.075.

3. What is the interest of \$300 for 7 years, at $12\frac{1}{2}$ per cent.?

Ans. \$262.50.

4. What is the interest and amount of \$800 for 12 years, at $8\frac{3}{4}$ per cent.?

Ans. Int. \$840; Amt. \$1640.

5. What is the interest and amount of \$1000 for 5 years, at $4\frac{7}{8}$ per cent.?

Ans. Int. \$243.75; Amt. \$1243.75.

6. What is the interest and amount of \$575 for 6 years, at 4 per cent.?

Ans. Int. \$138; Amt. \$713.

7. What is the interest and amount of \$860 for 10 years. at $7\frac{1}{2}$ per cent.?

Ans. Int. \$645; Amt. \$1505.

CASE 2. *To calculate interest for years and months, at 6 per cent.*

1. What is the interest of \$400 for 1 year and 6 months, at 6 per cent?

At 6 per cent., the interest of \$1 for 1 year is 6 cents. Then as there are 12 months in a year the interest for 1 month is $6 \div 12 = \frac{6}{12} = \frac{1}{2}$ a cent or 1 cent for every 2 months. Then if it is 1 cent for every 2 months, for 8 months it will be 4 cents; for 10 months, 5 cents; for 6 months, 3 cents, &c.; that is, *the interest of 1 for any number of months, is half as many cents as there are months in the given time.*

Operation.

$$\begin{array}{r} \$400 \\ .09 \\ \hline \text{Ans. } \$36.00 \end{array}$$

Therefore, in the above example, 1 yr. 6 mo. = 18 mo, and the interest of \$1 for 18 mo. is 9 cts.; and the interest of \$400, is $.09 \times 400 = \$36$. Hence—

How do we find the amount? To what is the interest of \$1 for any number of months equal? Show why this is correct.

To find the interest of any sum for years and months, at 6 per cent.

Obs. 8. *Call half the number of months cents; and multiply the principal by this, expressed decimally.*

2. What is the interest of $350 for 3 yrs. 8 mo., at 6 per cent.?
Ans. $77.

3 What is the interest and amount of $1000 for 5 yrs.?
Ans. Int. $300 ; Amt. $1300.

When the rate per cent. is not mentioned in these examples, 6 per cent is understood.

4. What is the interest of $1200 for 4 yrs. 7 mo.?
Ans. $330.

5. What is the interest of $750 for 15 yrs. Ans. $675.

6. What is the interest of $225.50 for 2 yrs. 4 mo.?
Ans. $31.57.

7. Required the interest of $120 for 7 mo.? Ans. $4.20.

8. Required the interest of $284.66 for 3 yrs. 9 mo.?
Ans. $64.048

9. What is the amount of $427.86 for 7 yrs. 5 mo.?
Ans. $618.257 +

10. What is the amount of $368.27 for 3 yrs. 3 mo.
Ans. $440.082.+

11. What is the amount of $218.75 for 10 yrs. 2 mo.?
Ans. $352.187.+

12. What is the amount of $768,25 for 4 yrs. 10 mo.?
Ans. $991.042.+

Case. 3. *To calculate the interest on any sum, for any number of days at 6 per cent.*

1. What is the interest of $80 for 24 days?

Operation.
$80
.004
———
Ans. $0.320

At 6 per cent the interest on $1 for 1 month is $\frac{1}{2}$ a cent; (solution of Ex. 1. Case 2.) In interest 30 days are reckoned as a month; therefore the interest of $1 for 1 day is $0.005 $\div 30 = \frac{5}{30} = \frac{1}{6}$ of a mill; or it is 1 mill for every 6 days. Then as it is 1 mill for every 6 days, for 24 days it would be $3\frac{1}{2}$ mills; for 12 days, 2 mills; for 15 days, $2\frac{1}{2}$ mills, &c.; that is, *the interest of $1 for any number of days, is one sixth as many mills as there are days given.*

How do we find the interest of any sum for years and months, at 6 per cent.? To what is the interest of $1 for any number of days equal? Show why this is correct.

Therefore, in the above example the interest of \$1 for 24 days is 4 mills, and the interest of \$80 is $.004 \times 80 = \$0.32$. Hence—

To find the interest on any sum for any number of days:

Obs. 9. *Call one sixth of the number of days mills; and multiply the principal by this, expressed decimally.*

2. What is the interest of \$50 for 28 days? Ans. \$0.233.+
3. What is the interest of \$120 for 20 days? Ans. \$0.40.
4. What is the interest of \$145 for 18 days? Ans. \$0.435.
5. What is the interes of \$180 for 17 days? Ans. \$0.51.
6. What is the interest of \$200 for 15 days? Ans. \$0.50.
7. Required the interest of \$250 for 12 days? Ans. \$0.50.
8. Required the interest of \$300 for 7 days? Ans. \$0.35.
9. Required the interest of \$900 for 5 days? Ans. \$0.75.
10. Required the interest of \$1000 for 1 day? Ans. $\$0.16\frac{2}{3}$.

CASE 4. *To find the interest of any number, for any time, at 6 per cent.*

NOTE.—This case is simply the three preceding cases united into one.

1. What is the interest of \$1200 for 2 yrs. 5 mo. 18 da. at 6 per cent.

Operation.

$$\begin{array}{r} \$1200 \\ .148 \\ \hline 9600 \\ 4800 \\ 1200 \\ \hline \end{array}$$

Ans. \$177.600

2 yrs. 5 mo. = 29 mo.; $29 \div 2 = \$0.14\frac{1}{2} = \$0.145$; $18 \div 6 = 3$ mills. $\$0.145 + \$0.003 = \$0.148$, the interest of \$1 for the given time, and the interest of \$1200 is $.148 \times 1200 = \$177.60$.

NOTE.—We generally use the interest of \$1 as the multiplier, as it is usually the smallest number and it makes no difference which factor we use as the multiplier. (Sect. IV. Art. 2, Obs. 5: Rem.)

Hence—To calculate interest at 6 per cent. we have the following

GENERAL RULE.

I. *Reduce the given number to months and days: then take half the number of months, and call it cents;* (Obs. 8.) *and if there is an odd month call it 5 mills, and count another mill for every* 6 *days;* (Obs. 9.) *this will give the interest of* \$1 *for the given time.*

How do we find the interest of any sum for any number of days? Which number do we generally use as the multiplier? Why? What is the General Rule for calculating interest at 6 per cent.?

II. *Multiply the given principal by the interest of $1, already found, and point off as in multiplication of decimals.* (Sect. VIII, Art. 11. Rule.

NOTE.—If there is no odd month, and the number of days be less than 6, a cipher must be put in the place of mills.

For the convenience of the learner we subjoin the following

TABLE

By which to find the number of days from any given date to any required date; or to find any required date from a given date.

Ind'x.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	354	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	828	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	60	88	119	149	180	210	241	272	302	333	363
30	30	**	89	120	150	181	211	242	273	303	334	364
31	31	**	90	***	151	***	212	243	***	304	***	365

This table may be applied to the solution of two problems.

Problem 1. To find the time between any two dates:

How do we find the time between two dates by the table?

Obs. 10. *Find the day of the month in the index, and opposite this in the column of the given months will be found the day of the year upon which the different dates fall. The difference of these days, less 1, will express the time between the two dates.*

Required—the time between March 12th and June 26th.

Solution.—Opposite 12, in the column for March is found 71; opposite 26, and in the column for June is found 177. 177—71 = 106; 106 less 1 = 105. Ans. 105 days.

REMARK 1. In leap years when the 29th of Feb. comes between the dates the difference between the days is the time.

2. When the time overruns the end of the year, take the earlier date from 365, and to the difference add the number found against the latter date, less 1.

Required—the time between Jan 17th, and March 8th, 1848.
Ans. 50 days.

Required—the time between Dec. 14th, and Feb. 9th.
Ans. 56 days.

Required—the time between Aug. 23d, and Oct. 17th.
Ans. 54 days.

Problem 2. To find any required date:

Obs. 11. *Find the day of the year as before; to this add the time between the given and required dates, and find the sum in the table. The column in which the sum is found will express the month, and opposite the sum in the index will be found the day of the month, less* 1.

A note was given May 7th, to run 90 days; when does it become due? Ans. Aug. 6th.

Solution.—Opposite 7, in the column for May is found 127; 127 + 90 = 217; 217 is found in the column for Aug. and opposite 5. 5 + 1 = 6.

REMARK 1. In leap years, when the 29th of Feb. comes between the dates, the result found in the table is the correct date.

2. When the time overruns the end of the year, take 365 from the sum, and the difference will be the number to find in the table, for the required date.

A note given Jan. 16th, 1848, fell due in 2 months, at what time did it fall due? Ans. March 17th.

Solution.—2 mo. = 60 d. 16 + 60 = 76; 76 is in the column for March, and opposite 17.

What must be observed in Leap Years when the 29th of February comes between the dates? When the time overruns the end of the year how do we proceed? How do we find a required date by the table? When the 29th of Feb. comes between the dates how do we proceed? When the time overruns the end of the year, how do we proceed?

What date is 54 days later than the 22nd of Dec.?
Ans. Feb. 15th.

A note was given Aug. 29th, payable in 6 months; when did it fall due? Ans. Feb. 26th.

A note was given Oct. 11th, 1847, payable in 5 months; when did it fall due? Ans. March 10th, 1848.

A note was given Jan. 12th, 1848, payable in 9 mo.; when did it fall due? Ans. Oct. 9th, 1848.

NOTE.—In these examples we have calculated 30 days to the month. When the *exact time* is required, the surplus time must be added or the result be diminished for Feb. if required. The exact answer for the last example is Oct. 13th 1848.

Find the interest and amount of the following sums at 6 per cent.

2. $140 for 1 yr. 3 mo. 12 da. Ans. Amt. $150.78.

In the answers to the following questions the amount only is given.

3. $84.23 for 2 yrs. 4 mo. 18 da. Ans. $96.274.+
4. $300 for 1 yr. 8 mo. Ans. $330.
5. $24 for from June 12th, 1847 to April 26 1848. Ans. $25.272.
6. $650 for 3 yrs. 1 mo. 6 da. Ans. $770.90.
7. $0.96 for 20 days. Ans. $0.9632.
8. $175 for 1 yr. 1 mo. 1 da. Ans. $186.404.
9. $280 for 2 yrs. 4 mo. 15 da. Ans. $319.90.
10. $315.25 for 3 yrs. 24 da. Ans. $373.256.
11. $420.50 for 4 yrs. 8 mo. 10 da. Ans. $538.94.
12. $742.80 for 5 yrs. 2 mo. 20 da. Ans. $975.544.
13. $896.96 for 6 yrs. 8 mo. Ans. $1255.744.
14. $1457.12 for 6 yrs. 6 mo. 6 da. Ans. $2026.853.+
15. $212.25 for 4 yrs. 1 mo. 2 da. Ans. $264.322.
16. $320.75 for 6 yrs. 3 mo. 9 da. Ans. $441.512.+
17. $478.60 for 1 yr. 11 mo. Ans. $533.639.
18. $317 for 7 yrs. 9 mo. 16 da. Ans. $465.249.
19. $400 for from Jan 6th to May 17th. Ans. $408.666.
20. $700 for 7 yrs. 7 mo. 7 da. Ans. $1019.316.

Required the interest and amount of the following sums at 6 per cent:

21. $1250 for 9 yrs. 2 mo. 1 day. Ans. $1937.708.

a. When the interest is required for a large number of years, it is generally more convenient, to find the interest for the years separate from that for the months and days, and then add the two re-

sults together. Thus, the interest of \$1250 for 9 yrs. is \$1250 $\times$,06 $\times$ 9 = \$675; (Obs. 7.) and for 2 mo. 1 da. the interest is \$1250 $\times$ .010$\frac{1}{6}$ = \$12:708; and \$675 + \$12,708 = \$687.708 as the interest of \$1250 for 9 yrs. 2 mo. 1 da.; and the amount is \$687.708 + \$1250 = \$1937.708.

22, \$590 for 9 yrs. 5 mo; 28 da.	Ans. \$926.002.
23. \$440 for 10 yrs. 7 mo. 29 do.	Ans. \$721.526.
24. \$111.11 for 11 yrs. 11 mo. 11 da	Ans. \$189.647.
25. \$1200 for 11 yrs. 10 mo. 23 da.	Ans. \$2056.60.
26. \$1500 for 14 yrs. 7 mo. 17 da.	Ans. \$2816.75.
27. \$319.90 for 15 yrs. 8 mo. 26 da.	Ans. \$621.992.+
28. \$975.50 for 20 yrs. 10 mo. 14 da.	Ans. \$2197.151.+
29. \$500 for 40 yrs. 6 mo. 24 da.	Ans. \$1717.
30. \$1000 for 60 yrs. 12 da.	Ans. \$4602.

CASE 5. *To find the interest of any sum, for any time, at any rate per cent.*

FIRST METHOD.

REMARK. We have now considered how interest is computed at 6 per cent., but it often happens that we wish to find the interest on sums of money, at other rates per cent. than 6.

The interest of \$1 for 1 year at 6 per cent. is 6 cents;
At 1 per cent. it is 1 cent, or $\frac{1}{6}$ of 6 per cent.;
At 7 per cent., it is 7 cents, or 7 times 1 per cent.;
At 3 per cent. it is 3 cents, or 3 times 1 per cent; &c. Hence—

To find the interest of any sum, for any time, at any rate per cent,

Obs. 10. *Find the interest at 6 per cent. as usual; divide this by 6 and the result will be the interest at 1 per cent. then multiply the interest at 1 per cent. by the given rate.**

REMARK.—If the learner chooses, he can multiply the interest at 6 per cent. by the given rate, and divide the product by 6, as it will always produce the

---

When the interest is required for a large number of years, how do we proceed? How do we compute interest at other rates per cent. than 6? Show why this rule is correct. By what other method can the interest be found? Demonstrate this rule.

---

*When the rate per cent. is 3, we may multiply the principal by $\frac{1}{4}$ the number of months; when the rate per cent. is 4, we may multiply the principal by $\frac{1}{3}$ the number of months; when the rate per cent. is 8 we may multiply by $\frac{2}{3}$ the number of months; and when the rate per cent. is 9 we may multiply by $\frac{3}{4}$ the number of months; and in each case point off two decimals. These rules will only apply however, when the time can be reduced to months.

DEMONSTRATION.—3 per cent., is 3 cents for 100 cents for 12 months; that is 3 divided by 12 = $\frac{1}{4}$ of a cent per month; 4 per cent is $\frac{1}{3}$ of a cent per month; 8 per cent. is $\frac{2}{3}$ of a cent per month; and 9 per cent. is $\frac{3}{4}$ of a cent per month. Hence, the rules are evident.

same result and by this means fractional numbers may often be avoided in the operation.

1. What is the interest of $400 for 2 yrs. 6 mo. 18 da., at 4 per cent? Ans. $40.80.

Solution.—$400 × .153 = $61.20, the interest at 6 per cent: $61.20 ÷ 6 = $10.20, the interest at 1 per cent.; and $10.20 × 4 = $40.80 the interest at 4 per cent.

Find the interest and amount of the following sums:

2. $15.30 for 9 mo., at 7 per cent. Ans. Amt. $16.103.
3. $40 for 1 yr. 6 mo., at 8 per cent. Ans. $44.80.
4. $120.60 for 18 da., at 9 per cent. Ans. $121.142.
5. $200 for 2 yrs. 4 mo. 18 da. at 3 per cent. Ans. $214.30.
6. $250.75 for 1 yr. 8 mo. 24 da., at $4\frac{1}{2}$ per cent. Ans. $270.308.
7. $300 for 3 yrs. 2 mo. 12 da., at $2\frac{3}{4}$ per cent. Ans. $326.40.
8. $325 for 2 yrs. 6 mo. 16 da., at 5 per cent. Ans. $366.347.
9. $400.50 for 4 yrs. 11 mo. 29 da., at $7\frac{1}{2}$ per cent. Ans. $550.604.
10. $525.75 for 5 yrs. 9 mo. 21 da., at 9 per cent. Ans. $800.585.
11. $600 for 6 yrs. 7 mo. 11 da., at 11 per cent. Ans. $1036.516.
12. $1000 for 8 yrs. 5 mo. 15 da., at 12 per cent. Ans. $2015.

SECOND METHOD.

Since 1 month is $\frac{1}{12}$ of a year; 2 months, $\frac{2}{12}$ or $\frac{1}{6}$ of a year, &c., it follows that the interest of any sum for 1 month, is $\frac{1}{12}$ of the interest of the same sum for 1 year; for 2 months, $\frac{2}{12}$ or $\frac{1}{6}$ of the interest for 1 year; for 3 months, $\frac{3}{12}$ or $\frac{1}{4}$ of the interest for 1 year, &c. Also, since 1 day is $\frac{1}{30}$ of a month; 2 days, $\frac{2}{30}$ or $\frac{1}{15}$ of a month; 3 days, $\frac{3}{30}$ or $\frac{1}{10}$ of a month, &c., it is evident that the interest of any sum for 1 day is $\frac{1}{30}$ of the interest of the same sum for 1 month; for 2 days, $\frac{2}{30}$ or $\frac{1}{15}$ of the interest for 1 month; for 3 days, $\frac{3}{30}$ or $\frac{1}{10}$ of the interest for 1 month; &c. Hence—

To find the interest of any sum, for any time, at any rate per cent:

a. Find the interest for the years according to Case 1, Obs. 7; then find the interest for the months and days by taking aliquot parts.

13. What is the interest of \$360 for 3 yrs. 11 mo. 18 da., at 5 per cent? Ans. \$71.40.

Operation.

		\$360		
		.05		
		\$18.00	= Int. for 1 yr.	
		3		
		\$54.00	= Int. for 3 yr.	
6 mo. $=\frac{1}{2}$ a year.	2	9.00	= Int. for 6 mo.	6 + 4 + 1 = 11 [mo.
4 mo. $=\frac{1}{3}$ of a year.	3	6.00	= Int. for 4 mo.	
1 mo. $=\frac{1}{4}$ of 4 mo.	4	1.50	= Int. for 1 mo.	
15 da. $=\frac{1}{2}$ of a mo.	2	75	= Int. for 15 da	15 + 3 = 18 d.
3 da. $=\frac{1}{5}$ of 15 da.	5	15	= Int. for 3 da.	
	Ans.	\$71.40		

Find the interest and amount of the following sums:

14. \$360.80 for 2 yrs. 6 mo. 15 da., at 6 per cent. Ans. \$415.822.

15. \$212.50 for 3 yrs. 8 mo. 10 da., at 7 per cent. Ans. \$267.454.

16. \$400 for 5 yrs. 10 mo. 12 da., at 4 per cent. Ans. \$493.865.

17. \$525 for 4 yrs. 29 da., at 8 per cent Ans. \$696.383.

18. \$612.75 for 7 yrs. 1 mo. 20 da. at 9 per cent. Ans. \$1006.441.

19. \$700 for 5 yrs. 10 mo., at 5 per cent. Ans. \$904.166.

20. \$1240.60 for 12 yrs. 11 mo. 29 da., at $7\frac{1}{2}$ per cent. Ans. \$2449.926.

Case 6. *To find the interest on Notes, Bonds, &c., on which partial payments have been made.*

Obs. 11. A Note or Bond *is an instrument of writing, in which the debtor promises to pay the creditor his due, in such a manner, and at such a time as may be agreed upon.*

Remark 1. The *debtor* is the one who borrows the money, or in any way owes the other; the *creditor* is the one who lends the money, or to whom anything is due.

What is a note, or bond? Which person is the debtor? Which the creditor?

2. It often happens that the debtor pays only a part of his note, or obligation, at a time. When a payment is thus made it is called an *endorsement*, and is written on the back of the note.

Obs. 12. When the debtor pays the creditor a part of his note, or obligation, it is evident that the *sum paid*, should be deducted from the *sum due*; the sum due being *the face of the note, together with the interest of the same until the time of payment.*

Hence, the following

1. RULE.

I. *Find the amount of the principal to the first time when a payment was made. which either alone, or together with the preceding payments,* (*if any,*) *exceeds the interest then due.*

II. *From this amount subtract the payment, or sum of the payments made within the time for which the interest was computed, and the remainder will be a new principal, with which proceed as before.*

Note.—The above rule, with some slight alterations of phraseology, is adopted by the *Supreme Court of the United States*, and also by the different States of the Union, with but few exceptions.

In the following examples, 6 per cent. is understood unless otherwise stated:

(1.)

$800 Columbus, Ohio, Jan. 1st, 1836.

For value received I promise to pay Thomas Trueman, or order, eight hundred dollars, on demand, with interest.

Charles Paywell.

On this note were the following endorsements:

Sept. 15th, 1836,	$300.
July 9th, 1837.	$20.
Dec. 12th, 1837.	$400.

What remained due Aug. 7th. 1838? Ans. $159.655.

What is an endorsement? Where written? When part of a note or obligation is paid, what fact is evident? What is the sum due? What then is the rule for computing interest on notes, bonds, &c., when partial payments have been made?

Operation.

	Principal	$800.
Time from Jan. 1st, 1836 to Sept. 15th, 1836, 8 mo. 14 da.; (Sect. IX, Art. 5, Obs. 1,) interest of $800 for this time,		33.866.
	Amount,	$833.866.
Payment, Sept. 15th, 1836, exceeds the interest,		300
Remainder for a new principal,		$533.866.
Interest from Sept. 15th, 1836 to July 9th, 1837, (9 mo. 24 da.)	$26.159.	
Payment, July 9th, 1837, less than the interest,	20.	
Interest from Sept. 15th, 1836 to Dec. 12th, 1837, (1 yr. 2 mo. 27 da.)		39.772.
	Amount,	$573.638
Payment, July 9th, 1837, $20.		
Payment, Dec. 12th, 1837, $400.		
	Sum of these payments,	420.000
Remainder for a new principal,		$153.638
Interest from Dec. 12th, 1837 to Aug. 7th, 1838, (7 mo. 25 da.)		6.017
Amount due Aug. 7th, 1838,		$159.655

(2.)

$1200 CLEVELAND, June 29th, 1840.

For value received, I promise to pay Timothy Just, or bearer, twelve hundred dollars, with interest.

JAMES GOODMAN.

Endorsed,	Dec. 18th, 1840,	$100.
	Aug. 1st, 1841,	$250.75.
	Jan. 7th, 1842,	$10.25.
	Nov. 3d, 1842,	$600.
	July 1st, 1843,	$200.

What was due Jan. 1st, 1844, Ans. $205.889

(3.)

$623.75. CINCINNATI, May 2nd, 1843.

For value received, six months after date, I promise to pay Rob-

ert Brown, or bearer, six hundred and twenty-three dollars, and seventy-five cents. JOHN SMITH.

Endorsed,	Jan. 1st, 1844,	$75.25.
	Dec. 13th, 1844,	$200.
	Aug. 7th, 1845,	$10.75.
	Jan. 1st, 1846,	$350.

And April 4th 1846, he paid the balance
How much was this balance? Ans. $70.942.

REMARK 1. When a note is given without mention of interest, as the above, it is not customary to charge interest.

2. After a note becomes due, however, if payment is delayed, it will draw legal interest, although no mention be made of interest.

II. VERMONT RULE.

"*Find the amount of the principal from the time the note was given, till the time of settlement: next, find the amount of each payment from the time the payment was made till the time of settlement; finally, add together the amounts of the several payments and subtract their sum from the amount of the principal already found; the remainder will be the sum due.*"

(4.)

$1000. PITTSBURGH Feb. 2nd, 1840.

For value received, I promise to pay Amos Bush, or bearer, one thousand dollars, on demand, with interest.

CHESTER DALE.

Endorsed,	Nov. 1st, 1840,	$300.
	July 5th, 1841,	$275.
	Feb. 19th 1842,	$325.
	Oct. 12th 1842.	$100.

What was due Jan. 1st., 1843? $93.051.

Operation.

Principal,	$1000.
Interest of principal from Feb. 2nd, 1840 to Jan. 1st, 1843;	174.833
Amount of same;	$1174.833

What is said respecting interest on notes in which no mention is made of interest? What is said respecting interest on notes which are not paid when they become due? What is the Vermont rule for computing interest on notes?

First payment, made Nov. 1st, 1840.	$300.000
Interest of same till Jan. 1st, 1843.	39.000
Second payment made July 5th, 1841.	275.000
Interest of same till Jan. 1st, 1843.	24.566
Third payment, made Feb. 19th, 1842.	325.000
Interest of same till Jan. 1st, 1843.	16.900
Fourth payment made Oct. 12, 1842.	100.000
Interest of same till Jan. 1st. 1843.	1.316

Total amount of payments, and interest; $1081·782

Amount due; $93.051

(5.)

$710.40. BURLINGTON, July 4th, 1841.

For value received, on demand I promise to pay Edwin Farr, or order, seven hundred and ten dollars, and forty cents, with interest.

GEORGE HUNTER.

Endorsed, Jan. 1st, 1842, $400.
March 3d, 1843, $250.50.
Feb. 19th, 1844, $100.
The balance, July 4th, 1844.

Required the balance? Ans. $5.241.

(6.)

$400. BUFFALO, March 1st, 1842.

For value received, on demand I promise to pay Ira Jones, or bearer, four hundred dollars, with interest at 7 per cent.

LEWIS MAKER.

Endorsed, Jan. 1st, 1844, $215.87½.
April 19th, 1845, $212.37½.
The balance, Sept. 1st, 1845.
Required the balance? Ans. $39.115.

III. CONNECTICUT RULE.

I. "Find the amount of the principal to the time of the first payment, if it be a year or more from the time the interest commenced, and from this amount subtract the payment.

II. The remainder will be a new principal; find the amount of this to the next payment, from which subtract the payment as above. So continue to do from payment to payment, until all are employed, provided a year or more intervenes between each two payments.

III. But if the time between any two payments be less than a year, find the amount of the last principal for a year, and of the payments up to the

---

What is the Connecticut rule for computing interest on notes?

same time, and subtract the latter from the former. If, however, a year overruns the time of settlement, find the amounts up to that time instead of for a year.

IV. If any remainder, after subtraction, be greater than the preceding principal, the preceding principal is still to be continued as the principal for the succeding time, instead of the remainder; and the difference is to be regarded as so much unpaid interest which is to be added to the principal at the time of the next payment."

(7.)

$1200. HARTFORD, Oct. 7th. 1836.

For value received, on demand I promise to pay Henry Inman, or bearer, twelve hundred dollars, with interest.

PETER RUSS.

Endorsed, April 3d, 1838, $600.
Jan. 1st, 1839, $300.
Sept. 2d, 1839, $250.

What was due Jan. 12th, 1840? Ans. $210.309.

Operation.

Principal,	$1200.
Interest of the same to April 3d, 1838, (17 mo. 26 da.)	$107.20.
Amount,	$1307.20.
1st payment, deduct,	600.
Remainder for a new principal,	$707.20
Interest April 3d, 1839, (12 mo.) payment being made before a year has elapsed,	42.432
Amount,	$749.632
2nd payment made Jan. 1st, 1839, $300; Interest of same to April 3d, 1839, (3 mo. 2 da.) 4.60	
Amount of payment, deduct,	304.60
Remainder for a new principal,	$445.032
Interest from April 3d, 1839 to Jan. 12th, 1840, (9 mo. 9 da.) settlement being made within the year,	20.693
Amount,	$465.725
3d payment made Sept. 2d, 1839, $250; Interest of same to Jan. 12th, 1840, (4 mo. 10 da.) 5.416	
Amount of payment, deduct,	255.416
Remainder due Jan 12th, 1840,	$210.309

(8.)

$900. NEW HAVEN, May 3d, 1842.

For value received, on demand, I promise to pay Silas Weeks, or bearer, nine hundred dollars, with interest.

JOHN LEAK.

Endorsed.	June 1st, 1843,	$300.
	Feb. 2nd, 1844,	$250.
	Jan. 1st, 1845,	$300.
	Nov. 9th, 1846,	$100.

The balance Jan. 1st. 1847.

Required the balance. Ans. $76.422.

By each of the preceding rules let the learner find the balance due on the following notes:

(9.)

$1500. SANDUSKY, June 1st, 1830.

For value received, on demand, I promise to pay Jonas Trusty, or bearer, fifteen hundred dollars, with interest.

THEODORE HAND.

Endorsed.	Jan. 1st, 1831,	$500.
	July 4th, 1832,	$700.
	Feb. 2d, 1833,	$300.

What remained due April 1st, 1833?

Ans.	By the Mass. and N. Y.	Rule,	$164.878.
	By the Vermont	Rule,	$153.40.
	By the Conn.	Rule,	$165.065.

(10.)

$12500. LOUISVILLE, Sept. 1st, 1841.

For value received, I promise to pay Henry Loan, or bearer, twelve thousand, five hundred dollars, with interest.

JOSEPH FAIRMAN.

Endorsed.	Jan. 1st, 1842,	$1800.
	Oct. 9th, 1843,	$6000.
	Dec. 15th, 1843,	$ 50.
	Nov. 25th, 1844,	$3000.
	Sept. 30th, 1845,	$1500.

What remained due Jan. 1st, 1846?

Ans.	By the Mass. and N. Y.	Rule,	$2187.571.
	By the Vermont	Rule,	$1939.118.
	By the Conn.	Rule,	$2202.525.

CASE 7. *Problems in Interest.*

Obs. 13. A PROBLEM *is a question proposed requiring a solution.*

This question may apply either to the investigation of some truth, or principle, or to the operation of practical questions.

REMARK.—In the preceding examples we notice *five different terms,* or *parts* connected with Interest; viz: the *Principal,* the *Time,* the *Rate,* the *Interest,* and the *Amount.* The relation of these terms to each other, is such, that if any three are given, the other two can be found.

PROBLEM I.

The Principal, Rate per cent. and time being given, to find the Interest, and Amount.

This is deemed the most important problem. It has already been exemplified in the preceding Cases of this Article.

PROBLEM II.

The Principal, Interest, and Time being given, to find the rate per cent.

1. A man borrowed \$250 for 3 years, and paid \$45 interest.—What was the rate per cent.? Ans. 6.

Solution.—The interest of \$250 for 3 years at 1 per cent is \$7.50. (Obs. 10.) Therefore, as many times as \$7.50 is contained in \$45, so many times more than 1 per cent. was the per cent. paid; and $\$45 \div \$7.50 = 6$.

PROOF.—3 yrs. = 36 mo. $36 \div 2 = \$0.18$; $\$250 \times .18 = \$45$ interest. (General Rule, Case 4.) Hence—

To find the rate per cent. in such cases.

Obs. 14. *Divide the given interest by the interest of the given principal, for the given time, at* 1 *per cent.*

REMARK.—The *amount* can be found *by adding together the principal and interest;* (Obs. 7, Remark) also, the *principal* can always be found *by subtracting the interest from the amount* and the *interest* can always be found *by subtracting the principal from the amount.* (Sect. VI, Art. 1, Obs. 11.)

2. If I borrow \$420 for 2 yrs. 6 mo., and pay \$73.50 interest, what is the rate per cent.? Ans. 7.

What is a problem? To what may this question apply? How many terms are connected with interest? Name them. What relation have these terms to each other? Which is the most important problem? Why do you suppose it to be the most important? How do we find the rate per cent. when the principal, interest, and time are given? How can the amount be found in such cases? The principal? The interest?

3. If \$117.60 is paid for the use of \$700 3 yrs. 8 mo. 24 da., what is the rate per cent.? Ans. $4\frac{1}{2}$.

4. If a person pays \$151.20 for the use of \$810 2 yrs. 4 mo., what rate per cent does he pay? Ans. 8.

5. A man lent \$1000, and at the end of 3 yrs. 6 mo. received for his due \$1236.25. Required the rate per cent.? Ans. $6\frac{3}{4}$.

6. Lent \$750 for 6 mo., and received for principal and interest \$765. Required the per cent.? Ans. 4.

7. At what rate per cent. must \$1250 be loaned, in order to amount to \$1343.75 in 1 yr. 6 mo.? Ans. 5.

Problem III.

The Principal, Rate per cent., and Interest being given to find the Time.

1. A person borrowed \$400 for a certain time at 7 per cent. interest: at the end of the time the interest was \$70. How long did he have the money? Ans. 2 yrs. 6 mo.

Solution.—The interest of \$400 for 1 year, at 7 per cent., is \$28, (Obs. 7.) Then if the interest of \$400 for 1 year is \$28, it will take it $\$70 \div \$28 = 2\frac{1}{2}$ yrs to produce \$70 interest.

Proof.—$2\frac{1}{2}$ yrs. = 30 mo. $30 \div 2 = \$0.15$; $\$400 \times .15 = \$60$ interest at 6 per cent. (Obs. 8.) $\$60 \div 6 = \$10$ interest at 1 per cent. $\$10 \times 7 = \$70$ interest at 7 per cent. (Obs. 10.) Hence—

To find the time in such cases:

Obs. 15. *Divide the given interest by the interest of the principal for 1 year at the given rate.*

Remark.—If there is a remainder after dividing, it may either be considered as the fractional part of a year, or continued to decimals. In either case the value can easily be found in months and days. (Sect. IX, Art. 3, Case 2, Ex. 7, and Case 3. Rule.)

2. In what time will \$375 gain \$90 interest, at 6 per cent.? Ans. 4 yrs.

3. In what time will \$700 gain \$210 interest at 9 per cent? Ans. 3 yrs. 4 mo.

4. In what time will \$560 gain \$45.36 interest, at $4\frac{1}{2}$ per cent.? Ans. 1 yr. 9 mo. 18 da.

5. In what time will \$840 amount to $\$1015.08\frac{3}{4}$ at $5\frac{3}{4}$ per cent.? Ans. 3 yrs. 7 mo. 15 da.

How do we find the time, when the principal, rate per cent., and interest are given? If there is a remainder after dividing, what is done with it?

6. In what time will \$900 amount to \$1710, at 10 per cent.?

Ans. 9 yrs.

7. In what time will \$1200 amount to \$2200, at $8\frac{1}{3}$ per cent.?

Ans. 10 yrs.

8. In what time will any sum of money double itself, at 5 per cent.?

Suggestion.—Assume \$100 as the given sum. Then, in what time will \$100 gain \$100 interest, at 5 per cent.?

Ans. 20 yrs.

9. In what time will any sum of money double itself, at $8\frac{1}{3}$ per cent.?

Ans. 12 years.

10. In what time will any sum double itself, at 6 per cent.?

Ans. 16 yrs. 8 mo.

11. In what time will any sum double itself, at the following rates per cent: 1; 4; 7; 3; 9; 2; 8; 10; 12; 15; 20; 24; 25; 30; 40; and 50 per cent.? At $1\frac{1}{2}$; $2\frac{1}{2}$; $3\frac{1}{2}$; $4\frac{1}{2}$; $5\frac{1}{2}$; $7\frac{1}{2}$; $8\frac{1}{2}$; and $12\frac{1}{2}$ per cent.?

Ans. in order. 100 yrs.; 25 yrs.; $14\frac{2}{7}$ yrs. 33 yrs. 4 mo.; $11\frac{1}{9}$ yrs.; 50 yrs.; 12 yrs. 6 mo.; 10 yrs; 8 yrs. 4 mo.; 6 yrs. 8 mo.; 5 yrs.; 4 yrs. 2 mo.; 4 yrs.; 3 yrs. 4 mo.; 2 yrs. 6 mo.; 2 yrs.; 66 yrs. 8 mo.; 40 yrs.; $28\frac{4}{7}$ yrs. $22\frac{2}{9}$ yrs.; $18\frac{2}{11}$ yrs.; 13 yrs. 4 mo.; $11\frac{13}{17}$ yrs.; 8 yrs.

12. In what time will any sum treble, aud quadruple itself, at $12\frac{1}{2}$ per cent.

Ans. $\begin{cases}\text{Treble itself in 16 yrs.}\\ \text{Quadruple itself in 24 years.}\end{cases}$

Problem IV.

The Interest, Time, and Rate per cent. being given to find the Principal.

1. The interest of a certain note is \$30, it having been at interest 2 yrs. at 5 per cent. Required the face of the note, or principal?

Ans. \$300.

Solution.—The interest of \$1 for two years, at 5 per cent. is \$0.10. (Obs. 7.) Then the principal is as many times \$1, as \$0.10 is contained in \$30; and \$30 ÷ .10 = \$300. Hence—

To find the principal in such cases:

Obs. 16. *Divide the given interest by the interest of \$1 for the given time, and at the given rate per cent.*

How do we find the principal, when the interest, time and rate per cent. are given.

2. What principal at 6 per cent. will gain $27.45 in 1 yr. 6 mo.? Ans. $305.

3. What principal at $4\frac{1}{2}$ per cent. will gain $76.50 in 3 yrs. 4 mo. 24 da.? Ans. $500.

4. What principal at 7 per cent will gain $236.25 in 4 yrs. 6 mo.? Ans. $750.

5. What principal at 9 per cent will gain $237 in 2 yrs. 7 mo. 18 da.? Ans. $1000.

6. What principal at $12\frac{1}{2}$ per cent will gain $1265.625 in 6 yrs. 9 mo.? Ans. $1500.

Problem V

The Amount, Rate per cent.. and Time being given to find the Principal and interest.

The amount of a certain note is $660; it has been on interest 1 yr. 8 mo. at 6 per cent. Required the face of the note, and the interest? Ans. Face of the note $600; and the interest $60.

Solution.—The amount of $1 for 1 yr. 8 mo. at 6 per cent., is $1.10. Then the principal, or face of the note, must be as many times $1, as $1.10 is contained in $660; and $660 ÷ $1.10 = $600 the principal, and $660—$600 = $60 the interest. Hence—

To find the principal and interest in such cases:

Obs. 17. *Divide the given amount, by the amount of* $1 *for the given time, at the given rate per cent.*

To find the interest; *Subtract the principal from the amount.* (Obs. 14. Rem.)

2. What sum of money will amount to $77.76, in 1 yr. 4 mo. at 6 per cent? Ans. 72.

3. What sum of money will amount to $1190 in 5 yrs. at 8 per cent? Ans. $850.

4. The amount of a certain note is $776.32; it has been on interest 4 yrs. 8 mo. 24 da. at $4\frac{1}{2}$ per cent. Required the face of the note and the interest?

Ans. Face of the note $640. Interest $136.32.

5. The amount of a note, after having been on interest 3 yrs. 10 mo. 18 da. at 7 per cent was $1144.65. Required the face of the note, and the interest.

Ans. Face of the note $990. Interest $244.65.

How do we find the principal, when the amount, rate per cent and time are given? How do we find the interest?

6. If the amount of a note is $2139 after having been on interest 7 yrs. 9 mo. 27 da. at 10 per cent., what is the face of the note, and what the interest?

Ans. Face of the note $1200. Interest $939.

DISCOUNT.

Obs. 18. DISCOUNT *is a deduction made from a debt for paying it before it becomes due.*

Thus, if I owe a man $106, due 1 year hence, and he wishes me to pay him now, it is evident he ought to make some deduction for present payment. And I ought only to pay him such a sum as would amount to $106 in 1 year. If the discount is made at 6 per cent., for instance, I ought to pay him $100, because the interest of $100 for 1 year is $6, and the amount is $100 + $6 = $106.

Obs. 19. The sum, which in the given time, with its interest, would amount to the sum on which the discount is made, is called the PRESENT WORTH. Thus, in the example mentioned above, $106 is the sum on which the discount is made, $6 is the discount, and $ 100 the present worth.

1. What is the present worth of $440, payable in 1 yr. 8 mo., discounting at 6 per cent. What is the discount?

Ans. Present Worth $400. Discount $40.

Suggestion.—By examining this question attentively, we perceive that it does not differ from those under Problem V; the *Debt* corresponds to the *Amount;* the *present worth* to the *Principal;* and the *Discount* to the *Interest* for the given time, at the given rate per cent. Therefore, $440 ÷ $1.10 = $400 the present worth; and $440—$400 = $40 the discount.

REMARK.—It is not unfrequently supposed that if we find the interest on the given sum for the given time, that this interest will be the discount, which subtracted from the given sum, will give the present worth. But this error may be avoided if the pupil will recollect that the debt answers to the *amount* in interest, and the principal is *always* the sum on which the interest is calculated.

2. What is the present worth of $~~100~~ payable in 10 mo. at 6 per cent.? Ans. 952.38\frac{2}{21}$.

3. What is the present worth of $1488, due 4 years hence?

Ans. $1200.

In these examples, 6 per cent. is understood when no other rate is mentioned.

What is discount? What is the present worth? In the example given which is the sum on which the discount is made? Which is the discount? Which is the present worth? What error do many fall into respecting discount? How may this error be avoided?

4. What is the present worth of $1590 due 1 yr. hence?
Ans. $1500.

5. What is the present worth of $1360 due 6 yrs. hence?
Ans. $1000.

6. What is the discount of $872 due 2 yrs. hence?
Ans. $93.429.+

7. Whaat is the discount of $1736 due 3 yrs. 6 mo. hence?
Ans. $301.29.+

8. What is the discount of $2412.75 due 6 yrs. 1 mo. 24 da. hence.? Ans. $694.263 +

9. A merchant bought goods to the amount of $1968.75, payable in 10 mo. How much ready money should pay the debt?
Ans. $1875.

10. What is the present worth of $2303.40 due 2 yrs. 4 mo. hence, discount being made at 7 per cent? The discount?
Ans. Present worth $1980. Discount $323.40.

11. What is the present worth of $2156.37½ due 3 yrs. 7 mo. 18 da. hence, the money being worth 9 per cent.? The discount?
Ans. Present worth $1625. Discount $531.37.

12. A man sold a farm for $5836.80, payable in 3 yrs. 6 mo.; he could have received $4650 down for it. Did he gain or lose by selling on credit; and how much, the money being worth 8 per cent.?
Ans. He lost $90.

13. What is the difference between the interest of $2352 for 4 yrs. 8 mo. 12 da., at 10 per cent., and the discount of the same sum, for the same time, and at the same rate? Ans. $353.44.

14. A merchant bought a lot of goods for $2500 ready money, and sold them the same day for $3044.25 payable in 1 yr. 9 mo. 12 da. at 6 per cent interest. Did he gain or lose by the transaction, and how much? Ans. He gained $250.

When payments are to be made at different times, to find the present worth:

Obs. 20. *Find the present worth of each payment separately, and add together these results, for the present value of the debt.*

This rule is so evident that it needs no demonstration. Let the learner see if he cannot demonstrate it himself.

15. What is the present worth of $3000, one half payable in 2 yrs. 6 mo. 18 da., and the other half payable in 3 yrs. 4 mo. 24 da.? The discount?

Ans. Present worth, $2546.801+. Discount $453.199+.

16. A merchant bought goods to the amount of $2460; ⅓ of

When payments are made at different times, how do we find the present worth?

which was to be paid in 4 mo.; $\frac{1}{3}$ in 8 mo.; and $\frac{1}{3}$ in 12 mo. What sum should be paid at the present time to balance the debt?

Ans. \$2365.966+.

17. An agent has \$1260 to use in trade, and is to receive 5 per cent. on what he lays out. What sum does he lay out, and how much does he receive?

Ans. He lays out \$1200, and receives \$60.

Solution.—As often as he lays out \$1, he receives \$0.05. Therefore, \$1260÷ \$1.05 = \$1200 what he lays out, and \$1260—\$1200 = \$60 what he receives. Had we multiplied \$1260 by .05, (as many a c reless pupil would have done,) we should have given the agent 5 cents fcr every 95 that he laid out, instead of 100 as required by the quest on.)

PROOF.—\$1200×.05 = \$60. \$1200 + \$60 = \$1260

18. An agent has \$918 to use in trade, for which he receives 8 per cent on what he lays out. How much does he lay out for his employer, and how much does he receive?

Ans. He lays out \$850, and receives \$68.

CASE 8.—*Banking.*

Obs. 21. A BANK *is an institution that deals in money.* Its *Capital* or *Stock* is divided into parts called *Shares,* and owned by individuals called *Stockholders.* A share is usually \$100.

Obs. 22. The operations of a bank are conducted by *a President and Board of Directors.* It has a deposite of specie, and issues *notes,* or *bills,* intended to be used as a circulating medium instead of gold and silver. These bills are usually obtained from the bank in *loans.*

Obs. 23. In loaning money, the banks usually deduct the *legal interest* off the face of the note for the given time, and pay the holder the balance. The sum which the holder of the note receives, is called the *Proceeds of the note.* The sum deducted is called the *Bank Discount,* and is the same as simple interest in advance.

Obs. 24. The time from the date of a note until it becomes due, is called the *Days to run.* Besides the time mentioned in the note, *three additional days* are allowed before it is *legally* due. These are called *Days of Grace**; and as notes are not usually paid until the *last* of these dates, the banks charge interest on the days of grace.

What is a bank? How is the capital divided? What are stockholders? How much is a share? How are the operations of a bank conducted? What has it? What does it issue? For what? How are these bills usually obtained? What is usually done by the banks in loaning money? What are the proceeds of the note? The bank discount? Days to run? Days of grace?

*Some Banks allow 4 days of Grace.

Obs. 25. A note is said to be discounted when the *Bank Discount* has been deducted. The difference between the *Bank Discount* and the *true discount,* is the interest of the true discount for the given time. This gives an extra profit to the Bank. On small sums for a short time, however, the difference is trifling; but when the sum is large, and the time for which it is discounted, is long, the difference amounts to a considerable sum.

1. What is the bank discount on a note of \$200 payable in 1 yr. 6 mo. at 6 per cent.? What are the proceeds?

Operation.

The interest of \$200 for 1 yr. 6 mo. is	\$18.00
The " " " 3 days of grace is	.10
Bank discount,	\$18.10

Face of the note	\$200.00
Bank discount	18.10
Proceeds,	\$181.90

Hence—To find the bank discount of any note, draft &c.:

Obs. 26. *Add the days of grace to the time the note has to run and find the interest of the face of the note for this time.*

To find the proceeds—

Deduct the Bank discount from the face of the note.

2. What is the bank discount of a note of \$500, payable 6 mo. after date? Ans. \$15.25.

NOTE.—In these questions, 3 days of grace, and 6 per cent. are understood, unless otherwise stated.

3. What is the bank discount on a note of \$350 payable in 2 yrs. 4 mo.? Ans. \$49.175.

4. What are the proceeds of a note of \$400, payable 9 mo. after date? Ans. \$381.80.

5. What are the proceeds of a note of \$750 for 1 yr. 8 mo. 21 da.? Ans. \$672.

6. What is the bank discount on a note of \$900, payable 30 days after date, at 7 per cent? The proceeds?

Ans. { Discount \$5.775. Proceeds \$894.225.

7. What is the bank discount on a note of \$1500, payable 90

When is a note said to be discounted? What is the difference between the bank discount, and the true discount of a note? How do we find the bank discount of a note, draft, &c.? The proceeds?

days after date, at 5 per cent. and allowing 4 days of grace? The proceeds?

Ans. { Discount $19.58\frac{1}{3}$. Proceeds $1480.41\frac{2}{3}$.

8. What is the difference between the bank discount of $1200 for 3 yrs. 6 mo., and the true discount of the same sum for the same time? Ans. $44.335.

9. What is the bank discount of $1000 payable 6 months after date? Ans. $30.50.

10. What are the proceeds of $1260.50, payable 90 days after date? Ans. $1240.963.

11. A man bought 1250 barrels of flour at $4.50 a barrel, cash, and sold them the same day for $5 a barrel, payable in 9 months, without interest. He got his note discounted at the bank, when money was worth 6 per cent. Did he gain or lose by the operation, and how much?

Ans. He gained 340.62\frac{1}{2}$.

12. A man got a note of $1500 payable in 90 days, without interest, discounted at the bank at 6 per cent., and put the proceeds at interest for 1 year at 7 per cent. He renewed his note at the bank each 90 days, by paying the bank discount at each renewal. At the end of the year, he received his due on what he had lent, and paid his note at the bank. Did he gain or lose by the operation, and how much? Ans. He gained $6.71.

In this example the money is supposed to be worth 7 per cent., although he paid but 6 per cent, discount at the bank.

Case 9. *Equation of Payments.*

Obs. 27, Equation of Payments *is the method of finding the time, when two or more payments, due at different times, may be made at once, without injury to either party.* This time is usually called the *mean*, or *equated time.*

1. A. owes B. $60, to be paid as follows; $10 in 1 month; $20 in 2 months, and $30 in 3 months. He wishes to pay it all at once, in what time ought he to pay it? Ans. 2 mo. 10 da.

Solution.—The interest of $100 for 1 year is $6; for 2 years, the interest is $12, which is the same as the interest of $200 for 1 year. Also the interest of $100 for 3 yrs. is $18, which is the same as the interest of $300 for 1 year. Hence, universally—

Obs. 28. *The interest of any sum of money for any time, is equal to the interest of twice as much, for half this time; three times as much,*

What is equation of payments? What is the time usually called?

for one third of this time, &c. And conversely; *The interest of any sum of money for any time is equal to the interest of one-half as much for twice this time; one-third as much for three times this time, &c.* Therefore,

The int. of	\$10	for 1 mo.,	is the same as the int. of	\$1	for 10 mo.
The "	\$20	" 2 "	" "	\$1	for 40 mo.
The "	\$30	" 3 "	" "	\$1	for 90 mo.

And the int. of \$60 for the mean time, is the same as the int. of \$1 for 140 mo.

But \$60 is 60 times \$1, therefore, it will take it but $\frac{1}{60}$ part of this time to gain the same interest, and $\frac{1}{60}$ of 140 mo. is 2 mo. 10 da.

PROOF.—The interest of \$60 for 2 mo. 10 da. is \$60 $\times$ .011$\frac{2}{3}$ = \$0.70. (Case 4, Rule.) The interest of \$1 for 140 mo. is \$1 $\times$ .70 = \$0.70. (Case 4, Rule). Hence—

To find the mean or equated time of several payments:

Obs. 29. *Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments.*

REMARK 1. This rule is founded on the supposition that the interest of any sum of money for a certain time, is equal to the discount of the same sum for the same time. This is not the case, as *the discount is always less than the interest;* the difference being equal to the interest of the true discount for the given time. The difference in small sums, for a short time, however, is too slight to be noticed.

2. A merchant has owing to him \$1000, to be paid as follows: \$200 in 3 months; \$300 in 5 months; \$250 in 6 months, and \$250 in 8 months. It is agreed to make but one payment. In what time must this payment be made? Ans. 5 mo. 18 da.

3. A. owes B. \$120 to be paid in 5 months; \$84 to be paid in 8 months; \$132 in 9 months; and \$160 in 10 months. Required the mean time for the payment of the whole?

Ans. 8 mo. 5 da.+

4. A. owes B. \$1200 to be paid as follows: $\frac{1}{2}$ in 6 months; $\frac{1}{3}$ in 8 mo., and the balance in 10 mo. Required the equated time for the payment of the whole.

Ans. 7 mo. 10 da.

5. A merchant in Columbus, Ohio, orders goods from New York as follows:

To what is the interest of any sum of money for any time equal? How do we find the mean or equated time of several payments? Upon what is this rule founded? Is this correct? Why not? Why then is the rule used? What is the correct rule?

Oct. 12th, 1849,	to the value of		$100,	payable in	6 mo.
Oct. 25th	"	"	" 150.	"	4 mo.
Nov. 10th	"	"	" 300,	"	3 mo.

After ordering the latter quantity he wishes to give his note for the entire lot. Required the date to which the note should run that neither party may lose by the operation.

Ans. Feb. 25th, 1850.

Solution.—He evidently owes the whole amount from Nov. 10th. From Oct. 12th, to Nov. 10th, is 28 days; from Oct 25th to Nov. 10th is 14 days. The accounts then stand $100 for 152 da. (180–28); $150 for 106 da. (120–14); and $300 for 90 da. The equated time is then found as usual. The date is found by the table on page 273.

As we have before said, the rule given is not strictly correct. The following will give the correct mean time if no mistake is made in the operation.

Obs. 30. *Find the present worths of the several debts at the given rate. Then find in what time at the same rate the sum of the present worths would amount to the sum of the debts. The result will be the equated time.*

Remark. When the date of the several transactions is different, the date from which to make the calculations must be found as in Ex. 5. The date from which we calculate in this example is Feb. 10th. Find the answers to the following questions by both Rules.

6. Required the date for the payment of the following sums at one time.

$300,	payable in	8 mo.;	date Jan. 6th,	1849.
450,	"	6 mo.;	" Jan. 20th,	"
500,	"	4 mo.;	" Feb. 1st,	"
1000,	"	3 mo.;	" Mar. 3d,	"

Ans. By Obs. 29. June 24th, 1849. By Obs. 30, June 25th 1849.

7. Required the date for the payment of the following sums, at one time.

$800,	payable in	1 yr.;	date Aug. 12th,	1849.
1000,	"	9 mo.;	" Sept. 1st,	"
1500,	"	8 mo.;	" Sept. 15th,	"
1800,	"	6 mo.;	" Oct. 1st.,	"

Ans. By Obs. 29. May 16th, 1850. By Obs. 30, May 17th, 1850, nearly.

8. Required the date for the payment of the following sums at one time.

$1000,	payable in	11 mo.;	date	Jan. 1st,	1849.
1200,	"	10 mo.	"	Jan. 15th	"
1500,	"	9 mo.;	"	Feb. 1st	"
1800,	"	8 mo.;	"	Feb. 22d,	"
2000,	"	6 mo.;	"	March 18th	"
2500,	"	3 mo.;	"	April 9th,	"

Ans. By Obs. 29. Sept. 27th 1849; by Obs. 30. Sept. 28th, 1849.

COMPOUND INTEREST.

Obs. 31. COMPOUND INTEREST *is the interest, on both the principal and interest, when the latter is not paid at the time it becomes due.*

It is calculated by adding the interest to the principal at the end of each year, or other stated time, and making their *sum* the principal for the next succeeding year or stated time.

The *compound interest* is found by subtracting the *first principal* from the *last amount*.

1. What is the compound interest of $400 for 4 years. at 6 per cent? Ans. $104.99.

Operation.

	$400	= Principal.
$400×.06 =	24	= Interest for 1 year. (Obs. 7.)
	$424	= Amount for 1 year.
$424×.06 =	25.44	= Interest for 2d year.
	$449.44	= Amount for 2d year.
$449.44 × .06 =	26.966	= Interest for 3d year.
	$476.406	= Amount for 3d year.
$476.406 × .06 =	28.584	= Interest for 4th year.
	$504.930	= Amount for 4th year.
Deduct	$400.000	= First principal.
Ans.	$104.99	= Compound interest for 4 yrs.

2. What is the compound interest of $300 for 3 yrs. at 7 per cent? Ans. $67.5129.

3. What is the compound interest, and amount of $500 for 4 yrs, at 5 per cent?

Ans. Amount. $607.647. C. Int. $107.647:

What is compound interest? How is it calculated? When months and days are given, how do we proceed?

4. What is the compound interest, of $700 for 5 yrs., at 6 per cent.? The amount?

Ans. Amt. $936.762. Comp. Int. $236.762.

When months and days are given:

Obs. 32. *First find the amount for the required number of years, then on the last amount for the months and days.*

What is the compound interest of $800 for 5 yrs. 6 mo. 24 da., at 6 per cent.? The amount?

Ans. Amt. $1106.98. Compt. int. $306.98.

6. What is the compound interest of $500 for 3 yrs. 8 mo. 18 da., at 6 per cent.? The amount?

Ans. Amt. $621.114. Comp. int. $121.114.

7. What is the interest of $200 for 3 yrs., compounded every 6 months, at 6 per cent? Ans. $38.79.

Note.—The learner will observe to add the interest to the principal, at the end of each 6 months. (Obs. 30.)

8. What is the amount and interest of $600 for 5 yrs., compounded every 6 months, at 6 per cent?

Ans. Amount. $806.34. Comp. Int. $206.34.

TABLE

Showing the amount of $1, at 5, 6, and 7 per cent, for any number of years from 1 to 30.

Yrs.	5 per cent	6 per cent	7 per cent	Yrs.	5 per cent	6 per cent	7 per cent
1	$1.0500	$1.0600	$1.0700	16	$2.1828	$2.5403	$2.9521
2	1.1025	1.1236	1.1449	17	2.2920	2.6927	3.1588
3	1.1576	1.1910	1.2250	18	2.4066	2.8543	3.3729
4	1.2155	1.2624	1.3107	19	2.5269	3.0256	3.6165
5	1.2762	1.3382	1.4025	20	2.6532	3.2071	3.8696
6	1.3400	1.4185	1.5007	21	2.7859	3.3995	4.1405
7	1.4071	1.5036	1.6057	22	2.9252	3.6035	4.4304
8	1.4774	1.5938	1.7181	23	3.0715	3.8197	4.7405
9	1.5513	1.6894	1.8384	24	3.2251	4.0489	5.0723
10	1.6288	1.7908	1.9671	25	3.3863	4.2918	5.4274
11	1.7103	1.8982	2.1048	26	3.5556	4.5493	5.8073
12	1.7958	2.0121	2.2521	27	3.7334	4.8223	6.2138
13	1.8856	2.1329	2.4098	28	3.9201	5.1116	6.6488
14	1.9799	2.2609	2.5785	29	4.1161	5.4183	7.1142
15	2.0789	2.3965	2.7590	30	4.3219	5.7434	7.6122

The amount of \$2 (whether at simple or compound interest,) is twice as much as the amount of \$1 for the same time; the amount of \$3 is three times as much as \$1; &c. Hence—

To find the amount of any sum by the table:

Obs. 33. *Multiply the amount of* \$1 *for the required number of years, by the given sum* To find the interest: See Obs. 30.

9. What is the amount of \$2000 for 15 yrs., at 5 per cent., compound interest?

Solution. By the table, the amount of \$1 for 15 yrs. is \$2.0789 and \$2.0789 × 2000 = \$4157.80. Ans. \$4157.80.

10. What is the amount of \$2500 for 20 yrs., at 6 per cent. compound interest?

Ans. \$8017.75.

11. What is the amount of \$1500 for 30 yrs., at 7 per cent. compound interest? Ans. \$11418.30.

12. What is the amount of \$50000 for 18 yrs. 6 mo. 12 da., at 6 per cent., compound interest? (See Obs. 31.)

Ans. \$147281.88.

13: What is the amount of \$10000 for 8 yrs. 8 mo. at 6 per cent compound interest?

Ans. \$16575.52.

14. What is the amount of 3000 for 40 yrs., at 6 per cent., compound interest?

Ans. \$30855.842.

NOTE.—First find the amount for 30 yrs., and then on this amount for 10 yrs.

15. Required the amount of \$5000 for 37 yrs., at 1 per cent., compound interest?

Ans. \$61114.5477.

REMARK.—In several of the preceding examples the principal has doubled itself. Any sum at 5 per cent., compound interest, will double itself in 14 yrs 2 mo. 13 da.: at 6 per cent. in 11 yrs. 10 mo. 22 da.: and at 7 per cent. in 10 yrs. 2 mo. 27 da.

How do we find the amount of any sum by the table?

SECTION XIV.

DUODECIMALS.

Obs. 1. Duodecimals *are fractions of a foot; or a species of numbers, of which, the ratio of increase and decrease, is twelve.* The term is derived from the Latin numeral *duodecim,* which signifies *twelve.*

The denominations are Feet, Inches or Primes, Seconds, Thirds, Fourths, &c.

TABLE.

12 fourths ('''')	make	1 third,	marked	'''.
12 thirds,	"	1 second,	"	''.
12 seconds,	"	1 inch or prime,	"	in. or '.
12 inches,	"	1 foot,	"	ft.

Remark 1. The marks ', '', ''', '''', &c., which distinguish the different denominations, are called *indices.* The foot has no index, being considered as the *unit.*

2. Duodecimals may be added or subtracted in the same manner as other compound numbers. (Sect. IX, Arts. 4 and 5.)

MULTIPLICATION OF DUODECIMALS.

Obs. 2. Duodecimals are used principally *in measuring surfaces* or *in finding the solidity of bodies.* The former is ascertained by multiplying together the length and breadth. (Sect. IX, Art. 2, Obs. 15.) And the latter is found by multiplying together the length, breadth and thickness. (Sect. IX, Art. 2 Obs. 18.)

Remark 1. As 12 inches make 1 foot, 12 '' make 1 ', &c., it follows that

1 ' is - - - - - $\frac{1}{12}$ of a foot.

1 '' is $\frac{1}{12}$ of an inch, or $\frac{1}{12}$ of $\frac{1}{12}$ = - $\frac{1}{144}$ of a foot.

1 ''' is $\frac{1}{12}$ of 1 '', or $\frac{1}{12}$ of $\frac{1}{12}$ of 1 ', or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{1728}$ of a ft.

1 '''' is $\frac{1}{12}$ of 1 ''', or $\frac{1}{12}$ of $\frac{1}{12}$ of 1 '', or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 ', or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{20736}$ of a foot, &c.

The foot being regarded as the unit, and the other denominations as the fractional parts of this unit, or foot, it follows, that if we

---

What are duodecimals? From what is the term derived? What are the denominations? Repeat the table. What are the marks ', '', ''', '''', &c., called Why has the foot no index? How may duodecimals be added and subtracted? For what are duodecimals principally used? How is the former ascertained? How is the latter found? What part of a foot is 1 '? What part of a foot is 1 ''? Is 1 '''? Is ''''? If we multiply feet by feet, what shall we obtain as the result? Why?

multiply feet by feet, we shall obtain feet as the result; as $1 \times 1 = 1$. But if we multiply feet by inches (′) or inches (′) by feet, the result will be inches (′) because 1 inch is $\frac{1}{12}$ of a foot, and $1 \times \frac{1}{12} = \frac{1}{12} = 1'$.

Also, inches (′) multiplied by inches (′) produce seconds, (″) as $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144} = 1''$.

Again, feet multiplied by seconds (″) or seconds (″) by feet, produce seconds, (″) as $1 \times \frac{1}{144} = \frac{1}{144} = 1''$.

And inches (′) multiplied by seconds, (″) or seconds (″) by inches (′) produce thirds, (‴) as $\frac{1}{12} \times \frac{1}{144} = \frac{1}{1728} = 1'''$.

And seconds (″) multiplied by seconds (″) produce fourths, (⁗). as $\frac{1}{144} \times \frac{1}{144} = \frac{1}{20736} = 1''''$.—Hence—

Obs. 3. *The product of any two denominations is always of that denomination, denoted by the sum of their indices.*

Ex. 1. How many square feet in a room 14 ft. 8′ 10″ in length, and 12 ft. 7′ 9″ in width?

Ans. 186 sq. ft. 4′ 2″ 5‴ 6⁗.

1st Operation.

		14 ft.	8′	10″	
		12 ft.	7′	9″	
		11′	0″	7‴	6⁗
	8 ft.	7′	1″	10‴	
	176 ft.	10′	0″		
Ans.	186 ft.	4′	2″	5‴	6⁗

2d Operation.

	14 ft.	8′	10″		
	12 ft.	7′	9″		
	176 ft.	10′	0″		
	8 ft.	7′	1″	10‴	
		11′	0″	7‴	6⁗
Ans.	186 ft.	4′	2″	5‴	6⁗

In the 1st operation, we multiply first by 9″. 10″ × 9″ = 90⁗ (Obs. 3.) 90⁗ ÷ 12 = 7‴ 6⁗. We set down 6⁗ and carry 7‴

Feet and inches multiplied together, produce what? Why? Inches into inches produce what? Why? Feet into seconds produce what? Why?—Inches into seconds produce what? Why? Seconds into seconds produce what? Why? Of what denomination is the product of any two denominations? In the operations of Ex. 1, how are the partial products written?

$8' \times 9'' = 72'''$ and $7'''$ to carry make $79'''$. $79''' \div 12 = 6''\ 7'''$. We set down $7'''$ and carry $6''$. 14 ft. $\times 9'' = 126''$, and 6 to carry make $132''$. $132'' \div 12 = 11'\ 0''$, which we set down. We multiply by $7'$ and 12 ft., setting down and carrying in the same manner. Finally, we add the several products together, observing to carry 1 for every 12, as in multiplying.

In the 2nd operation, we multiply first by 12 ft., and afterwards by $7'$, and $9''$ setting down and carrying as in the first operation.

REMARK.—In both these operations the learner will perceive that *we place those numbers of the same denomination under each other.*

From the preceding remarks we derive the following

RULE. I. *Write the multiplier under the multiplicand.*

II. *Multiply each denomination of the multiplicand (commencing at the right,) by each denomination of the multiplier and write each product under its corresponding denomination.*

III. *Finally, add together the several partial products, as they stand, carrying for every 12 both in multiplying and adding.*

REMARK 1. The learner will observe that feet multiplied by feet produce square feet, and the same remark is observed of all the denominations of *linear measure.* (Sect. IX, Art. 2, Obs. 15. Rem). Also, *square measure* multiplied by *linear measure* produces *solid,* or *cublic measure.*

2. The *inches* in duodecimals are usually called *Carpenters' inches.* They are neither *linear, square,* nor *cubic.* In measuring *length,* an *inch* is one-twelfth of a foot. In measuring *surfaces,* an *inch* is one-twelfth of a *square foot;* and measures 1 *foot* in *length,* and 1 *inch* in *width.* In measuring *solids,* an *inch* is one-twelfth of a *solid,* or *cubic foot;* and measures 1 *foot in length,* 1 *foot* in *thickness,* and 1 *inch* in *width.*

2. How many square feet in 12 boards, each 18 ft. $7'\ 6''$ long, and 1 ft. $4'$ wide. Ans. 298.

3. How many square feet in a room 24 ft. $6'\ 9''$ in length, and 18 ft. $9'\ 10''$ in width? Ans. 462 sq. ft. $3'\ 0''\ 4'''\ 6''''$.

4. How many solid feet in a vat 9 ft. $6'$ long, 7 ft. $8'$ wide, and 6 ft. $9'$ deep? Ans. 491 sq. ft. $7'\ 6''$.

5. How many solid feet in a box, 17 ft. $8'\ 4''$ in length, 6 ft. $7'\ 3''$ in width, and 4 ft. $6'\ 6''$ in depth?

Ans. 530 sq. ft. $8'\ 8''\ 6'''\ 2''''\ 6'''''$.

Obs. 4. *Painters', Pavers'* or *Plasterers'* work is generally computed by the *square yard.* Square feet are reduced to square yards by dividing by 9. (See table, square measure.)

What is the rule? What does a linear measure multiplied by linear measure produce? What does square measure multiplied by linear measure produce? What are the inches in duodecimals usually called? What is an inch, and how much does it measure in measuring lengths, surfaces and solids? How is painters' pavers and plasterers' work computed?

6. How many square yards in the walls of a room which measures 61 ft. 4′ in compass, and 11 ft. 3′ in height? And what will it cost to paint the walls at $12\frac{1}{2}$ cents per square yard?
Ans. $76\frac{2}{3}$ sq. yds.; cost \$$9.58\frac{1}{3}$.

7. How much will it cost to paint a building measuring 96 ft. 9′ in compass, and 15 ft. 8′ in height, at 10 cents per square yard?
Ans. \$$16.84\frac{1}{6}$.

8. How much will it cost to pave a side walk 124 ft. 6′ in length, and 7 ft. 10′ in width, at 25 cents per square yard?
Ans. \$27.09.

9. How much will it cost to pave a yard 35 ft. 6′ in length, and 25 ft. 8′ in width, at 30 cents per square yard?
Ans. \$$30.37\frac{2}{9}$.

10. How much will it cost to plaster a ceiling, at 18 cents per square yard, it measuring 21 ft. 8′ in length, and 12 ft. 9′ in width?
Ans. \$$5.52\frac{1}{2}$.

11. How much will it cost to plaster a ceiling 18 ft. 6′ in length, and 10 ft. 2′ in width, at 20 cents per square yard?
Ans. \$4.18 nearly.

12. A certain room measures 25 ft. 3′ in length, 18 ft. 4′ in width, and 9 ft. in heighth. Required the cost of plastering it, at 25 cents per square yard, deducting 3 doors, each 6 ft. 6′ by 2 ft. 8′, a fire place 5 ft. 6′ by 4 ft. 4′, and 4 windows, each 5 ft. 8′ by 3 ft. 3′? Ans. \$32.032.

13. A certain building is three stories in height; each story has 12 windows, each 6 ft. 10′ by 3 ft. 6′. Required the cost of the glazing at 15 cents per square foot. Ans. \$129.15.

Obs. 5. Some parts of Carpenters' and Joiners' work are computed by the sq. yd.; other parts such as flooring, &c., are computed by the *square*. *A square* consists of 100 square feet.

14. How much will it cost to lay a floor 22 ft. 6′ in length, and 18 ft. 6′ in width, at \$3.50 per square? Ans. \$$14.56\frac{7}{8}$.

15. How much will it cost to lay a floor 30 ft. 8′ in length, and 26 ft. 6′ in width. at \$4.50 per square, deducting a place for the stairs 9 ft. 10′ by 4 ft. 6′; and a fire place 5 ft. 10′ by 4 ft. 6′.
Ans. \$$33.39\frac{3}{4}$.

Obs. 6. *Masons'* work is sometimes computed by the *solid foot* and at other times by the *perch*. *A perch* measures $16\frac{1}{2}$ feet in

---

How are square feet reduced to square yards? How is flooring, roofing, &c., computed? Of how much does a square consist? How is masons' work sometimes computed?

length, $1\frac{1}{2}$ ft. in breadth, and 1 foot in depth, and contains $16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$ solid feet. (Sect. IX, Art. 2, Obs. 18.) Hence—

To find the number of perches in any wall or solid body:

Obs. 7. *Find the contents in solid feet, and divide this by* $24\frac{3}{4}$.

16. How many perches in a wall 72 ft. 3′ long, 12 ft, 8′ high, and 2 ft. thick? Ans. 73.952+.

17. How many perches in a wall 48 ft. 10′ long, 15 ft. 6′ high, and 2 ft. 3′ thick? What will it cost to build it at $2.50 per perch? Ans. 68.81 + perches. Cost $172.025.

18. How many bricks in a wall 100 ft. 4′ long, 10 ft. 6′ high, and 1 f. 8′ thick, allowing 20 bricks to the solid foot, and how much will it cost to lay this wall, at $6 per thousand bricks? Ans. 35116 bricks. Cost $210.70.

19. How many bricks, each 8 inches long, 4 inches wide, and 2 inches thick will it take to build a wall 40 ft. 6′ long, 14 ft. 4′ high, and 2 ft. 8′ thick? Ans. 41796.

20. A certain brick building is 28 ft. 6′ in length, 24 ft. 6′ in width, and 15 ft. 9′ in heighth, and the walls are 1 foot thick. Now deducting 4 doors, each 6 ft. 9′ by 2 ft. 8′ and 10 windows, each 6 ft. 6′ by 3 ft. 6′, how many bricks, each 8′ long, 4′ wide, and 2′ thick, did it require for this building. and how much did they cost at $3.25 per thousand?

Ans. It took 36990 bricks, and they cost $$120.21\frac{3}{4}$

SECTION XV.

POWERS AND ROOTS.

Def. 1. A POWER *of any number, is the product arising from mutiplying it into itself.* Thus ; 4 is a power of 2, because $2 \times 2 = 4$; and 27 is a power of 3, because $3 \times 3 \times 3 = 27$, &c.

Def. 2. A ROOT *of any number, is a number which multiplied into itself a certain number of times will produce the given number.* Thus: 2 is a root of 4, because $2 \times 2 = 4$; 3 is a root of 27, because $3 \times 3 \times 3 = 27$; and 4 is a root of 256, because $4 \times 4 \times 4 \times 4 = 256$.

What does a perch measure? How many solid feet does it contain? How do we find the number of perches in a wall or solid body? What is a power? Give examples. What is a root? Give examples?

Article 1. Involution.

Obs. 1. Involution *is the method of finding the powers of numbers.*

Obs. 2. Powers are divided into several *orders*, called the *first, second, third, fourth power*, &c.

Obs. 3. The number which is to be involved is called the *root*, or *first power*. The other powers derive their name from *the number of times the root is employed as a factor, in producing this power.* Thus:

$$
\begin{array}{rl}
 & 2 \text{ is the first power of } 2. \\
2\times2= & 4 \text{ is the second power of } 2. \\
2\times2\times2= & 8 \text{ is the third power of } 2. \\
2\times2\times2\times2= & 16 \text{ is the fourth power of } 2.
\end{array}
$$

Remark.—The second power of a number is usually called the *square*
The third power is usually called the *cube*:
The fourth power is usually called the *biquadrate:*
The fifth power is usually called the *sursolid:*
The other powers generally receive no other name than their numeral distinction, as the *sixth power*, the *seventh power*, &c. of the root, or first power.

MENTAL EXERCISES.

1. What is the square of 3? Of 4; 5; 6; 8; 11; 7; 10; 9; 12?
2. What is the cube of 3? Of 4; 7; 6; 5; 12. 9; 11; 10; 8?
3. What is the fourth power, or biquadrate, of 3? Of 5; 4; 6?
4. What is the fifth power, or sursolid, of 2? Of 3; 4; 5; 7?
5. What is the sixth power of 2? Of 3?
6. What is the seventh power of 2? Of 3?

Obs. 4. The number that denotes the power to which the root is to be raised is called the *index*, or *exponent* of the power, and is usually written at the *right*, and a *little above* the root. Thus:

3^1 signifies 3, or the *first power* of 3.
3^2 signifies $3\times3=9$, or the *square* of 3.
3^3 signifies $3\times3\times3=27$, or the *cube* of 3.
3^4 signifies $3\times3\times3\times3=81$, or the *biquadrate* of 3, &c., &c.

Remark 1.—To find the second power, or square of 3, we use 3 as a factor *twice*; that is, we multiply it into itself *once*.

Thus: $3\times3=9$.

What is Involution? How are powers divided? What is the number to be involved called? From what do the other powers receive their name? Give examples. What is the second power of a number usually called? The third power? The fourth power? The fifth power? What name do the other powers receive? What is the number denoting the power to which the root is to be involved called?

2.—To find the cube, or third power of 3, we use 3 as a factor *three times;* that is, we multiply it into itself *twice.*

Thus: $3 \times 3 \times 3 = 27$.

3.—To find the biquadrate, or fourth power of 3, we use 3 as a factor *four times;* that is, we multiply it into itself *three times.*

Thus: $3 \times 3 \times 3 \times 3 = 81$. Hence—

To involve any number to any required power:

Obs. 5. *Multiply the given number into itself, until it has been employed as a factor as many times as there are units in the index, denoting the power to which it is to be raised.*

REMARK.—*The number of multiplications* in involving any number to a required power, is always 1 *less than the index.* Thus, 3^3 signifies that 3 is to be employed as a *factor* three times, but to be multiplied into itself but *twice.*

Thus: $3 \times 3 \times 3 = 27$, &c.

Obs. 6. *All the powers of* 1 *are the same*—that is, 1. For $1^2 = 1$; $1^4 = 1$, &c.

EXERCISES FOR THE SLATE.

1. What is the square of 14? Ans. 196.
2. What is the cube of 13? Ans. 2197.
3. What is the fourth power of 15? Ans. 50625.
4. How much is 13^2? 14^3? 18^2? 20^4? 30^5? 40^6?

Ans. in order. 169; 2744; 324; 160000; 24300000; 4096000000.

9 is the square of 3. To multiply 9 by 3, and that product by 3, we obtain $81 = 3^4$, which is the same as 9×9, or $3^2 \times 3^2$.

$3^3 = 27$; $27 \times 3 \times 3 = 243 = 3^5$, which is the same as 27×9, or $3^3 \times 3^2$.

$3^3 = 27$; $27 \times 3 \times 3 \times 3 = 729 = 3^6$, which is the same as 27×27, or $3^3 \times 3^3$, &c.

Obs. 7. Hence—*The fourth power is equal to the square multiplied by the square.*

The fifth power is equal to the cube multiplied by the square.

The sixth power is equal to the cube multiplied by the cube.

The seventh power is equal to the fourth power multiplied by the cube, &c.

That is—*The product of any two powers of a number, is equal*

How do we involve numbers to any required power? How many times is a number used as a multiplier in involving it to any required power? What are all the powers of 1? To what is the fourth power of any number equal? The fifth power? The sixth power? The seventh power? To what is the product of any two powers of a number equal?

to that power of the number denoted by the sum of their exponents.

From the above illustrations, the following proposition is also evident:

Obs. 8. *The quotient arising from dividing any power of a number by another power of the same number, is equal to that power of the number denoted by the difference of their exponents.*

Let the learner find the results of the following questions by the two preceding observations:

5. Required—the fourth power of 16? 18? 22? 25?
Ans. in o der. 65536; 104976; 234256; 390625.

6. Required—the fifth power of 17? 19? 21? 24? 30?
Ans. in order. { 1419857; 2476099. 4084101; 7962624; 24300000.

7. Required—the sixth power of 16? 18? 22? 25?
Ans. in order. { 16777216; 34012224; 113379904 244140625.

8. Required—the seventh power of 17? 19? 21? 24? 30?
Ans. in order. { 410338673; 893871739; 1801088541; 4586471424; 21870000000.

7. Divide the ninth power of 12 by the seventh power of 12.
Ans. 144.

10. Divide 14^5 by 14^3. Ans. $196 = 14^2$.

11. Divide 18^{12} by 18^9. Ans. $5832 = 18^3$.

12. Divide 36^{30} by 36^{25}. Ans. 60466176.

Obs. 9. *Fractions* may be involved as well as whole numbers, by *involving both the numerator and denominator to the power indicated by the exponent.*

REMARK 1.—A fraction to be involved is generally written within a parenthesis; thus: $(\frac{2}{3})^2$. This signifies that $\frac{2}{3}$ is to be multiplied into itself, or squared. Thus: $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$; and $(\frac{3}{4})^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$; but $\frac{2^3}{3} = \frac{8}{3} = 2\frac{2}{3}$: and $\frac{2}{3^3} = \frac{2}{27}$, &c.

2.—*Any power of a proper fraction is less than the fraction itself.* This is evident from the fact that the numerator is *less* than the denominator; (Sect. VIII. Art. 1. Obs. 11.) and therefore, when the fraction is *involved*, the multiplier of the denominator is *greater* than the multiplier of the numerator, and this being the case, the *ratio*

To what is the quotient arising from dividing any power of a number, by another power of the same number, equal? How may fractions be involved? How is a fraction to be involved generally written? What is the value of the power of a proper fraction, compared with the value of the fraction itself? Show why this is the case?

between the terms of the fraction *is diminished;* hence, as the *ratio* is the *quotient* of one number divided by another, (Sect. XII. Art. 1. Obs. 1.) and the *value of the fraction* answers to the *quotient* in division, (Sect. VIII. Art. 1. Obs. 9.) it is evident that if the *quotient*, or *ratio*, is diminished, *the value of the fraction* is also diminished.

3.—It is generally most convenient to reduce *mixed numbers* to *improper fractions*, or the fractional part to a *decimal*, before involving it to the power indicated by the index. Likewise, *compound* and *complex fractions* must be reduced to *simple ones* before involving them.

13. What is the square of $\frac{3}{4}$? $\frac{4}{5}$? $\frac{7}{8}$? $\frac{11}{12}$? $\frac{8}{15}$? $\frac{11}{18}$? $7\frac{1}{8}$? $\frac{3}{4}$ of $\frac{5}{6}$?

14. What is the cube of $\frac{5}{6}$? $\frac{9}{10}$? $\frac{7}{11}$? $\frac{11}{14}$? $\frac{15}{17}$? $\frac{13}{24}$? $4\frac{2}{7}$? $\frac{1}{2}$ of $\frac{7}{9}$?

15. How much is $\left(\dfrac{2\frac{1}{2}}{3\frac{2}{3}}\right)^2$? $\frac{4}{5}^2$? $\frac{3}{7}^2$? $\dfrac{(7\frac{1}{8})^3}{4}$? $\dfrac{2}{(1\frac{1}{4})^2}$? $(\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{9}{10})^2$?

16. What is the square of 25? Ans. 625.

Common Operation.

$$\begin{array}{r} 25 \\ 25 \\ \hline 125 \\ 50 \\ \hline \text{Ans. } 625 \end{array}$$

Analytic Operation.

$$\begin{array}{rl} 25=20+5 & \\ 20+\ 5 & \\ \hline 100+25 & = \text{Product of } 20+5 \text{ by } 5. \\ 400+100 & = \text{Product of } 20+5 \text{ by } 20. \\ \hline \text{Ans. } 400+200+25=625= & \text{the square.} \end{array}$$

Thus, we perceive the result to be equal to the square of the first part, $(20,)^2$ plus twice the product of the two parts, $(20 \times 5 \times 2,)$ plus the square of the second part, $(5)^2$. The operation may be contracted, thus:

$$\begin{array}{r} 25 \\ 25 \\ \hline 625 \end{array}$$

5×5, or $5^2 = 25$; set down 5 and carry 2. $2 \times 5 = 10$; $10 \times 2 = 20$, and 2 to carry $= 22$; set down 2, and carry 2. 2×2, or $2^2 = 4$, and 2 to carry $= 6$. (Sect. VI. Art. 2. Ex. 13. Hence—

To find the square of any number consisting of but two figures:

Obs. 10. *Square the right hand digit for the unit figure of the*

How do we proceed with mixed numbers, when involving them? With compound and complex fractions? In the analytic operation of Ex. 16, to what is the result equal? How then may we find the square of a number consisting of but two figures?

result, double the product of the two digits for the ten's figure of the result, and square the left hand digit for the remaining figures of the result, observing to carry in each case as in Multiplication of Simple Numbers.

NOTE.—By this method the learner can often find the square of numbers *mentally*, which would otherwise require the use of the slate.

17. How much is 99^2? 27^2? 46^2? 75^2?
Ans. 9801; 729; 2116; 5625.

18. How much is 92^2? 86^2? 50^2? 100^2?

19. How much is 31^3? 44^3? 37^4? 90^4? 60^5? 10^9? 20^7?

20. How much is 120^3? 144^2? 300^6? 1728^2? 256^4?

21. How much is 14^6? 9^{10}? 15^3? $(\frac{4}{7})^4$? $\frac{2^3}{3^2}$? $\frac{7^4}{8^6}$? $(4\frac{1}{2})^3$? $\left(\frac{1\frac{1}{4}}{2\frac{1}{3}}\right)^4$?

22. How much is $(\frac{2}{3})^6$? $(\frac{1}{4})^9$? $.5^2$? $.07^3$? $.001^{10}$? 4.75^3? 1.1^5?

23. How much is $.4^2$? $.02^6$? 1.0002^2? 3.0702^2? $(10\frac{2}{3})^3$? $(8\frac{1}{6})^1$?

24. How much is 7.9^2? 1.16^3? 211^4? 200.002^3? $.0001^6$?

25. How much is $(10\frac{3}{4})^4$? 17.9^3? 216.01^2? 100.01^5?

ARTICLE 2. EVOLUTION.

Obs. 1. EVOLUTION *is the method of finding the roots of numbers.*

REMARK 1.—*Evolution* is the opposite of *Involution*, and each reciprocally proves the other. By the one, we multiply a number into itself to find the *power:* by the *other*, we resolve the power into *equal factors* to find the *root.*

2.—Different roots have different names, corresponding to those of the different powers.

Thus: 2 is the second root of 4, because $2 \times 2 = 4$; and 3 is the third root of 27, because $3 \times 3 \times 3 = 27$. Hence—

Obs. 2. *A root takes its name from the number of times it is employed as a factor to produce the given power.*

REMARK 1.—The *second root* is usually called the *square root.*
The *third root* is usually called the *cube root.*
The other roots generally receive no other name than their *numeral* distinction—as the *fourth root*, the *fifth root*, &c.

What is the advantage of this rule? What is Evolution? What relation has Evolution to Involution? How is this? From what does a root take its name? What is the second root usually called? The third root? What name do the other roots generally receive?

2.—*Powers and roots are correlative terms;* because, if one number is a *power* of another, the latter is a root of the former. Thus, 8 is the cube of 2, and 2 is the cube of 8.

MENTAL EXERCISES.

1. What is the square root of 9? Of 16; 25; 49; 36; 81; 100?
2. What is the cube root of 8? Of 27; 64; 125?
3. What is the fourth root of 16? Of 81?
4. What is the square root of 144? The cube root of 729?

Obs. 3. This character ($\sqrt{}$) is the sign of the *square root.*

Other roots are indicated by the same character, with the index of the root placed above it. Thus: $\sqrt[3]{}$ indicates the cube root; $\sqrt{}$ indicates the fourth root, &c.

REMARK 1.—Roots are also indicated by a *common fraction,* the numerator of which is 1, and the denominator the *index of the root.* This is written in the same manner as the exponents of powers.

Thus: $4^{\frac{1}{2}}$ is the same as $\sqrt{4}$; $8^{\frac{1}{3}}$ is the same $\sqrt[3]{8}$; $16^{\frac{1}{4}}$ is the same as $\sqrt[4]{16}$, &c.

2.—As the square multiplied by the square produces the fourth power, (Art. 1. Obs. 7.) it is evident that the *square root of the square root is the fourth root.*

Thus: $\sqrt[3]{16} = 4$; $\sqrt{4} = 2 = \sqrt{16}$.

Likewise, the *sixth root* is equal to *the square root of the cube root, or the cube root of the square root.*

Thus: $\sqrt[3]{64} = 4$; $\sqrt{4} = 2 = \sqrt[6]{64}$. And $\sqrt{64} = 8$; $\sqrt[3]{8} = 2 = \sqrt[6]{64}$. Hence—

Obs. 4. *When the index of the root can be resolved into factors, these factors denote the roots, which being successively found, will give the required root.*

Obs. 5. The process of finding the roots of numbers is called the *Extraction of Roots.* It consists in finding such a number, as multiplied into itself a certain number of times will produce the power. (Definition 2.)

5. How much is $\sqrt{144}$? $\sqrt{81}$; $\sqrt[3]{216}$; $\sqrt[3]{344}$; $\sqrt[4]{81}$?

What kind of terms are powers and roots? Why? What is the sign of the square root? How are other roots indicated? By what other method are roots indicated? How is this fraction written? To what is the square multiplied by the square equal? To what is the square root of the square root equal? To what is the sixth root equal? When the index of root can be resolved into factors, what do these factors denote? What is the process of finding the roots of numbers called? In what does it consist?

6. How much is $\sqrt{121}$? $\sqrt{100}$; $\sqrt[3]{512}$; $\sqrt[3]{729}$; $\sqrt[5]{243}$?

7. How much is $\sqrt{64}$! $\sqrt{36}$; $\sqrt[3]{125}$; $\sqrt{16}$; $\sqrt[6]{64}$; $\sqrt[6]{729}$?

Obs. 7. *Roots of Fractions* may be extracted, by finding the *roots of both the numerator and the denominator.*

Thus: $\sqrt{\frac{4}{9}}=\frac{2}{3}$; $\sqrt[3]{\frac{8}{27}}=\frac{2}{3}$; but $\frac{\sqrt{4}}{7}=\frac{2}{7}$; and $\frac{5}{\sqrt[3]{8}}=\frac{5}{2}=2\frac{1}{2}$, &c.

a. *Mixed numbers* must be reduced to *improper fractions* before extracting their roots.

8. How much is $\sqrt{\frac{81}{100}}$? $\sqrt{\frac{49}{64}}$; $\sqrt{\frac{36}{144}}$; $\sqrt{\frac{121}{144}}$?

9. How much is $\sqrt[3]{\frac{27}{64}}$! $\sqrt[4]{\frac{16}{81}}$; $\sqrt[3]{\frac{27}{3}}$; $\frac{1}{\sqrt[3]{64}}$.

10. How much is $\sqrt{12\frac{1}{4}}$? $\sqrt{13\frac{4}{9}}$; $\sqrt[3]{3\frac{3}{8}}$; $\sqrt[3]{2\frac{10}{27}}$?

Obs. 8. From these examples we notice this consideration:

If the power contains a proper fraction, the root also contains a proper fraction. And conversely:

a. *If the root contains a proper fraction, any power of this root must also contain a proper fraction.* This is evident from Art. 1. Obs. 9. Rem. 2. Also:

Obs. 9. *If the power contains a decimal, its root must also contain a decimal.* And conversely—

a. *If the root contains a decimal, any power of this root must contain a decimal.*

This is evident from the fact, that if we multiply by a decimal, the product must contain a decimal. (Sect. VIII. Art. 11.)

Remark.—From the fact that the product must contain as many decimals as both factors, it is evident that *the root must contain one half as many decimals as its square, one third as many decimals as its cube, &c.*

11. How much is $\sqrt{.25}$? $\sqrt{.64}$? $\sqrt[3]{.027}$?

12. How much is $\sqrt[3]{.343}$! $\sqrt[3]{.729}$? $\sqrt{.16\times9}$? $\sqrt[3]{30-3}$?

Obs. 10. A number whose root can be exactly found, is called a *perfect power*, and its root is called a *rational number*. Thus: 9, 16, 25, 27, &c., are perfect powers, and their roots, 3, 4, 5, and 3, (the cube root of 27,) are rational numbers.

How may roots of fractions be extracted? How do we proceed with mixed numbers when they occur? What consideration is noticed from Ex. 8, 9, and 10? Show why this is correct? What is said respecting decimals in the power or root? Show why this is correct. How many decimals must the root contain, compared with those of the square? Of its cube? Show why this is correct. What is a perfect power? A rational number?

Obs. 11. A number whose root *cannot* be exactly ascertained, is called an *imperfect power*, and its root is called a *surd*, or *irrational number*.

Thus: 11, 17, 45, &c., are imperfect powers, and their roots 3.3+, 4.1+, 6.7+ are surds.

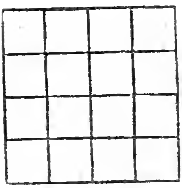
REMAKK.—A number may often be a *perfect power* of one order, and an *imperfect power* of another order. Thus, 25 is a perfect power of the *second order*, and an imperfect power of the *third order*. Also, 64 is a perfect power of the *second, third*, and *fifth orders*, and an imperfect power of the *fourth order*.

Obs. 12. *Every root and power of* 1 *is the same—that is,* 1. Thus, 1^2, 1^3, 1^4, $\sqrt{1}$, $\sqrt[3]{1}$. $\sqrt[5]{1}$, $\sqrt[1]{1}$, &c., are all equal.

NOTE.—The preceding exercises in Evolution are designed to be performed mentally; but it frequently happens that the numbers are too large to ascertain their roots in this manner, and we have to use the slate. In such cases there is a particular process by which the roots are found. The methods of extracting the square we will now explain. As the other roots seldom occur in practical business, they are not treated of in this work.

Case 1.—*Extraction of the Square Root.*

Obs. 13. *To extract the square root of a number, is to find a number, which being multiplied into itself, will produce the given number.* (Art. 1. Obs. 4. Rem.)



REMARK.—The terms *Square* and *Square Root* are derived from Geometry. The *sqnare* may be considered as a square figure, made up of several smaller squares, and the *square root* as the number of smaller squares on one side of this figure. Thus, in the diagram, the large square is made up of 16 smaller ones, and there are 4 of these on a side; therefore, 16 is the *square*, and 4 is the *square root*. Hence—

Obs. 14. *The extraction of the Square Root may be defined, as finding one side of a square, when the square contents are given.*

Obs. 15. Take the following numbers:

	1,	2,	3,	4,	5,	6,	8,	9,	10,	99,	100.
Their factors are	1,	4,	9,	16,	25,	36,	64,	81,	100,	9801,	10000.

The numbers in the first line may also be regarded as the *square roots* of those in the second line. (Obs. 2. Rem. 2.)

What is an imperfect power? A surd or irrational number? Can a number be a perfect power of one order, and an imperfect power of another order? Give examples. What is every root and power of 1? What is it to extract the square root of a number? From what are the terms square and square root derived? What may the square be considered? The square root? How then may the extraction of the square root be defined?

From this we notice the following considerations:

a. 1st. *The square of no digit exceeds two figures.*

b. 2nd. *When a number contains but two figures, its square root contains but one.* Also: *When the number contains three or four places of figures, its square root contains 2.*

c. 3rd. *The square of no number contains more than double the number of figures in the number squared, and but* 1 *less.*

Obs. 16. Hence—*When we extract the square root of a number, we first point it off into periods of two figures each, commencing at the right hand.* By this means we ascertain—

a. 1*st. How many figures the left hand period contains, or how many figures must be taken to find the first figure of the root.* This is the most important object gained.

b. 2nd. *Of how many figures the root will consist.*

REMARK.—From Obs. 15, *c.*, it is evident that the square root must contain as many figures as there are periods in the square. If the *square* contains an *odd* number of figures, the *root* will contain 1 more than half as many; if the *square* contains an *even* number of figures, the *root* will contain just half as many. Thus, the square root of 144 is 12; and of 6400, the square root is 80. These numbers when pointed off into periods stand thus: $\dot{1}4\dot{4}$; $6\dot{4}0\dot{0}$.

Ex. 1. A man has a square field containing 625 square rods. How many rods does it measure on each side? Ans. 25.

Operation.

```
    6̇25̇ (25
    4
    ---
45)225
   225
   ---
   000
```

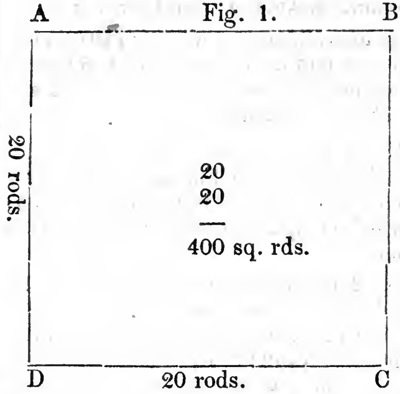
1*st step.*—In this example we wish to find one side of a square, having given its contents; that is, we wish to extract the square root of 625. (Obs. 14.) We first point off the given number into periods, and then ascertain that the *left hand period* contains *one figure,* and that the *root* will contain *two figures.* (Obs. 16.) These two figures (of the root) of course are a *ten* and a *unit.*

2*nd step.*—We now seek the greatest square in the left hand period, (6) and find it to be 4, the square root of which is 2. This we place at the right, with a line to separate it from the power.

As this is the root of the left hand period, (6 hundreds) it is in reality 2 *tens,* (because the square of 2 *tens,* or 20, is 4 hundreds, or 400,) and is equal to one side of a square measuring 2 tens (20) rods on a side. This may be shown by a diagram:

What is the first consideration that we notice from examining the given numbers and their squares? The second? The third? What is the first thing we do in extracting the square root of a number? What do we ascertain by this means? How many figures must every root contain? If the square contains an odd number of figures, how many figures will the root contain? If the square contains an even number of figures, how many will the root contain?

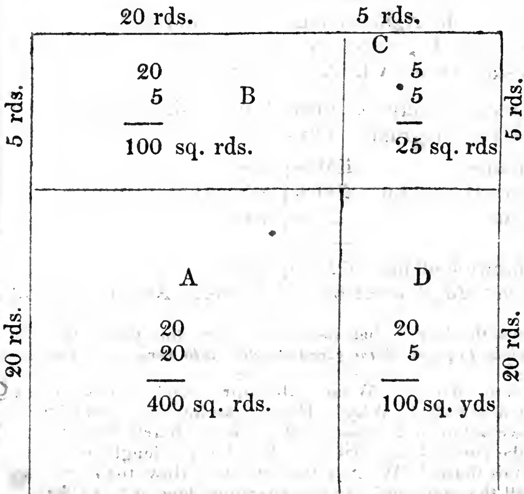
Fig. 1.



The contents of this figure we find to be $20 \times 20 =$ 400 sq. rds., now disposed of, and which we will deduct from the whole number of rods in the field, (625) showing 225 sq. rds. remaining. This deduction is readily effected by subtracting the 4 (hundreds) the square of the 2 (tens,) from the left hand period, 6, (hundreds,) leaving 2 (hundreds,) and bringing down the next period, (25,) making 225 as seen in the operation.

3d step.—We now wish to enlarge our figure by the addition of 225 sq. rds., and it is evident that this addition must be made on *two sides*, in order to preserve the square. This is shown in the following diagram :

FIGURE 2.



Now as the addition is made on two sides of the square, if we divide the number of rods to be added, (225,) by the length of the two sides, ($20 + 20 = 40$,) the quotient will evidently be the width of the additions made to the two sides A B, B C, Fig. 1. Accordingly, we double

In the operation of Ex. 1, what is the first step? What do we ascertain by this? What is the second step? What is this root? Why? To what is it equal? What is done with the contents of this figure?

the root (2 tens,) already found, making 4 (tens) for a divisor.

4th step.—We now seek how many times our divisor (40) is contained in our dividend, (225) and find it to be contained 5 times; therefore, 5 rds. is the width of the additions, and $40\times5=200$ additional rods disposed of, and 25 rds. remaining.

5th step.—But looking at Fig. 2 we perceive that there is still wanting in the corner, C, a small square, equal in size to the width of the additions at the sides, (5 rds.) As this is a square, its contents are $5\times5=25$ sq. rds., which completes the square, and gives 25 rds. as the length of one side.

If the learner examines Fig. 2 attentively, he will perceive the additions measure 45 rds. in length, (20 rds. on each side, and 5 rds. at the corner,) and 5 rds. in width; and therefore, if we multiply 45 by 5, we shall obtain the same result (225.) This is effected, in our operation, by rejecting the cipher at the right of our divisor, (40,) and also the right hand figure (5) of our dividend, (225,) and dividing the remaining figure of our dividend (22) by the remaining figure (4) of the divisor, and writing the quotient figure both in the root, and also at the right of our divisor (4.) Next, we complete our dividend, (225,) and multiply our divisor (45) by the last figure of the root, (5.) and subtract the product from our dividend, which in this case leaves no remainder.

Rejecting a figure at the right of our divisor and dividend, is in effect dividing both by 10, (Sect. V. Art. 4. Obs. 1.) and this cannot affect the result. (Sect. VI. Art. 1. Obs. 28.)

PROOF.—The operation may be proved by adding together the several parts of the last diagram. Thus:

The square A contains	400	sq. rds.
The additions B and D contain	200	sq. rds.; that is, 100 sq. rds. each.
The corner C contains	25	sq. rds.
And the entire figure contains	625	sq. rds.

Or, by Involution: $25\times25=625$, as before. (Art. 1. Obs. 5.)

REMARK 2.—Perhaps the learner has inquired, before this time—*Why do we commence at the right to point off the given number into periods?* The an-

How is this deduction effected? What is the third step? On how many sides must this addition be made? Why? How do we find the width of these additions? How do we obtain our divisor? What is the fourth step? How are the contents of the corner C ascertained? What is the length of all the additions? How is this found? What is their width? How then may we find the contents of all the additions? In the operation, how is this effected? What effect does it have upon the divisor and dividend to reject a figure from the right of each? Does this effect the result? Why not? How do we prove the operatiot? By what other method? Why do we commence at the right to point off the given sum into periods?

is—Because, after the first figure of the root is obtained, there must be two figures in the given sum for every additional figure in the root. [Rem. 1] Hence, we must point off from the right, as the first figure of the root is found from the left hand period. We commence our operations at the left on account of the remainders which occur. [Sect. V. Art. 2. Rule for Short Division. Rem. 1.]

From these remarks we derive the following

RULE FOR EXTRACTING THE SQUARE ROOT.

I. *Point off the given sum into periods of two figures each, placing a dot over the unit figure, another over the hundreds, and so on.* (Obs. 16.)

II. *Find the greatest square in the left hand period, and place its root at the right, like a quotient in Division. Subtract the square of the root from the left hand period, and bring down the first figure of the next period for a partial dividend.*

III. *Double the root already found for a defective divisor; seek how many times this defective divisor is contained in the partial dividend, and place the result in the root, and also at the right of the defective divisor to complete it. Likewise, bring down the remaining figure of the period at the right of the partial dividend, for a true dividend.*

IV. *Multiply the true divisor by the last figure of the root, subtract the product from the dividend, and to the right of the remainder bring down the first figure of the next period for another partial dividend.*

V. *Double the root already found for another defective divisor, with which proceed as before, until all the periods have been brought down and divided.*

VI. *If at any time the defective divisor is not contained in the partial dividend, write a cipher in the root, and bring down the remaining figure of the period, together with the first figure of the next period, for another partial dividend, with which proceed as before.*

Proof.—*Multiply the root into itself, and if the product is equal to the given sum, the work is correct.*

2. What is the square root of 41616? Ans. 204.

Operation.	*Proof.*
$\dot{4}1\dot{6}1\dot{6}$(204	204
4	204
404)1616	816
1616	408
0000	41616 (Art. 1. Obs. 5.)

The operation may also be demonstrated in the following manner:

3. What is the square root of 1296? Ans. 36.

Operation.	*Proof.*
$\dot{1}2\dot{9}6(36$	$36=30+6$
9	$30+6$
$66)396$	$180+36$
396	$900+180$
000	$900+360+36=1296.$
	(Art. 1. Obs. 10.)

1*st step.*—We find the greatest square in the left hand period to be 9, and write its root (3) in the quotient, subtract the square (9) from the period (12), and bring down the next period for a dividend.

2*nd step.*—As the root consists of but two figures, the second figure must be such, that twice the product of the first and second terms, together with the square of the second, must complete the square [Art. 1. Obs. 10.]; therefore we divide our dividend [396] by twice the figure of the root already found, viz.: 3 [tens]$\times$2= 6 [tens], or 60, and find the second figure of the root to be 6.

3*d step.*—We then add the last figure of the root [6] to our divisor [60], making 60+6=66, (or merely write the 6 in the place of the cipher,) and multiply this by the last figure of the root [6], and subtract the product from the dividend.

By this means we obtain twice the product of the two terms, and the square of the second term; because 60 is twice the first term, which being multiplied by 6 gives twice the product of the two terms, and 6, multiplied by 6, gives the square of the second term. It produces the same result to multiply 66 by 6, because 66=60+6, and it is an established principle in mathematics that *if equals are multiplied by equals, their products will be equal.*

Obs. 17. From examining the first nine digits and their powers, we perceive that *every perfect square must end with* 0, 1, 4, 5, 6, or 9; because *the right hand figure of any root must be one of the nine*

Why do we commence at the left to extract the root? What is the first step in extracting the square root? The second? Third? Fourth? Fifth? If at any time the defective divisor is not contained in the partial dividend, how do we proceed? How do we prove the operation? What is the first step in the operation of Ex. 3? The second? The third? What do we obtain by this method? Explain why this is correct? By what other method can we obtain the same result? Why is this correct? What consideration do we notice from examining the first nine digits and their squares? Why is this correct?

digits, or a cipher; and the square of no digit ends with any other figure than one of the above.

Again—as no digit, multiplied by itself, will produce 0 as the unit figure of the result, if the right hand place of the root is a cipher, the last *period* must be ciphers; that is, if a *perfect power must end with* 1, 4, 5, 6, 9, *or* 00; likewise, *if a perfect power ends with* 00, *the remaining figures must be a perfect power.* This is evident without further demonstration.

Also—since every defective divisor is an even number, (it being twice the root,) the product of this defective divisor, multiplied by 5, must end with a *cipher; therefore, if the root ends with* 5, *the square must end with* 25; because the square of 5 is 25, and the 2 occupies the place of the cipher above mentioned. Hence, *every perfect square must end with* 00, 1, 4, 25, 6, *or* 9, *and if a perfect square ends with* 00, *the remaining figures must also be a perfect square.*

EXERCISES FOR THE SLATE.

1. What is the square root of 502681? Ans. 709.
2. What is the square root of 15876? Ans. 126.
3. What is the square root of 75076? Ans. 274.
4. What is the square root of 18541636? Ans. 4306.
5. What is the square root of 81072016? Ans. 9004.
6. What is the square root of 73960000? Ans. 8600.
7. What is the square root of 174740720400? Ans. 418020.

Obs. 18. *If there is a remainder after the last period has been divided, we may form new periods, by annexing ciphers, and continue the root to decimals, to any degree of exactness.*

The learner must not expect to find an exact root in such cases, however, as the square of no digit ends with a cipher; but we can approximate sufficiently near by the aid of decimals. To prove the operation, he must add the remainder to the square of the root.

8. Required, the square root of 8? Ans. 2.828+.
9. Required, the square root of 590? Ans. 24.289+.
10. Required, the square root of 46780? Ans. 216.286+.

Obs. 19. To extract the square root of decimals—

Commence at the hundredth's place, and point towards the right.

The square of no digit ends with a cipher; what is the inference deduced from this fact? If a perfect square ends with 00, what must the remaining figures be? Why? What is every defective divisor? Why? What inference is deduced from this fact? If the root ends with 5, with what must the square end? Why? With what then must every square end? If there is a remainder after the last period has been divided, how do we proceed? Can we find an exact root in such cases? Why not? How can we approximate sufficiently near? How prove the operation? How do we extraet the square root of decimals?

When the power consists of whole numbers and of decimals:

Point both ways from the unit's place.

REMARK.—The reason why we point decimals from the left towards the right, is evident from the fact, that *every perfect square must contain twice as many decimals as its root.* (Obs. 9. Rem.) Therefore, for every decimal in the root, there must be *two* decimals, or a full period, in the square; and if the periods are not complete, a cipher must be annexed to the decimal. Hence—we must commence at the left to point off decimals; because if we prefix a cipher to the left to complete the period, the value of the decimal is altered, whilst if a cipher is annexed to the right it is not. (Sect. VIII. Art. 7. Obs. 6 and 7.)

11. What is the square root of 356.164? Ans. 18.872+.
12. What is the square root of .0225? Ans. .15.
13. What is the square root of 225.6004? Ans. 15.02.
14. What is the square root of .0000163216? Ans. .00404.
15. What is the square root of 18.211? Ans. 4.26731+.
16. What is the square root of 1.004030076? Ans. 1.002012+.

b s. 20. To extract the square root of a common fraction:

First reduce the fraction to its simplest form; then if both terms are perfect squares, extract the square root of each (Obs. 7.); *but if both terms are not perfect squares, reduce it to a decimal, and then extract the square root, according to* Obs. 19.

17. What is the square root of $\frac{256}{576}$? Ans. $\frac{16}{24}$.
18. What is the square root of $\frac{121}{58564}$? Ans. $\frac{11}{242}$.
19. What is the square root of $\frac{18}{144}$? Ans. .3535+.
20. What is the square root of $\frac{32}{1250}$? Ans. .16.
21. What is the square root of $10\frac{9}{16}$? (Obs. 7. *a.*) Ans. $\frac{13}{4} = 3\frac{1}{4}$.
22. What is the square root of $18\frac{4}{5}$? ($18\frac{4}{5} = 18.8.$) Ans. 4.335+.
23. What is the square root of $200\frac{409}{1024}$? Ans. $14\frac{5}{32}$.
24. If an army of 186624 men were drawn up in a perfect square, how many men would there be on a side? Ans. 432.
25. A certain square field contains 86436 hills of corn. How many are there on a side? Ans. 294.
26. A company of men paid $2304 to a charitable institution, each man paying as many dollars as there were men in the company. How many men were there in the company? Ans. 48.
27. A. has two lots, one measuring 55 rods in length, and 40 rods in width, the other measuring 70 rods in length, and 20 rods

---

When the power consists of both whole numbers and decimals, how do we proceed? Why do we point decimals from the left hand towards the right? How we extract the square root of a common fraction?

in width. B. offers him for these, a square lot containing as many rods as both. How many rods square is B.'s lot? Ans. 60.

28. What is the difference between a square half foot, and half a square foot? Ans. 36 sq. in.

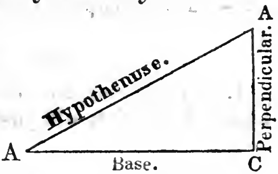
29. If it takes 4356 square tiles to pave a square floor, how many are there on a side? Ans. 66.

30. A nurseryman wishes to set out 1521 fruit trees, so as to form a perfect square. How many must he put in a row? Ans. 39.

Case 2.—Application of the Square Root. Triangles.

Obs. 21. A Triangle *is a figure bounded by three straight lines.*†

Obs. 22. When one line meets another so as to form a *square corner*, or *right angle*, it is called a *Right Angled Triangle*.

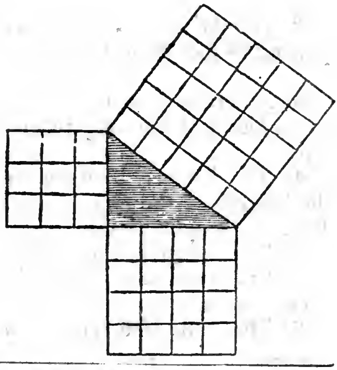


Thus: A B C is a right angled triangle, of which the right angle is B. The side A B is called the *base;* the side B C is called the *perpendicular;* and the side A C is called the *hypothenuse.* The *hypothenuse is always the side opposite the right angle;* the hypothenuse A C is opposite the right angle B.

Obs. 23. *In every right angled triangle, the square described on the hypothenuse is equal to the sum of the squares of the other two sides.*

Thus: if the base is 4 feet, and the perpendicular 3 feet, the hypothenuse is $\sqrt{4^2+3^2}=5$ feet.

This point is also illustrated by the adjoining diagram.



What is a triangle? A right angled triangle? Which side is the hypothenuse? To what is the square of the hypothenuse equal?

† This definition has reference only to rectilinear triangles; a curvilinear triangle is bounded by curved lines.

From this we derive the following considerations :

a. 1st. *If we add the square of the base to the square of the perpendicular, and extract the square root of their sum, the result will be the hypothenuse.*

b. 2nd. *If we subtract the square of the base from the square of the hypothenuse, and extract the square root of the remainder, the result will be the perpendicular.*

c. 3d. *If we subtract the square of the perpendicular from the square of the hypothenuse, and extract the square root of the remainder, the result will be the base.*

REMARK 1.—When the base and perpendicular are equal, the hypothenuse may be found by multiplying the base (or perpendicular) by 1.4142. This is the hypothenuse of such a triangle, the equal sides of which are unity, or 1.

2—The base and perpendicular are sometimes called the *legs* of the triangle

EXERCISES FOR THE SLATE.

1. A room measures 16 feet in length, and 12 feet in width. How long is a line that will just reach from corner to corner?

Operation.

$$\begin{array}{l} 16^2=256 \\ 12^2=144 \\ \hline \quad\quad \dot{4}0\dot{0}(20 \text{ feet. Ans.} \\ \quad\quad 4 \\ \hline \quad\quad 00 \end{array}$$

2. A ladder 30 feet long will just reach the top of a wall by placing its foot 24 feet distant. Required—the heighth of the wall. Ans. 18 feet.

3. A certain pole is 60 feet high. How far from the foot of it will a line 100 feet long reach the ground, by being fastened at the top? Ans, 80 feet.

4. The distance between the foot of two rafters is 32 feet, and the heighth of the ridge above the plate on which the rafters rest is 12 feet. Required—the length of the rafters. Ans. 20 feet.

5. If a square field contains 35 acres, 25 sq. rds., how many rods does it measure on a side? What is the distance from the centre to each corner? Ans. to the last. 53.033+ rds.

6. The walls of a certain building are 32 feet in length, 30 feet

---

How then do we find the hypothenuse when the other two sides are given? How do we find the perpendicular when the other two sides are given? How do we find the base when the other sides are given?

in width, and 24 feet in heighth. Required—the length of a line that will connect the two corners farthest distant from each other.

Ans. 50 feet.

7. Suppose a pole 60 feet long to be so planted between two straight trees, as to reach a limb of one 48 feet high, and without moving the foot, to reach a limb of the other 36 feet high. How far apart are the trees? Ans. 84 feet.

Obs. 24. *A mean proportional* between two numbers is found by *extracting the square root of their product.*

Thus, a mean proportional between 2 and 8 is 4, because $8\times2=16$; and $\sqrt{16}=4$.

PROOF.—2 : 4 : : 4 : 8; $2\times8=4\times4$.

REMARK.—This rule is evident from the fact that a mean proportional forms the two means of a proportion. (Sect. XII. Art. 2. Obs. 7.) But the product of the extremes is equal to the product of the means. (Sect. XII. Art. 2. Obs: 9.) Therefore, the square root of the product of the extremes is the mean proportional.

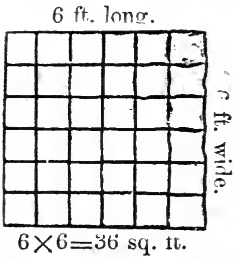
8. What is a mean proportional between 16 and 64?

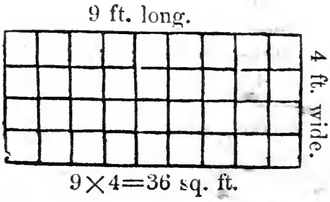
Ans. 32.

9. What is a mean proportional between 68 and 612?

Ans. 204.

To find the side of a square equal in area to a given surface.





We perceive from the diagrams that a square 6 ft. long, and 6 ft. wide, contains $6\times6=36$ sq. ft. We also perceive that a rectangle 9 ft. long, and 4 ft. wide, contains $9\times4=36$ sq. ft.; (Sect IX. Art. 2. Obs. 15.) or a square 6 ft. long and 6 ft. wide, is equal in area to a rectangle 9 ft. long, and 4 ft. wide. But the side of a square is formed by extracting the square root of its contents, or superficial area. (Obs. 14.)

How do we find a mean proportional between two numbers? Show why this rule is correct. How do we find the side of a square equal in area to a given surface? Show why this is correct.

Therefore, *if we extract the square root of the area of a rectangle, we shall have one side of a square of equal area, as the result.* It is also evident, from Obs. 14, that *the square root of the area of any surface is one side of a square surface of equal area.* Hence—

To find the side of a square equal in area to a given surface:

Obs. 25. *Extract the square root of the given area.*

10. A certain field is 48 rods long, and 12 rods wide. Required—the length of one side of a square field containing the same quantity of land? Ans. 24 rds.

11. A parallelogram is 16 ft. long, and 9 ft. wide. What is one side of a square equal in area? Ans. 12 ft.

12. A circular field contains 2025 sq. rds. How many rods on one side of a square field containing the same quantity? Ans. 45.

13. A certain triangular field contains 10 acres, 128 sq. rds. Required—one side of a square field containing the same quantity. Ans. 41.569+ rds.

14. If a certain square field measures 30 rds. on a side, what will be the length of one side of a square field containing 4 times as much? Ans. 60 rods.

Operation.

30
30
———
900 sq. rds. = contents of the given field. (Sect. IX. Art. 2. [Obs. 15.)
4
———
3600 sq. rds. = contents of the required field.
———
$\sqrt{3600}$ = 60 rds., one side of the required field. (Obs. 14.)

15. If the above field measured 28 rods on a side, what is the length of one side of a square field containing 9 times as much? 16 times as much? 25 times as much? 64 times as much? Ans. in order. 84 rds.; 112 rds.; 140 rds.; 224 rds.

16. If a hall measures 40 feet in length, and 10 feet in width, what is the length of one side of a square room 9 times as large? $\frac{1}{4}$ as large? $\frac{9}{16}$ as large? Ans. in order. 60 ft.; 10 ft.; 15 ft.

17. I have 512 sq. rds. of land in a field which is twice as long as it is wide. Required its length and width?

Suggestion.—If the field is divided at the middle of the sides, it

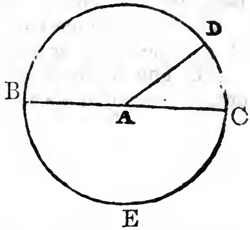
will form *two equal squares*, each containing $512 \div 2 = 256$ square rods.
Ans. Length 32 rds.; width 16 rds.

18. A nurseryman wishes to set out 1280 fruit trees, having the length of his orchard 5 times its width. How many rows must he have, and how many trees in a row?
Ans. 16 rows, and 80 trees in a row.

19. A certain room contains 300 sq. ft., and its width is $\frac{3}{4}$ of its length. Required its length and width?
Ans. Length 20 ft., Width 15 ft.

CIRCLES.

Obs. 26. A CIRCLE *is a portion of space enclosed by a curved line, called the* CIRCUMFERENCE, *every part of which is equally distant from a point within, called the* CENTER.



A straight line passing through the center from one part of the circumference to another is called the DIAMETER.

A straight line from the center to any part of the circumference is called a RADIUS.

Thus, in the above figure, the point A is the Center; the line B E C D is the circumference; the line B A C is the Diameter; and the line A D is the Radius.

REMARK 1. It will be perceived that A B, and A C are *radii*, and that *the radius is half the diameter*. For this reason all radii of the same circle are equal.

2. The learner must not confound the circle with the circumference. The *circle* is the *surface enclosed*, and the *circumference* is the *line* which enclosesf it.

Obs. 27, *The areas of circles, (and also of all other surfaces,) are to each other as the squares of their like dimensions.*

NOTE.—This proposition cannot be demonstrated without a knowledge o geometry.

To find the dimensions of a circle which shall contain 2, 3, 4, $\frac{1}{2}$, $\frac{1}{4}$, &c. times as much as a given circle:

Obs. 28. *Square the diameter, or circumference (whichever is given,) of the given circle, multiply the square by the given proportion.*

What is a circle? What is the circumference? The diameter? The radius? What is the ratio of the radius to the diameter? What is the difference between the circle and the circumference? What relation have the areas of circles to each other? How do we find the dimensions of a circle which shall contain 2, 3, 4, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., times as much as a given circle?

and extract the square root of the product; the result will be the similar dimensions of the required circle.

20. A circle measures 6 inches in diameter. Required the diameter of one 4 times as large? Ans. 12 inches.

21. The circumference of a circle is 8 feet. Required the circumference of one 9 times as large? Ans. 24 feet.

22. The circumference of a circle is 15 feet. Required the circumference of one 27 times as large? Ans 75 feet.

23. A gentleman has two circular ponds on his farm; one of which is 16 yards in diameter, and the other is 16 times as large. Required the diameter of the larger? Ans. 64 yds.

24. The diameter of a circle is 16 feet. Required the diameter of a circle but $\frac{1}{4}$ as large? Ans. 8 feet.

25. The circumference of a circle is 64 feet. Required the circumference of a circle but $\frac{1}{16}$ as large? Ans. 16 feet.

SECTION XVI.

MENSURATION.

Def. Mensuration *teaches the art of finding the area of surfaces or the contents of solids. The area of a surface is the space enclosed by its boundaries.* (Sect. IX, Art. 2, Obs. 14.)

Article 1. Mensuration of Surfaces.

Obs. 1. When we speak of a *surface* we mean the *face* of any thing. A Surface, therefore has *length* and *breadth*, but not *thickness.*

Case 1. *To find the area of a Square, Rectangle, &c.*

Obs. 2. *Multiply together its length and breadth.* (Sect. IX, Art. 2, Obs. 15.)

Remark. The area of a surface is expressed by *Square Measure;* as, square feet, square rods, square miles, &c.

Ex. 1. How many square feet in the floor of a room 22 ft. long and 16 ft. wide? Ans. 352.

What is Mensuration? What is the area of a surface? When we speak of a surface what do we mean? What then has a surface? How do we find the area of a square, rectangle, &c.

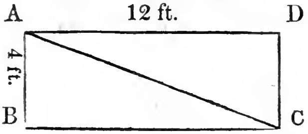
2 A square field measures 34 rds. on each side. Required, its contents. Ans. 1156 sq. rds. = 7 A. 36 sq. rds.

3. A certain field is 125 rods long and 65 rds. wide. Required, its contents, it being in the form of a rectangle.

Ans. 50 A. 125 sq. rds.

CASE 2. *To find the area of a Triangle.*

1. How many square feet in a board 12 ft. long, and 4 ft. wide at one end, and tapering to a point at the other? Ans. 24.



The surface of the board would evidently describe a triangle (which we will suppose to be right-angled,) like the triangle A B C in the diagram. Now by drawing the lines A D and D C, parallel to B C and A B, we form another equal and similar triangel A D C, and also the rectangle A B C D. The length and width of this rectangle are the same as the base and perpendicular of the triangle A B C; that is 12 feet long, and 4 feet wide; and its area is $12 \times 4 = 48$ sq. ft. (Obs 2.) Therefore, as the rectangle is composed of two equal triangles, the area of each must be $48 \div 2 = 24$ sq. ft. Hence—

To find the area of a right angled triangle:

Obs. 3. *Multiply together the base and perpendicular, and take half their product.*

REMARK 1. *Parallel lines* are those which are every where equally distant from each other.

2. From the above demonstration we perceive that *every Triangle is half a Rectangle havingthe same base and altitude.*

2. What is the area of a triangle the base of which is 40 feet, and the perpendicular 24 feet? Ans. 480 sq. ft.

3. How many acres in a triangular field the side or base of which is 124 rods, and the end or perpendicular, 40 rods?

Ans. 15 A. 80 sq. rds.

4. How many acres in a triangular field, the base of which is 150 rods, and the perpendicular 50 rds?

Ans. 28 A. 20 squa;e rods.

When the triangle is not a right angled triangle, and we have the three sides given we proceed as follows:

How is the area of a surface expressed? How do we find the area of a right angled triangle? What are parallel lines? What is every triangle?

Obs. 4. *Add together the three sides and take half their sum.*

From the half sum take the three sides severally:

Finally, multiply together the half sum and the three remainders, and extract the square root of the product.

REMARK. When the perpendicular height is given we work by Obs. 3.

NOTE.—The demonstration of this rule cannot be understood without a knowledge of geometry.

5. Required the area of a triangle the sides of which are 6, 7 and 8 feet, respectively? Ans. 20.333 + sq. ft.

6. Required the area of a triangle the sides of which are 24, 30, and 36 feet respectively?

Ans. 357.176 + sq. ft.

7. Required the area of a triangle the sides of which are 26, 28, and 30 feet, respectively? Ans. 336 sq. ft.

8. How many sq. yds. of plastering are there in a triangle whose sides are 30, 40, and 50 feet, respectively? Ans. $66\frac{2}{3}$.

CASE 3. *To find the area of a Trapezoid.*

Obs. 5. *A figure having two sides parallel, and the other two not is called a Trapezoid.*

1. A man has a board in the form of a trapezoid; the length of one side is 60 inches, and of the other side 48 inches, and its width is 24 inches. Required its area? Ans. 9 sq. ft.

Operation
$60\times48=108$ inches=the length of both sides.
$108\div2=54$ inches=the mean, or average length. (Sect. IX, Art. 2, Obs. 24.)
$54\times24=1296$ sq. in.=the area of the surface of the board. (Obs. 2.)
$1296\div144=9$ sq. ft. Hence—

To find the area of a trapezoid:

Obs. 6. *Multiply half the sum of the parallel sides by the width.*

2. Required the area of a trapezoid whose parallel sides are 64 and 57 feet, and the width, 39 feet. Ans. $2359\frac{1}{2}$ sq. ft.

3. A farmer has a field in the form of a trapezoid: the parallel sides measure 40 and 50 rods, and the width measures 20 rods.—How many acres does it contain? Ans. 5 A. 100 sq. rds.

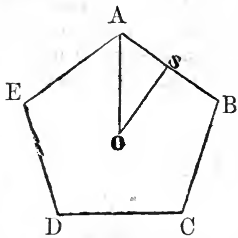
CASE 4. *Circles and Polygons.*

When the triangle is not right angled how do we find its area? What is a Trapezoid? How do we find the area of a Trapezoid?

Obs. 7. A Polygon *is a figure bounded by straight lines. A Triangle* is a polygon ; also, a *square.*

Problem 1. To find the area of a regular polygon :

Obs. 8. *Multiply the perimeter by half the perpendicular, let fall from the centre to the middle of one side.*



Demonstration.—In the diagram we draw the right angled triangle O S A within the polygon A B C D E. In the same manner the entire polygon may be divided into right angled triangles. But the area of the triangle A S O is found by multiplying the base A S by half the the perpendicular S O; (Obs. 3.) and the area of the other triangles is found by multiplping their bases by half of an equal perpendicular. Therefore, since the sum of their bases forms the perimeter of the polygon, the rule is evident.

Remark 1. *A Regular Polygon* is one in which each corner is equally distant from the center. The middle point of each side is also equally distant from the center, and the sides are all of equal length.

2. A polygon of five sides is called a *Pentagon*; of six sides, a *Hexagon;* of seven sides, a *Heptagon;* of eight sides, an *Octagon;* of nine sides, a *Nonagon;* and of ten sides a *Decagon.*

1. How many square feet in the floor of a school house in the form of an octagon, each side measuring 10 feet, and the line from the center to the middle of one side measuring 12 feet?

Ans. 480 sq. ft.

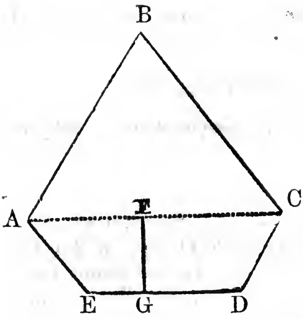
2. A man has a field in the form of a hexagon, each side of which is 60 rods, and the distance from the center to each corner is 60 rods. Required its area? Ans. 58 A. 8 sq. rds., nearly.

Problem 2. To find the area of irregular Polygons :

Obs. 9. *Draw diagonals dividing the polygon into triangles, rectangles, &c. Then find the area of each of these, separately, and add these several areas together.*

Remark. A Diagonal is a line joining two angles not adjacent to each other.

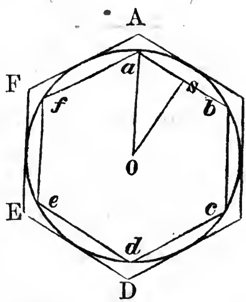
What is a Polygon? Give examples. How do we find the area of regular polygon? Demonstrate this rule? What is a regular polygon? What is a polygon of five sides called? Of six sides? Of seven sides? Of eight sides? Of nine sides? Of ten sides? How do we find the area of an irregular polygon?



3: There is a certain field in the form of the adjoining diagram. The side A B measures 26 rds.; the side B C measures 28 rds.; the side C D measures 12 rds.; the side D E measures 20 rds.; and the side E A mea-15 rds.; diagonal A C measures 30 rds.; and the line F G measures 10 rds. How many acres in the field?

Ans. 3 A. 106 sq. rds.

Problem 3. Circumscribed and Inscribed Polygons.



Obs. 10. When a Polygon is drawn about a circle so that the middle point of each side touches the circumference, it is called a *Circumscribed Polygon.*

When a polygon is drawn within a circle so that each corner shall touch the circumference, it is called an *Inscribed Polygon.*

Thus, in the diagram, A B C D E F is a circumscribed polygon, and *a b c d e f* is an inscribed polygon. Also, in the same diagram, the circle is said to be inscribed within the polygon A B C D E F, and to be circumscribed about the polygon *a b c d e f.*

Remark 1. The term *Circumscribed* is derived from two Latin words, *Circum, about,* and *Scribo, to write,* or *draw,* The term *Inscribed* is derived from the same word *Scribo,* and *In, within.*

2. The lines which bound the polygon are termed the *Perimeter of the polygon.*

If in the above diagram we *bisect,* or divide into two equal parts each side of each polygon it is evident that each polygon will approach nearer the circumference, and consequently nearer each other. Therefore, if the sides of each polygon be bisected indefinate-

---

What is a diagonal? What is a circumscribed Polygon? An Inscribed Polygon? What is said respecting the circle in the diagram? From what is the term circumscribed derived? What does it mean? From what is the term inscribed derived? What does it mean? What is the Perimeter of a Polygon? What effect does it have upon the perimeter of a circumscribed and inscribed polygon, to bisect each side indefinately? Where will the perimeters meet? Why?

ly, the perimeters of the two polygons will meet. But as the *inscribed polygon* cannot fall without the circumference, (for then it would be a circumscribed one,) and as the *circumscribed polygon* cannot fall within the circumference, (for then it would be an inscribed one,) they must meet upon the circumference. Hence—

Obs. 11. *A Circle may be defined as a polygon with an indefinite number of sides.*

REMARK. This definition is not strictly correct, as the side of a polygon, and the circumference of a circle can never coincide so as to form one and the same line. It is sufficiently correct, however, for all practical purposes.

PROBLEM 4. The Quadrature of the Circle.

The problem is not only difficult, but impossible to solve exactly, although we can approximate very nearly. What is meant by it, is, to determine the area of a circle, the diameter of which is equal to the side of a given square; that is, to find the area of a circle inscribed in a square.

The first step is to find the *ratio of the circumferance to the diameter*. The process used to obtain this is too complicated for a treatise on Arithmetic; therefore we will merely tell how it is done, and take the result as correct without further trouble. The learner must, however, bear in mind, that the *exact ratio* can never be ascertained, neither the area of the circle, and all good mathematicians have long since abandoned both as impossible.

The process used, is, to inscribe and circumscribe a circle with regular polygons, and then to bisect these until the sides of the two polygons and the circumference of the circle all seem to unite in one common line. They do not exactly coincide, (Obs. 11. Rem.) but come sufficiently near to render accuracy almost absolutely certain. The circumference of the circle must always be between the perimeters of the two polygons, and since the ratio of the diameters to the perimeters of the polygons can be ascertained, the ratio of the diameter to the circumference of the circle must lie between these two ratios.

ARCHIMEDES a *Grecian*, first commenced the investigation of this problem, and by increasing the number of sides of the polygon to 32768, he found the ratio to be between $3\frac{10}{71}$ and $3\frac{10}{70}$. $3\frac{10}{70} = \frac{22}{7}$, or the ratio of the circumference to the diameter is as 22 to 7. METIUS

How then may a circle be defined? Is this strictly correct? Why not? What is meant by the quadrature of the circle? Is this problem capable of being solved exactly? What is the first step in solving it approximately?— What is the process used to find this? Can the perimeters of the polygons, and the circumference of the circle ever coincide? Where does the circumference of the circle lie? Where then must the ratio of the diameter to the circumference be? Why?

a *German,* afterwards found the ratio to be as 355 to 113. Van Ceulen, a Dutch mathematician, carried the process much farther, and found that if the diameter was 1, the circumference would be less than 3.14159265358979323846264338327950289, and greater than 3.14159265358979323846264338327950288. Later mathematicians have carried the process still farther, and have ascertained the ratio of the diameter to the circumference to be as 1 to 3.-14159265358979323846264338327950288419716939937570582097494459230781640628620899862803482534211706798214808651328230664709384464609550518223172535940812848802.

The ratio then may safely be put down as 3.141592. This gives the exact ratio to 5 decimal places, and making the error as small as $\frac{1}{1000000}$. For all practical purposes, 3.1416 may be used, changing the 9 in the fifth order, into 6 in the fourth order of decimals. Hence—

To find the circumference of a circle when the diameter is given:

Obs. 12. *Multiply the diameter by* 3.1416; *or, for greater accuracy, by* 3.141592.

1. A certain wheel is 36 in. in diameter. Required its circumference. Ans. 9 ft. 5.0976 in.

2. If the diameter of a circle is 15 ft., what is the circumference? Ans. 47.124 ft.

3. If the earth is 7912 miles in diameter, what is its circumference? Ans. 24856.3392.

To find the diameter of a circle when the circumference is given.

Obs. 13. *Divide the circumference by* 3.1416. *Or, Multiply the circumference by* .31831.

REMARK. Since the circumference is 1 multiplied by 3.1416, the diameter is 1 divided by 3.1416. 1 divided by 3.1416 equal .31831.

4. If the circumference of a circle is 37.6992 ft., what is the diameter? Ans. 12 ft.

5. If the circumference of a circle is 60 ft., what is its diameter? Ans. 19.0985 ft.

6. If the circumference of a circle is 42 ft., 6 in., what is its diameter? Ans. 13 ft. 6.3376 in.

7. If a tree measures 12 ft. 9 in. around, what is the distance though it. Ans. 4.0584 ft.

To what may this ratio be safely put down? How small is the error in this case? What number is used for practical purposes. How then do we find the circumference of a circle when the diameter is given? How do we find the diameter when the circumference is given?

PROBLEM 5. The area of the Circle.

In the diagram (Obs. 8.) we have shown the right-angled triangle *a.* S O, the area of which is equal to one-half the perpendicular (O S) multiplied by the base *a* S. (Obs. 3). The whole polygon may be separated in triangles in the same manner, and the area of each is found by the same process. Now if the sides of the polygon are bisected until they are indefinitely increased, its perimeter will be the circumference of the circle, and the perpendicular O S will be the radius of the circle. (Sect. XV, Art. 2, Obs. 26.) Hence—

Obs. 14. *The area of a circle is found by multiplyiug the circumference by half the radius.**

1. If a circle is 15 ft. in diameter, and 47.124 feet in circumference how many square feet does it contain? Ans. 176.715.

The learner will remember that the radius is half the diameter Sect. XV, Art. 2, Obs. 26, Rem. 1.)

2. If the diameter of a circle is 25 ft., and the circumference 78.54 ft., what is its area? Ans. 490 sq. ft., 126 sq. in.

As the radius is half the diameter, half the radius is $\frac{1}{4}$ of the diameter, the area of a circle is equal to the product of the circumference into $\frac{1}{4}$ the diameter; or which is the same thing $\frac{1}{4}$ of the product of the circumference and diameter. But the circumference is equal to the diameter multiplied by 3.1416; therefore, (Obs. 12.) the circumference multiplied by the diameter is the same as 3.1416 times the diameter into itself, or 3.1416 times the square of the diameter; and the area of the circle is $\frac{1}{4}$ of this, or $3.1416 \div 4 =$ .7854 times the square of the diameter. Hence—

a. The area of a circle may be found by multiplying the square of the diameter by .7854.

REMARK. The learner will observe from this that when the diameter of the circle is 1, the area is .7854.

How do we find the area of the circle when the circumference and diameter are both given? Demonstrate this rule. When the diameter only is given how do we find the area? Demonstrate this rule. When the circumference only is given how do we find the area? Demonstrate this rule? When we have given the area how do we find the dimensions?

---

*In this case as the circumference is the perimeter of a polygon with an indefinite number of sides, the circle is composed of an indefinite number of triangles, of which the area of each is found by Obs. 3, and the sum of all these areas is equal to the area of the circle. (Sect. IV, Art. 4, Obs. 4, Rem. 2). Or, as the perpendiculars to all these triangles are equal, the sum of their bases, (or circumferences) multiplied by half the perpendicular (or radius) must give the area.

3. Required—the area of a circle the diameter of which is 50 ft.
Ans. $1963\frac{1}{2}$ sq. ft.

When the circumference is 1, the diameter is .31831, (Obs. 13, Rem.) and the area $.31831^2 \times .7854$. But $.31831^2 \times .7854 = .08$ nearly. Hence—

b. The area of a circle may be found by multiplying the square of the circumference by .08; or, for greater acuracy by .0795.

4. If the circumference of a circle is 15 ft. what is its area?
Ans. 18 sq. ft.

5. If the circumferance is 28 ft. what is the area?
Ans. 62.72 sq. ft.

6. The area of a circle is 176.715 sq. ft., what is its diameter and circumferance?
Ans. Dia. 15 ft., Cir. 47.124 ft.

Operation $\begin{cases} 176.715 \div .7854 = 225; \sqrt{225}, = 15 \text{ ft. diameter.} \\ 15 \times 3.1416 = 47.124 \text{ ft. circumference.} \end{cases}$

Remark. This question is exactly the reverse of those under Obs. 14, *a*. Therefore, the rule is evident. Hence—

When we have the area of a circle given to find the dimensions:

Obs. 15. *Divide the area by .7854, and extract the square root of the quotient for the diameter. The circumference is then found by Obs.* 12.

7. A square field contains 490.875 sq. rds. Required—the diameter of a circular field containing the same quantity. The circumference.
Ans. Dia. 25 rds., Circle 78.54 rds.

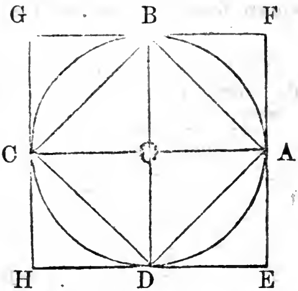
8. How long must a halter be, which being fastened to a post at the center, will just allow a horse to feed on half an acre of ground?
Ans. 83 ft. 3.1476 in.

Problem 6. To find the Area of a Square inscribed in a Circle.

The rule is deduced from the following proposition:

1. *The area of a square inscribed within a circle is one-half the area of a square circumscribed about the same circle.*

From what principle is the Rule for finding the area of a square inscribed in a circle deduced? Demonstrate this principle. To what is the area of an inscribed square equal?



Demonstration.—Let A B C D be the inscribed, and E F G H the circumscribed square. It will be perceived that the angles of the inscribed square are located at the middle of the sides of the circumscribed square. Draw the diagonals B O D and C O A. These divide the circumscribed square into four equal smaller squares. The sides of the inscribed square divide each of these smaller squares into two equal and right angled triangles. (Obs. 3, Rem.) Therefore, since both of these triangles are included in the circumscribed square, and but one of them in the inscribed square, the circumscribed square contains eight triangles, and the inscribed square but four—hence, the proposition is correct.

Now the side of the circumscribed square is equal to the diameter of the circle, and its area is the square of this diameter.—(Obs. 2.) Hence—

Obs. 16. *The area of a square inscribed within a circle is one half the square of the diameter of the same circle.*

a. The area of an inscribed square is also equal to the diameter of the circumscribed circle multiplied into one half of itself.

This is evident from an examination of the above diagram. We perceive that the inscribed square is composed of the two equal triangles, A B C and A D C. (Obs. 3, Rem. 2.) Now the area of each of these triangles is equal to the hypothenuse A C, multiplied by half the perpendicular height B O, or O D, (Obs. 3.) But $\frac{1}{2}$ of B O + $\frac{1}{2}$ of O D is equal to either B O or O D, since B O and O D are equal. Hence—the area of the inscribed square is equal to A C (the diameter of the circumscribed circle) multiplied by B O, or O D, (half this diameter.)

1. Required— the area of a square inscribed in a circle 12 feet in diameter: Ans. 72 sq. ft.

Solution.—$12 \times 12 = 144$; $144 \div 2 = 72$. Or, $12 \times 6 = 72$.

2. Required—the cost of a log when squared, at \$0.40 per

To what else is it equal? Why? How do we find the side of an inscribed square? Why is this correct? When the diameter of the circle only is given how do we find the side of the inscribed square? Demonstrrate this rule. When the circumference only is given how do we find the area? Demonstrate this rule? To what may this principle be applied?

square foot, which measures 3 ft. in diameter, and 22 ft. in length.
Ans. \$39.60.

Suggestion.—Find the size which the log will square, and multiply this by the length for the number of sq. ft.

3. How much would a log cost at \$4.50 per 100 ft., which is $4\frac{1}{2}$ ft. in diameter, and 48 ft. in length? Ans. \$21.87.

2. How much would a log cost which is $4\frac{1}{2}$ ft. in diameter and 64 ft. long, at \$50 per M.? Ans. \$32.40.

5. How much would a log cost which is $5\frac{1}{4}$ ft. in dia., and 80 ft. long at \$75 per M.? Ans. $\$82.68\frac{3}{4}$.

PROBLEM—To find the side of a Square inscribed in a Circle,

The side of a square is equal to the square root of its are a (Sect. XV, Art. 2, Obs. 25). Hence—

To find the side of an inscribed square :

Obs. 17. *Extract the square root of its area found according to Obs.* 16.

From examining the diagram for illustrating the preceding case, we perceive that the inscribed square consists of four equal right-angled triangles, and that each of these triangles has two of its sides equal, since these sides are radii of the same circle (Sect. XV, Art. 2, Obs. 26, Rem. 1). The hypothenuse of these triangles then is equal to one of these sides (or radii) multiplied by 1.4142. (Sect. XV, Art. 2, Obs. 23, Rem 1). But this hypothenuse is the side of the inscribed square, and twice the other side of the triangle, (radius) is the diameter of the circumscribed circle. (Sect. XV, Art. 2, Obs. 26, Rem. 1). Therefore, this diameter multiplied by .7071 (1.4142 ÷ 2) must be the side of the inscribed square.—(Sect. VI, Art 1, Obs. 30.)

Again, when the circumference is 1 the diameter is .31831, (Obs. 13, Rem) and when the diameter is 1 the side of the inscribed square is .7071; therefore when the circumferance is 1, the side of the inscribed square is $.31831 \times .7071 = .2251$ nearly. Hence—

To find the side of an inscribed square :

a. Multiply the diameter by .7071, *or the circumference by* .2251.

1. How large a square can be hewn from a log 2 ft. in diameter.

How many dimensions are used in measuring lumber? What are they?—What is the thickness considered? How is lumber more than an inch in thickness counted? When the waste by sawing is considered how do we proceed? When the lumber is less than an inch in thickness how do we proceed? How do we find the dimensions of a circumscribed circle when the side of the inscribed square is given? Demonstrate this rule.

Solution.—$2^2=4$; $4\div 2=2$; $\sqrt{2}=1.4142$. Or, $.7071\times 2=$ 1.4142 as before. Ans. 1.4142 ft.

2. How many inches will a log square that is 120 inches circumference? Ans. 27.012 in.

Solution.—$120\times .2251=27.0120$.

3. I wish a sill 18 inches square, and find a tree 25 in. in diameter. Will it answer my purpose or not?

Ans. It will not as it will square but $17.67\frac{3}{4}$ in.

4. I find another tree measuring 80 in. in circumference. Will this answer my purpose? Ans. It will, as it squares 18.008 in.

This principle may be applied to finding the quantity of lumber a log will make.

5. A log measures 24 ft. in length, and 15 inches in diameter how many feet of inch lumber will it make, allowing no waste for sawing?

Solution.—$15\times .7071=10.6065$ inches that the log will square. Then 24 ft. = 288 in.; $10.6065^2=$ area of the end, and $10.6065^2\times 288=$ square contents in inches, which divided by $144=214.389+$ sq. ft. Ans.

The operation may be shortened somewhat by canceling.

REMARK 1. In measuring lumber but two dimensions are generally used, viz: length and width. The thickness is considered a unit 1 inch being the standard. Lumber more than an inch in thickness, is counted an additional thickness for every additional inch. When the waste by sawing is considered, it should be deducted from the result.

6. A log measures 25 feet in length, and 12 inches in diameter. How much two inch lumber will it make allowing it to lose $\frac{1}{4}$ of an inch in sawing? Ans. 132.3225 sq. ft.

Solution.—We find the contents of the log, as in the last example, to be 150 sq. ft. nearly. The log squares 8.4852 in., and as the lumber is 2 inches thick, the saw passes through four times, which makes 1 inch waste. This 1 inch measures 8.4852 in. in width, and 300 in, (25 ft.) in length which makes 17.6775 sq. ft. loss by sawing, and $150-17.6775=132.3225$ sq. ft of lumber.

REMARK 2. Had the width of the lumber been given, and the log been sawed both ways across the end, an additional waste would have accrued from sawing which would have been deducted. When the lumber is less than an inch in thickness the calculations for the quantity are made by the inch as usual, although the allowance for waste in sawing must be made according to the thickness of the lumber sawed.

7. A log measuring 2 ft. 8 in. in diameter and 24 ft. in length, is to be sawed into lumber, 6 inches wide, and 1 inch thick. Required the cost of this lumber at $1.75 per C., the waste being $\frac{5}{16}$ of an inch for sawing. Ans. $5.17, nearly.

8. Twelve logs, the average diameter of which is 2 ft. 6 in., and the lengths, 12, 15, 10, 22. 14, 16, 20 18, 15, 14, 24 and 20 ft. respectively are to be sawed into lumber $1\frac{1}{2}$ inches in thickness, and 6 inches in width. Required the cost of the lumber at $22.50 per M, allowing $\frac{3}{16}$ of an inch waste for sawing.

Ans. $142.392+.

9. How much half inch lumber, 4 inches in width, can be sawed from a log 3 ft. in diameter, and 25 ft. long, allowing $\frac{1}{4}$ of an inch waste for sawing? Ans. 589.488+sq. ft.

PROBLEM 8. To find the Circumference or Diameter of the Circumscribed Circle when the side of the Inscribed Square is given.

By examining the diagram for finding the area of the inscribed square, we perceive that the diagonal of this square, is the hypothenuse of a right-angled and equal sided triangle, the legs of which are the sides of the inscribed square. But this diagonal is the diameter of the circumscribed circle, and is equal to either of the sides of the inscribed square multiplied by 1.4142. (Sect. XV. Art. 2, Obs. 23, Rem. 1).

Again, the circumference is equal to the diameter multiplied by 3.1416. (Obs. 12.) But the diameter is equal to the side of the inscribed square multiplied by 1.4142. therefore the circ. is equal to the side of the inscribed square $\times$ 1.4142 $\times$ 3.1416. 1.4142 $\times$ 3.1416 $=$ 4.443 nearly; therefore the circumference is equal to the side of the inscribed square multiplied by 4.443. Hence—

To find the dimensions of circle when the side of an inscribed square is given:

Obs. 18. *Multiply the side of the inscribed square by* 1.4142 *for the diameter, or, by* 4.443 *for the circumference.*

1. How large must a tree be in diameter and circumference to square 10 inches?

Solution.—10 $\times$ 1.4142 $=$ 14.142 in. dia. and 10 $\times$ 4.443 $=$ 44.43 in. Circ. Ans.

How do we find the side of a square of equal area to a given circle? Demonstrate this rule. How do we find the dimensions of a circle of equal area to a given square? Demonstrate this rule.

2. I wish a stick of timber 12 inches square, and find a tree 16 inches in diameter. Will it answer my purpose or not.

Ans. It will not, as to square 12 in. it must be 16.9704 inches in diameter.

3. How large must a tree be in circumference to square 12 inches?
Ans. 53.316 inches.

4. How large must a tree be in diameter to square 15 inches?
Ans. 21.213 inches.

We learn (Obs. 14. *a.* Rem.) that if the diameter of a circle is 1, the area is .7854; therefore, the side of a square of equal area is $\sqrt{.7854} = .8862$; and $.8862 \div 3.1416 = .2821$ nearly. Hence—

To find the side of a square equal in area to a given circle:

Obs. 19. *Multiply the diameter of the circle by* .8862; *or the circumference by* .2821.

5. A circle is 12 ft. in diameter. Required the side of a square of equal area. Ans. 10.6344 ft.

6. A circle is 40 ft. in circumference. Required—the side of a square of equal area. Ans. 11.284 ft.

Since the diameter of a circle multiplied by .8862, or the circumference multiplied by .2821, will give the side of a square of equal area, it is evident that the side of a square divided by .8862 will give the diameter, or by .2821 will give the circumference of a circle of equal area. Then assuming unity, or 1 as the standard, $1 \div .8862 = 1.128$ as the diameter, and $1 \div .2821 = 3.545$ nearly, as the circumference of a circle, the area of which is 1, or equal to the area of a square the side of which is 1. Hence—

To find the dimensions of a circle, the area of which shall be equal to the area of a given square.

Obs. 20. *Multiply the side of a given square by* 1.128 *for the diameter; or, by* 3.545 *for the circumference.*

7. The side of a square measures 150 feet. Required—the diameter and circumference of a circle of equal area.
Ans. Dia. 169.2 ft. Circ. 531.75 ft.

ARTICLE 2. *Mensuration of Solids.*

Obs. 1. In Mensuration of solids, two things are to be considered:

1st. *The Mensuration of their surfaces;* and

How many things are to be considered in the mensuration of solids?

2nd. *The mensuration of their Solidities.*

A Solid has *length, breadth,* and thickness.

Remark. It is an established theorem in Geometry, that *all solid bodies are to each other as the cubes of their like dimensions.*

1. If a ball weighing 6 lbs., be 4 inches in diameter, what is the diameter of a ball of the same metal weighing 48 lbs.

Ans. 8 inches.

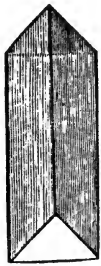
Operation.—6 lbs. : 48 lbs. : : 64 (4^3) : $512 = \sqrt{512} = 8$.

2. If a ball 8 inches in diameter weighs 48 lbs., what is the weight of a ball 4 inches in diameter? Ans. 6 lbs.

3. If a cubical vessel is 8 inches in length, what is the side of a cubical vessel that shall contain 27 times as much?

Ans. 24 inches.

Case 1. *Prisms.*



Obs. 2. A Prism *is a solid having its ends, or bases equal and parallel.* It is said to be *Triangular, Quadrangular, &c.,* according as its sides are triangles, squares, &c.

To find the area of the surface of a prism :

Obs. 3. *Multiply the perimeter of the base by its altitude, or heighth, and to the product add the area of the bases when the entire surface is required.*

Demonstration.—The sides of the prism are parallelograms, (Obs. 2.) and the area of each of these parallelograms is found by multiplying together its length and width ; (Art. 1, Obs. 2). But as their lengths are equal, their area is equal to the sum of their width, or bases multiplied by their length, or altitnde; hence the above rule is evident

Ex. 1. What is the entire surface of a triangular prism, whose sides are 3, 4, and 5 feet, and whose altitude is 6 feet?

Ans. 84 sq. ft.

What are they? What has a solid? What relations have solid bodies to each other? What is a prism? When is a prism triangular, quadrangular, &c.? How do we find the surface of a prism? Demonstrate this rule?

2. What is the entire surface of a triangular prism the sides of which are 8, 10, and 12 feet, and its altitude 9 feet?

Ans. 349.36+sq. ft.

3. What is the convex surface of a quadrangular prism, each side of which is 5 feet, and its altitude 6 feet? Ans. 120 sq. ft.

To find the solid contents of a prism :

Obs. 4. *Multiply the area of its base by its altitude, or heighth.*

Demonstration.—The area of the base comprises the two dimensions, breadth and thickness, and is expressed by square measure. (Art 1. Obs. 2. Rem.) But the altitude is the length of the prism; and square measure multiplied by linear measure gives solid or cubic measure. (Sect. XIV. Rem. 1., under the Rule.) Hence, the rule is evident.

4. Required, the solid contents of a triangular prism, whose length is 12 inches, and each side of its base 3 inches?

Operation.
- $3+3+3=9$; $9\div2=4.5=$ the half sum of the sides of the base.
- $4.5-3=1.5$; the three remainders are equal; since the sides are equal.
- $4.5\times1.5\times1.5\times1.5=15.1875$; $\sqrt{15.1875}=$ area of base. (Art. 1. Obs.)
- $\sqrt{15.1875}\times12=$ contents of the prism. $12=\sqrt{144}$.
- $\sqrt{15.1875}\times\sqrt{144}=\sqrt{2187}=46.7653+$ sq. in. Ans.

5. Required, the solid contents of a quadrangular prism 36 inches in length, and each side of its base 8 inches?

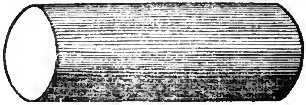
Ans. 1 s. ft. 576 s. in.

6. Required, the solid contents of a triangular prism, whose sides are 4, 6, and 8 feet, and its length 15 feet?

Ans. 174.2812+ s. ft.

Case 2.—Cylinders.

Obs. 5. *A round prism having equal circles for its ends, is called a* Cylinder.



To find the surface of a Cylinder :

Obs. 6. *Multiply the circumference of its end by its length.*

How do we find the solidity of a prism? Demonstrate this Rule. What is a Cylinder? How do we find the surface of a Cylinder?

Demonstration.—The surface of any prism whose base is a polygon, is found by multiplying the perimeter of its base by its altitude. (Obs. 3.) Now if the sides of a base are bi-ected indefinitely, the perimeter will become a circle, and therefore the prism will become a cylinder. Hence, the rule is evident from Art. 1. Obs. 9.

REMARK.—The above rule only gives the *convex* surface. When the *entire* surface is required, the area of the *bases* must be added to the result.

1. A cylinder measures 6 inches in circumference, and 18 inches in length. Required, its convex surface. Ans. 108 sq. in.
2. A cylinder is 60 inches in diameter, and 15 feet in length. Required, its entire surface. Ans. 274.89 sq. ft.

To find the solidity of a cylinder:

Obs. 7. *Multiply the area of its base by its length.*

Demonstration.—The solidity of any prism whose base is a polygon, is found by multiplying the area of its base by its altitude. (Obs. 4.) Now if the sides of the prism are bisected indefinitely, the prism will become a cylinder. (Art. 1. Obs. 9.) Hence, the rule is evident.

3. Required, the solid contents of a cylinder 12 feet in length, and 5 feet in diameter? Ans. 235.62 s. ft.
4. How many solid inches in a common, or Winchester bushel, the diameter of which is is $18\frac{1}{2}$ inches, and the depth 8 inches. Ans. 2150.4252 s. in.

Case 3.—CONTENTS OF BOILERS.

Obs. 8. A BOILER *may be regarded as a cylinder, with several smaller cylinders (flues) within it.*

A cubic foot contains 1728 solid inches; a wine gallon contains 231 cubic inches, and a beer gallon contains 282 cubic inches. Therefore a solid foot contains $1728 \div 231 = 7.48$ wine gallons; and $1728 \div 282 = 6.127$ beer gallons. $1728 \times .7854 = 1357.1712$ cubic inches in a cylindric foot; and $1357.1712 \div 231 = 5.875$ wine gallons, and $1357.1712 \div 282 = 4.812$ beer gallons in a cylindric foot. $5.875 = \frac{5875}{1000} = \frac{47}{8}$; and $4.812 = \frac{4812}{1000} = \frac{24}{5}$. Hence—

To find the contents of Boilers:

Obs. 9. *Mutliply the square of the diameter by the length and*

Demonstrate this rule. What surface does this rule give? When the entire surface is required, how do we proceed? How do we find the solidity of a cylinder? Demonstrate this rule. As what may a boiler be regarded? How do we find the contents of a boiler? Demonstrate this rule.

47, and divide the result by 8. The result will be in wine gallons. For beer gallons use 24 and 5, instead of 47 and 8. Find the contents of the flues in the same way, which subtract from the entire contents.

REMARK 1.—The dimensions must be taken in feet by this Rule

1. How many wine gallons in a boiler 30 inches in diameter, and 25 feet long, having two flues, each 9 inches in diameter?

Operation.

30 in.$=2\frac{1}{2}=\frac{5}{2}$ ft.; $\overline{(\frac{5}{2})^2\times25\times47}\div8=917.96875$ entire contents.
9 in.$=\frac{3}{4}$ ft.; $\overline{(\frac{3}{4})^2\times25\times47\times2}\div8\;=165.234375$ flues.

Ans. 752.734375 gallons.

We multiply by 2 because there are are 2 flues.

2. Required, the contents, in wine and beer gallons, of a boiler 36 in. in diameter, and 45 ft. long, having 3 flues, each 8 inches in diameter? Ans. 2026.875 wine gallons; 1656 beer gallons.

REMARK 2.---The contents of circular cisterns may also be found by the above rule. When the cistern is square, rectangular, &c., multiply its solid contents by 7.48 for wine gallons, and 6.127 for beer gallons. In this case we do not use the numbers 47 and 8, or 24 and 5. Notice that the dimensions must all be taken in feet.

NOTE.—The wine measure of 231 cubic inches to the gallon, is generally used as the standard in the United States, and is so understood in the following examples, unless otherwise mentioned.

3. Required, the contents, in wine and beer gallons, of a circular cistern, 6 ft. 8 in. in diameter, and 8 ft. deep.
Ans. $2088\frac{8}{9}$ wine gallons; $1706\frac{2}{3}$ beer gallons.

4. Required, the contents, in wine and beer gallons, of a rectangular cistern that contains 300 solid feet?
Ans. 2244 wine gallons; 1838.1 beer gallons.

5. A rectangular cistern is to be made containing 1400 gallons; its length and breadth are 8 feet each. Required, its depth?
Ans. 3 ft. nearly.

Suggestion.—Find the contents of the cistern by multiplying 1400 by 231. We then have the contents, and two of the dimensions given.

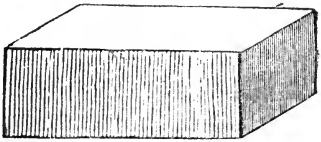
6. A circular cistern measures 5 ft. in diameter, and contains 1000 gallons. Required, its depth? Ans. $6\frac{38}{47}$ ft.

What is the standard for Liquid measure in the United States?

7. A circular cistern is to be made to contain 1692 gallons, and to be 8 ft. deep. Required, its diameter? Ans. 6 ft.

Suggestion.—By putting some character, as x, in the place of the dimension wanted in the last two examples, the method of working will be at once perceived.

Case 4. PARALLELOPIPEDS.



Obs. 10. A PARALLELOPIPED *is a solid or prism having six faces, of which those opposite each other are equal and parallel.*

REMARK.—A CUBE is a parallelopiped, each face of which is a square. (Sect IX, Art. 1, Obs. 17.) A common brick, or chest, is a parallelopiped, each side of which is a rectangle.

To find the surface of a parallelopiped :

Obs. 11. *Find the surface of each side separately, and add the several results together.*

REMARK.—The surface of any solid bounded by plane, or straight edged surfaces, is found by the same Rule.

1. Required, the surface of a parallelopiped 6 ft. long, 4 ft. wide, and 4 ft. deep? Ans. 128 sq. ft.

2. Required, the surface of a solid 4 ft. 9 in. long, and 3 ft. 10 in. wide, and 2 ft. 8 in. thick? Ans. 68 sq. ft. 104 sq. in.

To find the solid contents of a parallelopiped :

Obs. 12. *Multiply together its length, breadth, and thickness.* (Sect. IX. Art. 2. Obs. 18.)

3. What is the solidity of a parallelopiped 12 ft. long, 6 ft. wide, and 4 ft. deep? Ans. 288 s. ft.

4. Required, the contents of a solid 24 ft. long, 20 ft. wide, and 16 ft. thick? Ans. 7680 s. ft.

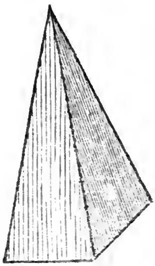
Case 5. PYRAMIDS.

Obs. 13. A PYRAMID *is a solid which decreases gradually from its base, until it comes to a point at the top.* This point is called the VERTEX.

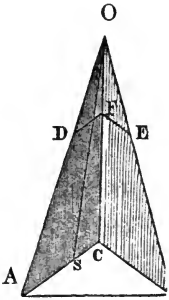
What is a Parallelopiped? Give examples. How do we find the surface of a Parallelopiped? What other solids can have their surfaces found by the same rule? How do we find the solidity of a Parallelopiped? What is a Pyramid? What is the point at the top called?

Pyramids are *triangular*, *quadrangular*, *circular*, &c., according as their bases are triangles, squares, circles. &c. A circular pyramid is called a Cone.

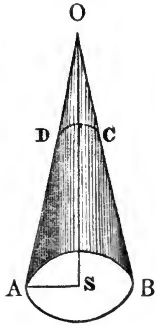
Obs. 14. A Frustrum *is what remains after the top of the pyramid or cone has been taken off.* The top and bottom of the frustrum are called its *bases*.



Sq. Pyramid.



Triangular Pyramid.



Cone.

If in the above diagrams, the part O-D E F be taken from the triangular pyramid, the remaining part ABC-DEF will be the frustrum.

Also, if the part O-CD be taken from the cone, the remaining part AB-CD will be the frustrum.

Obs. 15. The *Altitude* of a pyramid, or cone, is a line passing through the centre from the vertex to the base. A line drawn from the vertex perpendicularly to one side, is called its *Slant Heighth.* Thus, OS is the slant heighth of the pyramid O-ABC.

To find the surface of a pyramid, or cone:

Obs. 16. *Multiply the perimeter of its base by half its slant heighth.*

Demonstration.—The sides of pyramids are triangles, which the slant heighth divides into right angled triangles, of which the area is found by multiplying the base by half the perpendicular. (Art. 1. Obs. 3.) But the sum of all the bases of these triangles is the perimeter of the pyramid, which multiplied by half the common perpendicular, or slant heighth of the pyramid gives the surface.

When are pyramids said to be triangular, quadrangular, &c.? What is a Cone? A Frustrum? What are the top and bottom of the frustrum called? What is the altitude of a Pyramid, or Cone? Its slant heighth? How do we find the surface of a Pyramid, or Cone? Demonstrate this rule?

Again: if the bases of these triangles are bisected indefinitely, they will become a circle, and the pyramid will become a cone.

Hence—The surface of a cone is found by the same process. (Art. 1. Obs. 9.)

1. Required, the area of the surface of a triangular pyramid, the slant heighth of which is 30 ft, and each side of its base 12 ft.?

Ans. 540 sq. ft.

2. Required, the surface of a quadrangular pyramid, each side of which is 9 ft., and the slant heighth 15 ft.? Ans. 270 sq. ft.

3. Required, the area of the surface of a cone 3 ft. in diameter, at the base, and its slant heigth 12 ft.? Ans. 56.5488 sq. ft.

4. Required, the area of the curved surface and base of a cone, the slant heighth of which is 12 ft. 11 in., and the diameter of its base 4 ft. 7 in.? Ans. 109 sq. ft. 70.905 sq. in.

To find the surface of the frustrum of a pyramid or cone:

Obs. 17. *Add together the perimeters of the upper and lower bases, and multiply half their sum by the slant heighth.*

Demonstration.—From the diagram under Obs. 12, we perceive the sides of the frustrum of a pyramid are trapezoids. (Art. 1. Obs. 5.) But the area of these is found by multiplying half the sum of the parallel sides (or bases,) by their altitude, (or slant heighth.) (Art. 1. Obs. 6.) Hence, as the sum of the several bases forms the perimeters of the two bases of the pyramid, and their altitude, or slant heighth is the same in all, the rule is evident.

Again: If the ends, or bases of these trapezoids are bisected indefinitely, they will become circles, and the frustrum of a pyramid will become the frustrum of a cone. Hence, the surface of the frustrum of a cone is found by the same process. (Art. 1. Obs. 9.)

5. Required, the convex surface of the frustrum of a triangular pyramid, of which the upper base measures 6 ft., and the lower base 9 ft. on a side, and the slant heighth is 12 ft.?

Ans. 270 sq. ft.

6. Required, the convex surface of the frustrum of a quadrangular pyramid, of which the slant heighth is 36 inches, and the upper and lower bases 14 and 26 inches on a side? Ans. 20 sq. ft.

7. Required, the convex surface of the frustrum of a cone of which the slant heighth is 30 ft., the diameter of the upper base 12 ft., and that of the lower base 18 ft.?

Ans. 1263 sq. ft. 103.68 sq. in.

To find the solidity of a pyramid or cone:

Obs. 18. *Multiplg the area of its base by one-third of its altitude.*

How do we find the surface of the frustrum of a Pyramid, or cone? Demonstrate this rule. How do we find the solidity of a pyramid, or cone?

The demonstration of this and the next rule in this case, and also of the rules of the next case, is too complex to be understood without a knowledge of Geometry.

8. Required, the solid contents of a triangular pyramid, whose base is 6 ft. on each side, and altitude 25 ft.?

Ans. 130 s. ft. nearly.

9. One of the Egyptian pyramids is square at the base, measuring 720 ft. on a side, and its heighth is 477 ft. Required, its contents?

Ans. 82425600 s. ft.

10. Required, the solid contents of a cone 20 ft. in heighth, and 5 ft. in diameter at the base?

Ans. 130.9 s. ft.

11. How many times can a conical vessel 5 inches in diameter, and 8 inches deep, be filled from a hogshead of cider?

NOTE.—The learner will recollect that a gallon of cider contains 231 cubic inches.

Ans. $277\frac{15}{17}$ times.

To find the solidity of the frustrum of a pyramid or cone:

Obs. 19. *Find the area of each end, or base; Multiply the two areas together, and extract the square root of their product. Finally, add together the areas of the upper base, the lower base, and the root just found, and multiply their sum by one-third of the altitude.*

12. Required, the solidity of a block whose ends are squares, each side of the lower base being 4 ft., each side of the upper base 3 ft., and its length 15 ft.?

Ans. 185 s. ft.

13. Required, the solidity of a block whose ends are triangles, the sides of the top end being 2, $2\frac{3}{4}$, and $3\frac{1}{2}$ feet, the sides of the lower end being $3\frac{1}{2}$, 4, and $5\frac{1}{4}$ ft., and its length being 12 ft.?

Ans. 56.5+ s. ft.

14. Required, the solid contents of a glass in the form of a frustrum of a cone, the diameter of the top being 4 inches, and of the bottom 5 inches, and its depth 9 inches?

Ans. 143.72823 in.

15. Required, the solid contents of a mound in the shape of a frustrum of a cone, the diameter of the upper base being 15 ft., of the lower base 20 ft., and its heighth 30 ft.?

Ans. 7264.95 s. ft.

How do we find the solidity of the frustrum of a pyramid or cone?

Case 6. Spheres, or Globes.

Obs. 20. A Globe, or Sphere, *is a solid, in which all parts of the surface are equally distant from a point within called the centre;* as a cannon ball, a ball of yarn, &c.

To find the area of the surface of a Globe, or Sphere:

Obs. 21. *Multiply the circumference by the diameter;* or
Multiply the square of the diameter by 3.1416; or,
Multiply the square of the circumference by .3183.

1. Required, the area of the surface of a globe 14 in. in diameter? Ans. 4 sq. ft. 39.7536 sq. in.

2. What is the area of the surface of a sphere 47.124 ft. in circumference? Ans. 706.86 sq. ft.

To find the solidity of a Globe, or Sphere:

Obs. 22. *Multiply the cube, or diameter, by* .5236: or,
Multiply the surface by one-sixth of its diameter.

3. Required, the solidity of three globes, whose diameters are 12, 16, and 20 inches respectively?
Ans 904.7808; 2144.6656; and 4188.8 s. in. respectively.

4. If the diameter of the earth is 7912 miles, what is its solidity, supposing it to be a perfect sphere?
Ans. 259333411782.8608 s. miles.

Case 7. Tonnage of Vessels.

Obs. 23. The first thing necessary in ascertaining tonnage, is to find the number of *cubic feet of water* displaced by the vessel. This is done by finding the solid contents of the hull, which is equal to the product of the *length*, *width*, and *depth* of the vessel.

A body floating on a fluid will displace as much of the fluid as is equal to its own weight; and a cubic foot of water weighs 1000 oz. Avor.; therefore, 95 cubic feet, (which is allowed for a ton of 2240 lbs.), will weigh $5937\frac{1}{2}$ lbs.; that is, nearly three times the weight of the freight in water is allowed to the ton.

What is a Globe, or Sphere? How do we find the surface of a globe, or sphere? How do we find its solidity? What is the first thing necessary in ascertaining tonnage? How is this found? What quantity of fluid will a body floating on its surface displace? How much does a cubic foot of water weigh? How many cubic feet are allowed to the ton? How much will a ton weigh?

There are two rules for calculating tonnage—the *Government Rule*, and the *Carpenter's Rule*.

GOVERNMENT RULE: *For Double-decked Vessels.*

"*If the vessel be double-decked, take the length thereof, from the fore part of the main stem to the after part of the stern part, above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel; and then deduct from the length three-fifths of the breadth, multiply the remainder by the breadth, and the product by the depth, and divide this product by 95, the quotient whereof shall be deemed the true contents, or tonnage of such ship or vessel.*"

For Single-decked Vessels.

"If the ship or vessel be single-decked, take the length and breadth, as above directed, deduct from the said length three-fifths of the breadth, and take the depth from the under side of the deck plank, to the ceiling of the hold; then multiply and divide as aforesaid and the quotient shall be deemed the tonnage."

REMARK 1.—The Carpenter's Rule is the same as the Government Rule, except that the $\frac{3}{5}$ of the breadth is not deducted from the length, but all is calculated.

2.—The *Keel* of a vessel is its main bottom in length; the *beam* is its greatest width from side to side of the hull; and the *hold*, the depth from the main deck to the bottom of the hull.

2. Required, the tonnage of a single-decked vessel, by both rules, 125 ft. keel, 38 ft. beam, and 10 ft. hold?

Ans. By Gov. Rule, $208\frac{1}{5}$ T.; By Car. Rule, 500 T.

Operation.—$\frac{3}{5}$ of $38 = 22\frac{4}{5}$; $125 - 22\frac{4}{5} = 102\frac{1}{5}$; $(102\frac{1}{5} \times 38 \times 10) \div 95 = 208\frac{4}{5}$.

By Carpenter's rule: $(125 \times 38 \times 10) \div 95 = 500$.

2. Required, the tonnage, by the Government rule, of a vessel 228 ft. long, 35 ft. wide, and $14\frac{1}{4}$ ft. deep? Ans. $1086\frac{3}{4}$ T.

3. Required, the Government tonnage of a double-decked vessel 300 ft. long, and 35 ft. wide? Ans. $1798\frac{31}{38}$ T.

4. Required, the Government tonnage of a double-decked vessel 379 ft. in length, and 30 ft. in width? Ans. 1710 T.

How many rules are there for calculating tonnage? What is the Government Rule for double-decked vessels? For single-decked vessels? What is the Carpenter's rule? What is the the keel of a vessel? The beam? The hold?

Case 8. Irregular Bodies.

To find the solidity of any irregular body which cannot be reduced to a regular form :

Obs. 22. *Immerse it in a vessel partly filled with water; then the solid contents of that part of the vessel filled by the rising of the water, will be the solidity of the body immersed.*

This rule is so evident that it needs no demonstration.

1. A stone of irregular form being put into a tub partly filled with water, raised the water $4\frac{1}{2}$ inches. Required, the solidity of the stone, the diameter of the tub being 18 inches? Ans. 1145.1132 s. in.

2. A lobster being put into a bucket partly filled with water, raised the water 3 inches. The diameter of the bucket at the surface of the water, before the lobster was put in, was 9 inches, and the diameter at the surface after the lobster was put in was 10 inches. Required, the solidity of the lobster? Ans. 212.8434 s. in.

3. Required, the solidity of a brush-heap, which being put into a conical cistern, raised the water 25 inches, the diameters being 5 ft. 10 in. and 6 ft. 3 in. respectively? Ans. 59 s. ft. $1295\frac{3}{8}$ s. in.

Article 3. Gauging.

Obs. 1. *The method of finding the contents of any vessel in gallons, bushels, &c., is called* Gauging.

To find the number of bushels which any vessel will contain :

Obs. 2. *Find the solid contents of the vessel in feet; then multiply this by* 45, *and divide the product by* 56.

Demonstration.—In a solid foot are 1728 solid inches; (see table Cubic Measure;) in a bushel there are 2150.4 solid inches; (see table Dry Measure;) therefore a bushel contains $\frac{2150.4}{1728.0} = \frac{56}{45}$ solid feet. Hence, the vessel will contain as many bushels as $\frac{56}{45}$ is contained in its solid contents. That is, we divide by $\frac{56}{45}$.

a. For heaped measure use 9 and 14 instead of 46 and 56, because $\frac{2688}{1788} = \frac{14}{9}$.

Ex. 1. How many bushels will a box hold that is 6 feet long, 4 ft. wide, and 6 ft. deep? Ans. $115\frac{5}{7}$.

2. How many bushels will a gum contain which is 12 ft. deep, and 4 ft. in diameter? Ans. 121.176.

How do we find the solidity of irregular bodies? What is guaging? How do we find the number of bushels any vessel will contain? How do we proceed with heaped measure? Why?

3. How many bushe's would a gum contain, which is 8 ft. deep, and measures 6 ft. in diameter at the top, and 5 ft. in diameter at the bottom? Ans. 153.155.

4. How many bushels of apples can be put in a bin 35 ft. long, 6 ft. wide, and 5 ft. deep? Ans. 675.

Note.—The learner will observe that this is heaped measure.

5. How many bushels of potatoes can be put in a bin 25 ft. long, 7 ft. wide, and 6 ft. deep? Ans. 675.

6. A bin is 6 ft. wide, and 6 ft. deep; how long must it be to contain 810 bushels of apples? To contain 810 bushels of wheat? Ans. 35 ft. for apples; and 28 ft. for wheat.

Remark.—When it is desired to obtain the contents in barrels, we divide the result obtained by 5. If corn is the thing measured, and it is in the ear, we divide by 10; if in the husk, by 20.

7. How many gallons of water will a conical cistern contain, that is 7 ft. 9 in. deep, and the diameter 5 ft. 6 in. at the bottom, and 4 ft. 9 in. at the top? Ans. 1217.0636.

Note.—We find the solid contents in inches, and divide this by 231, because 231 cubic inches make a gallon. (See table Wine measure.)

To find the number of wine and beer gallons contained in a cask, or barrel:

Obs. 3. *Take the dimensions of the cask in inches: that is, the diameter at the bung and head, and the length of the cask.*

If the staves are MUCH *curved, multiply the difference between the bung and head diameter by .7; if but* LITTLE *curved, by .6; if they are of a* MEDIUM *curve, by .65; or, if they are* ALMOST STRAIGHT *by .55; in either case, add the product to the head diameter, and the sum will be the mean diameter, and the cask will be reduced to a cylinder.*

Multiply the square of the mean diameter by the length of the cask, and that product by 34 for wine measure, and 28 for beer measure, and point off four decimals in the product; the result will be in gallons, and decimals of a gallon.

Demonstration.—The demonstration of this rule, until the cask is reduced to a cylinder, cannot be understood, without a knowledge of the higher departments of mathematics. After it is reduced to a cylinder, we find its solidity by multiplying the square of the mean diameter by .7854, and its length; (Art. 1. Obs. 11. and Art. 2.

How do we find the contents in barrels? How do we find the number of wine and beer gallons contained in a cask or barrel? Demonstrate this rule.

Obs. 7.) and this is reduced to gallons by dividing by 231 for wine measure, and 282 for beer measure. (Obs. 3.) Thus, in wine measure, .7854 is a multiplier, and 231 is a divisor; that is, we multiply by $\frac{.7854}{231}$; but $\frac{.7854}{231} = .0034$. Also, in beer measure, we multiply by $\frac{.7854}{282}$, and $\frac{.7854}{282} = .0028$ nearly. Hence, this part of the rule is evident.

REMARK.—Guaging of casks by computation is of but little utility, as it is now mostly done by a rod, with calculations already made, in tabular form.

8. Required, the contents, in wine and beer gallons, of a cask, of which the staves are much curved, the diameter at the head being 20 inches, and at the bung 30 inches, and its length 40 inches?
Ans. 99.144 wine gallons; 81.648 beer gallons.

9. How many wine and beer gallons in a cask 45 inches in length, the bung diameter 30 inches, and the head diameter 25 inches, the staves being a medium curve?
Ans. 122.1035+ wine gallons; 100.5558 beer gallons.

10. Required, the number of wine and beer gallons in a cask of which the staves are nearly straight, and the dimensions as follows: bung diameter, 20 inches; head diameter, 16 inches; length, 50 inches? Ans. 56.3108 wine gallons; 46.3736 beer gallons.

11. How many wine and beer gallons will a hollow cylinder contain, which is 4 feet deep, and 1 ft. 6 in. in diameter?
Ans. 52,8768 wine gallons; 43.5456 beer gallons.

SECTION XVII.

PHILOSOPHICAL AND MISCELLANEOUS QUESTIONS.

ARTICLE 1. THE MECHANICAL POWERS.

Obs. 1. THE MECHANICAL POWERS *consist of six simple instruments, viz.:* the *Lever*, the *Wheel and Axle*, the *Pulley*, the *Inclined Plane*, the *Wedge*, and the *Screw*.

Obs. 2 A MACHINE *is an instrument combining one or more of the mechanical powers.* It may be very simple, as a pin; or very complicated, as a steam-engine.

Obs. 3. THE POWER is the body which *applies* the force; and the WEIGHT is the body which *resists* the force.

What are the Mechanical Powers? What is a Machine? The power? The weight?

Case 1. The Lever.

Obs. 4. The Lever *is simply a bar, used in raising weights.* It is moved about a fixed point, called its *Fulcrum*, or *Prop.*

Remark.–There are three kinds of levers. Force exerted by the first kind is called *prying*. In this the weight and power are at the ends of the lever, and the fulcrum between them. Force exerted by the second kind is called *lifting*. In this the power and fulcrum are at the ends of the lever, and the weight between them. Force exerted by the third power is called *lifting*, or *raising*. In this the weight and fulcrum are at the ends of the lever, and the power between them. Raising a ladder to the roof of a house is an example of this kind of lever.

2.—The *arms* of the lever are the parts between the fulcrum and its ends.

Obs. 5. A power and weight acting upon the arms of a lever will balance each other, *when the product arising from multiplying the power by its distance from the fulcrum, is equal to the product arising from multiplying the weight by its distance from the fulcrum.*

By keeping this principle in view, we can always find any one of these four things: the power, the weight, the distance of the weight from the fulcrum, and the distance of the power from the fulcrum, if the other three are given.

Note.—By this rule, the power and weight exactly balance; in order to raise the weight, there must be a small increase of power. It is also customary in practice, when estimating the mechanical powers, to deduct *one-third* for *friction.*

Ex. 1. Suppose a man weighing 140 lbs. rests on a lever 8 feet long, what weight will he balance on the other end, supposing the fulcrum to be 6 inches from the weight? Ans. 2100 lbs.

2. Given, the weight, d stance of the weight from the fulcrum, and the distance of the power from the fulcrum, in the last example, to find the power? Ans. 140 lbs.

3. Suppose a power of 150 lbs. be applied to a lever, 15 ft. from the fulcrum, what weight will it balance at the other end of the lever, which is 4 inches from the fulcrum? Ans. 6750 lbs.

What is a lever? How many kinds of levers are there? What is force exerted by the first kind called? How are the power, weight and fulcrum arranged with respect to each other? What is force exerted by the second kind of lever called? How are the power, weight, and fulcrum arranged with respect to each other? What is force exerted by the third kind of lever called? How are the weight, power, and fulcrum arranged with respect to each other? Give an example of this kind of lever. What are the arms of a lever? When will the power and weight acting upon the arms of a lever balance each other? What is necessary in order to raise the weight? What deduction is made for friction in the practical application of the mechanical powers?

4. By a lever 30 ft. long, what weight will 400 lbs. sustain, the weight being 6 inches from the fulcrum? Ans. 23600 lbs.

5. By a lever 12 ft. long, what weight will 200 lbs. balance, the power being applied 18 inches from the fulcrum? Ans. $28\frac{4}{7}$ lbs.

Case 2. The Wheel and Axle.

Obs. 6. The Wheel and Axle *consists of two wheels, one of which is larger than the other, but the smaller one passes through the larger, and thus both have a common centre on which they turn.*

The *power* is applied to the circumference of the *larger wheel*, and the *weight* to that of the *smaller wheel* (or axle), by means of cords.

Obs. 7. The power, when applied to this machine, will exactly balance the weight, *when the product arising from multiplying the power by the diameter of the wheel, is equal to the product of the weight by the diameter of the axle.*

Hence, if any three of these are given, the other can be easily found.

1. If the diameter of the wheel is 4 ft., and the diameter of the axle 3 inches, what weight on the axle will balance 125 lbs, applied to the wheel? Ans. 2000 lbs.

2. If the diameter of the wheel is 6 ft. 8 in., and the diameter of the axle 10 inches, what power must be applied to the wheel to balance 2600 lbs. at the axle? Ans. 325 lbs.

3. If the diameter of the wheel is 5 ft., the power 224 lbs., and the weight 2688 lbs., what is the diameter of the axle?

Ans. 5 inches.

4. If the diameter of the axle is 8 inches, the power 340 lbs., and the weight 2975 lbs., what is the diameter of the wheel?

Ans. 5 ft. 10 in.

5. Suppose there are two wheels: one 6 ft. 9 in. in diameter, with an axle 9 inches in diameter, and the other 5 ft. 5 in. in diameter, with an axle 6 inches in diameter; if the power cord of the smaller be attached to the axle of the larger, what weight at the axle of the smaller wheel can be supported by 275 lbs. at the power cord of the larger? Ans. $26812\frac{1}{2}$ lbs.

Case 3. The Pulley.

Obs. 8. A Pulley *is a grooved wheel, which turns about its axis by means of a cord passing over it.* It may be either simple or compound, fixed or movable.

What is a Wheel and Axle? How is the power applied? The weight? When will the weight and power exactly balance? What is a Pulley?

Obs. 9. The *simple pulley* consists of a single wheel and its cord, to one end of which the *weight* is attached, and to the other end the *power.* No advantage is gained from this machine, as the power and weight must always be equal. When a number of pulleys are used, one half of them are movable, and the whole is called *a system of pulleys.* It is evident that the weight is divided between the number of strings, as there are always double the number of movable pulleys: Hence—

Obs. 10. *The weight divided by twice the number of movable pullies, will give the power; and the power multiplied by twice the number of movable pulleys, will give the weight.*

1. What weight will be balanced by a power of 200 lbs. attached to a cord, which passes over 4 movable pulleys?

Ans. 1600 lbs.

2. What power must be applied to a cord passing over 6 movable pullies to balance 3600 lbs.? Ans. 300 lbs.

3. What number of movable pulleys would be necessary to raise a weight of 1000 lbs. by a power of 125 lbs.? Ans. 4.

4. If a cord that passes over 5 movable pullies be attached to an axle 4 inches in diameter, and if the wheel is 6 feet in diameter, what weight can be raised by the pulley, by applying 200 lbs. to the wheel? Ans. 36000 lbs.

Case 4. The Inclined Plane.

Obs. 11. An Inclined Plane *is a plane surface which inclines to or from the earth.* Thus, a board with one end on the gronnd, and the other end on a block of wood, forms an inclined plane.

Obs. 12. The power when applied to this machine will exactly balance the weight, *when the weight multiplied by the heighth of the plane, is equal to the power multiplied by its length.*

1. If an inclined plane is 150 ft. long, and 15 ft. high, what weight will 200 lbs. sustain? Ans. 2000 lbs.

2. If an inclined plane is 80 ft. long, and 12 ft. high, what power will sustain 2880 lbs.? Ans. 432 lbs.

3. What power would be required to draw a train of cars weighing 50000 lbs. up an inclined plane 3 miles in length, and 100 ft. in heighth? Ans. $315\frac{2}{3}$ lbs. nearly.

What is a Simple Pulley? What is a system of pulleys? How is the weight divided? To what are these equal? How do we find the power? The weight? What is an inclined plane? Give an example. When will the power balance the weight in this machine?

Case 5. The Wedge.

Obs. 13. The Wedge *may be regarded as two inclined planes placed base lo base.*

Obs. 14. From Obs. 12 we conclude the force will balance the power *when the product a ising from multiplying the force produced at the side, by the breadth of the head of the wedge, is equal to that of multiplying the power acting against the head by the length of its side.*

Note 1.—This force only respects one side of the wedge; when the force against both sides are required, only half the breadth of the head must be taken into consideration.

2.—In applying the wedge, the friction is at least equal to the force to be overcome; and therefore not less than one-half of the power is lost, for which no allowance is made in the above rule. The advantage ef the wedge arises from the force of percussion or blow with which it is struck, which is much greater than that of any dead weight or pressure, such as is usually employed on the other mechanical powers.

1. What weight of force would be effected on either side of a wedge, the head of which is 3 inches broad, and the side 15 inches long, by a power of 85 pounds? Ans. 425 lbs

2. What power must be applied to a wedge 18 inches long and 6 inches broad at the head, to effect a force of 1575 lbs., allowing no loss for friction? If the friction is equal to the force to be overcome, what power must be employed?

Ans. { 525 lbs. allowing no loss for friction.
1050 lbs. allowing loss for friction.

Case 6. The Screw.

Obs. 15. The Screw *may be considered as a thread, or groove, running spirally around a cylinder.*

Remark.—The principle of the screw is the same as that of the inclined plane; the distance between the threads being the heighth, and the circumfereece of the cylinder the base of the plane.

Obs. 16. The power when applied to this machine will exactly balance the weight, *when the power multiplied by the circumference of the circle described by the power, is equal to the weight multiplied by the distance between the threads.*

What is a wedge? When will the force balance the power? What has this force reference to? When the force against both sides is required, how do we proceed? What part of the power is lost in applying the wedge to practical purposes? How so? What advantage then has the wedge? What is a screw? What is the principle of the screw? Show the comparison. When will the power balance the weight when applied to this machine?

Hence, if any of these four quantities are given, the other can be easily found.

1. What weight may be supported by a screw, the lever of which is 6 feet long, and the distance between the threads of a screw $\frac{1}{2}$ of an inch, by a power of 125 lbs.? Ans. 113097.6 lbs.

The learner will observe that the length of the lever is but half the diameter of the circle.

2. What power would be required to balance 12000 lbs., if the distance between the threads of the screw was $\frac{5}{8}$ of an inch, and the length of the lever 8 feet? Ans. 12.43 lbs:

Case 7. Machinery.

Obs. 17. *By* Machinery *is understood the connection of two or more of the mechanical powers, by means of belts, bands, cogs, &c.*

Remark.—*The velocities of wheels are to each other as their like dimensions.*

1. A cog wheel 150 inches in diameter, runs into another but 12 inches in diameter. Required, the number of revolutions the smaller wheel makes per minute, the larger performing 8? Ans. 100.

Operation.—12 : 150 : : 8 : 100.

Or, whilst the larger wheel makes 1 revolution, the smaller one makes $150 \div 12 = 12\frac{1}{2}$; and $12\frac{1}{2} \times 8 = 100$, as before.

2. A wheel 56 inches in diameter is connected with another 6 inches in diameter, and this is attached to a shaft, on the end of which a wheel 42 inches in diameter is joined to another 4 inch s in diameter. Required, the velocity of the latter wheel per minute the larger one making 50 revolutions. Ans. 4900 revolutions.

3. How many revolutions will a spindle 2 inches in diameter make per minute, which is connected with a drum 4 ft. in diameter, that performs 40 revolutions per minute? Ans. 960.

4. A belt connects a drum 5 feet in diameter, making 50 revolutions per minute, with a spindle 3 inches in diameter. What is the velocity of the spindle per minute? Ans, 1000 revolutions.

5. If a drum 5 feet in diameter performs 40 revolutions per minute, what is the diameter of a cylinder connected with it that performs 600 revolutions per minute? Ans. 4 inches.

6. If the spindle of a common spinning wheel is $\frac{1}{4}$ of an inch in diameter, and the rim is 4 ft. 9 in. in diameter, how many revolutions will the spindle perform whilst the rim performs 15?

Ans. 3420.

What is understood by Machinery? What is the ratio of the velocities of wheels to each other?

Article 2. Method of keeping Books, Forms of Notes, &c.

Remark.—It is not our design to present here an elaborate treatise on book-keeping. We shall only give a form or two for the benefit of those whose entries are few, and who have but little business to transact. Those who desire further information are referred to treatises on book-keeping, where they will find the subject treated upon more fully.

Obs. 1. It is necessary that every person should have some regular, systematic method of keeping his accounts, because the law requires in all cases of dispute, that the book in which the charges were originally made, be produced before the court, and legal evidence given of their correctness.

The following form is recommended for farmers and mechanics, whose entries are but few, and generally made at the close of the day or week. It consists in writing the name of the person on the left hand page, *Dr.* (debtor) *for* the sum he is owing you, and on the right hand page, *Cr.* (creditor) *by* the sums you owe him. The difference between the *Dr.* and *Cr.* sides will at once show how much is due from one to the other.

Henry Worthy, Dr. Henry Worthy, Cr.

1848		$	cts.	1848		$	cts.
Jan. 1.	For 8 bushels of Wheat, at $1 per bushel,....	8	00	Jan. 3.	By Cash	5	00
" 4	For one day's work, Chopping, ...		75	" 6	By Goods	4	75
" 7	For 12 lbs. butter, at 16⅔ cts. per lb......	2	00	" 7	By Cash	1	00
" 8	For 2 doz. Eggs, at 12½ cts. per doz.		25	" 8	By balance, being the amouut now due me,		25
		11	00			11	00
1848							
Jan. 9.	For bal'nce due me from old account		25				

Note.—It is immaterial whether we place the debits and credits both on the same page as above, or on opposite pages. The same rules are observed in both cases; that is, write the name of the person on the left hand page, Dr., and on the right hand page, Cr. The pages should be ruled as above.

Obs. 5. The following remarks may also be of use to the learner:

a. The person who receives any thing from you is *Dr. to* you *for* the amount he receives from you; and the person from whom you receive any property is *Cr.* by the amount you receive from him.

b. *Places of residence* should be named when they are not the same as that where the book is kept. Also, if different persons bear the same name, it is best to designate the *occupation* or particular

place of residence. The name of the person who owns the book, and his place of residence should be written on the first page.

c. The *date* of each transaction should be written against the entry of it in the column for that purpose.

d. When the account of any person is closed *To* (or *For*) *Balance,* you are debtor to him, and when it is closed *By Balance,* he is debtor to you for the amount of balance ; and in opening a new account, he must be *credited By Balance,* or made *Dr. To* (or *For, Balance* from one account to another, the balance always crossing from the *Dr.* to the *Cr.* side, or from the *Cr.* to the *Dr.* side, in passing from the old account to the new.

e. When a person has dealings with several individuals, he should have an index to his book, in which the name of every person with whom he transacts business should be written under its nitial, and the page where the account is kept should be noted down.

f. Care should be taken to preserve a book free whom blots, and mistakes, and every entry should be made in a full, bold and legible hand.

REMARKS.—For drovers, and farmers who raise considerable stock, or grain, and are in the habit of receiving and paying out money, frequently, the following form is recommended:

CASH ACCOUNT,		Dr.		CASH ACCOUNT,		Cr.	
1848		$	cts.	1848		$	cts.
Jan. 1	To Cash on hand	200	00	Jan. 7	By Cash paid for Goods	300	00
" 4	" Sales of Cattle ..	75	50	" 1'	" Cash paid for work	15	25
" 9	" Sales of Wheat ..	50	00	" 25	" Cash p'd for Cattle	66	75
" 12	" Cash rec,d of Thomas True on his note	15	75	" 28	" Cash paid for wheat	20	00
" 30	" Cash rec'd for horses	425	75	" 30	" Balance, being the cash on hand...	365	00
		767	00			767	00
1848							
Feb. 1	To bal. from old acc't.	356	00				

Obs. 3. The learner will now notice the following suggestions, with reference to the *Cash-Book:*

a. The CASH-BOOK is kept by making Cash *Dr. to* what is on hand, and what is received, and *Cr. by* whatever is paid out.

b. The excess of the *Dr.* side over the *Cr.* side ought always be equal to the cash on hand.

c. When the cash on hand is counted, it should be entered on

the *Cr.* side; but when a balance is struck, the cash on hand should be entered on the *Dr.* side.

NOTE.—Ornament and perspicuity being the object of many accountants, they often use *capital letters*, notwithstanding the rules they frequently see to the contrary. They likewise, seldom use the preposition *of*, writing 12 lbs. Butter, 6 yds. Sheeting, &c., instead of 12 lbs. of Butter, 6 yds. of Sheeting, &c.—But the learner need observe but one particular viz: to have a good pen, and clean fingers, and to preserve his book from blots, mistakes, and every thing else but his entries.

FORMS OF ORDERS, RECEIPTS, AND NOTES.

ORDERS.

COLUMBUS, June 7th, 1848.

Mr. Silas *Brown*—Please pay the bearer, Mr. *Samuel Jones*, six dollars and twenty cents, and charge the same to my account.

THOMAS HENRY.

NEWARK, Sept. 1st, 1848.

Mr. *Simon Dealer*—Please pay Mr. *Chares Lane* such goods as he may call for, not exceeding the sum of fifteen dollars, and charge the same to your humble servant. THOMAS THRIFTY.

REMARK 1.—If the persons on whom the order is made resides in another town from the one in which the order is dated, the name of the town in which they reside should be written with their names.

RECEIPTS.

GENOA, Oct. 3d, 1848.

Received of *Henry Martin* two dollars in full of all accounts.

THOMAS STONE.

Received of *Moses Hale* five dollars in full of all demands.

Dublin, Dec. 4th, 1848. PETER GOODRICH.

REMARK 2.—When a receipt is given "in full of all accounts," it cuts off only the claims to accounts; but when it is given "in full of all demands," it cuts off all claims of any kind.

Received of *Thomas Mosely*, five dollars and fifty cents on his note for ten dollars, dated Sept. 2nd, 1847.

CHARLES WAKEFIELD.

CHILLICOTHE, June 6th, 1848.

Received of *William Jones* twelve dollars, to pay on the account of John Smith. HENRY MARKLEY.

DUE-BILLS.

Obs. 4. A Due-bill *is a mere pledge to pay a certain amount of money or other property therein specified, in consideration of an equivalent specified.*

Form of a Bill.

Due *Isaac Dealer*, or bearer, twelve dollars for value received.
Columbus, Dec. 4th, 1847. Jacob Faithful.

Note.—If payment is to be made in any thing besides money, it should be so specified in the due-bill.

NOTES.

No. 1. *Payable to Order.*

$200. Columbus, Feb. 2nd, 1848.

For value received, on demand, I promise to pay *Abraham Drive-all*, or order, two hundred dollars.

James Constance.

No. 2. *Note payable to Bearer.*

$500. Columbus, March 4th, 1848.

For value received, on or before the first day of February next, I promise to pay Joseph Goodwill, or bearer, five hundred dollars, with interest from date.

Ezra Fairface.

No. 3. *Note by two Persons.*

$75. Cleveland, April 3d, 1847.

One year after date, for value received, we, jointly and severally promise to pay Charles Good, or bearer, seventy-five dollars, with interest after thirty days.

Peter Trueman.

Attest: Henry Jones, Samuel Albright.

REMARKS ON NOTES.

1. The *drawer* or *maker* of the note, is the one who *signs* it; the *holder* of the note is the one who has possession of it.
2. No note is negotiable or transferable, unless the words *"or order,"* or the words *"or bearer"* are inserted in it.
3. When the holder of a negotiable note payable to order, (Note 1) wishes to transfer it, he must *endorse it;* that is write his name on the back of it. The holder of the note is then authorized

to collect it of the *drawer*, but if the drawer refuses to pay it, he can collect it of the *endorser*.

4. When a note is given with *surety*, the surety is not responsible only through the inability of the principal, or drawer.

5. When a note is made payable to *bearer*, (Note 2.) any person who has the note can collect it of the drawer.

6. The words "*For value received*" should be written in every note, and the amount of the note should always be written in words.

7. Every note should be made payable on demand, or at some specified time.

8. When the time expires for which the note, was given, the note will draw interest, although no mention be made of interest.—Also:

A note payable on demand, will draw interest after a demand of payment has been made, for it is then due.

9. When the time in which a note becomes due is not specified, it becomes due as soon as a demand of payment is made.

10. All notes have three days of grace after they are nominally due, before they are legally due.

11. The rate of interest is always understood to be the *legal rate*, unless otherwise specified in the note.

12. When a note is given by two persons, jointly and severally, (Note 3,) it may be collected of either of them, but not of both.

13. If a note is payable by installments, the amount of each installment, and the time when it is to be paid should be specified in the note.

14. If a note is given for specific articles, as wheat, corn, &c., payable at some fixed time, and payment is not made at this time, the holder can claim and recover the value of the note in money.

Article 3. Miscellaneous Problems and Rules.

Ex. 1. A vessel 6 feet square is filled with water to the depth of 10 feet, what pressure does the bottom sustain, a cubic foot of water weighing 1000 ounces Avoirdupois?

Solution. $\begin{cases} 6 \times 6 = 36 \text{ sq. ft. area of the bottom of the vessel.} \\ 36 \times 10 = 360 \text{ cubic feet of pressure.} \qquad [\text{Ans.} \\ 360 \times 1000 = 360000 \text{ oz.}; 360000 \div 16 = 22500 \text{ lbs.} \end{cases}$

Obs. 1. *The sides of a vessel sustain a pressure equal to the area of the sides multiplied by half the depth of the water.*

2. What would be the pressure of water against the gates of a sluice 18 ft. deep, and 24 ft. wide, when filled?

Ans. 243000 lbs.

3. What would be the pressure of water against the gates of a sluice 18 ft. wide, and 12 ft. deep, when full? Ans. 81000 lbs.

To find the height of an object by knowing its distance:

Obs. 2. *Take the distance in miles: two thirds of the square of this will be the height of the object, in feet.*

NOTE.—In this calculation, of course there must be no obstruction to the range of vision more than the natural convexity of the earth.

4. Looking across a plain, I saw the top of a pole exactly 10 miles distant. Required the height of the pole? Ans. 66 ft. 8 in.

5. I saw the top of a mast which I knew to be just 12 miles distant. Required the height of the mast? Ans. 96 feet.

To find the distance of an object by knowing its height:

Obs. 3. *Take the height in feet; increase this by one-half of itself and extract the square root of the sum. The result will be the distance in miles.*

6. I saw the top of a mast, which I knew to be 150 ft. in height. Required its distance? Ans. 15 miles.

7. To what distance on a level plain could a pole be seen which measured 80 ft. 8 in. in height? Ans. 11 miles.

To find the time a body has been falling:

Obs. 4. *Divide the velocity by* $32\frac{1}{6}$. Or,

Divide the space reduced to feet by $16\frac{1}{12}$, *and extract the square root of the quotient.*

8. How long will a body be in falling, to acquire a velocity of 193 feet per second? Ans. 6 seconds.

9. I dropped a stone into a pit 400 ft. deep. How long was it in falling? Ans. 4.98 + sec.

10. How long would it take a body to fall 2 miles? Ans. 25.62 + sec.

To find the velocity, which a body has acquired in falling:

Obs. 5. *Multiply the space reduced to feet by* $16\frac{1}{2}$, *and double the square root of the product.* Or,

Multiply the time by $32\frac{1}{6}$.

11. A stone was dropped from the top of a ledge of rocks 228 ft. in height. Required its velocity per second when it had reached the bottom? Ans. 121.11 ft.

12. What velocity per second, would a stone acquire in falling 1 mile? Ans. 582.82 ft. per sec.

17

13. A body has been falling 12 seconds. Required the velocity it has acquired? Ans. 386 ft. per sec.

To find the space a body has fallen through:

Obs. 6. *Multiply the square of the time by* $16\frac{1}{12}$. Or, *Divide the square of the velocity by* $64\frac{1}{3}$.

14. A body has been falling 9 seconds: Required the space it has passed through? Ans. $1302\frac{3}{4}$ ft.

15. Wishing to ascertain the depth of a chasm, I dropped a stone from the top and saw it strike the bottom in 4 seconds. Required the depth of the chasm?

Ans. 257 ft. 4 in.

16. A body has fallen so far as to have acquired a velocity of 579 ft. per second. Through what space has it passed?

Ans. 5211 ft.

NOTE.—In the above rules, no allowance is made for the resistance of the air.

Given the base (or perpendicular) of a right-angled triangle, together with the sum (or difference) of the other two sides, to find those sides:

Obs. 7. *To the square of the given side, add the square of the sum (or difference) of the other two sides, and divide the result by twice this sum (or difference;) the quotient will be the length of the longest side.*

17. The base of a right angled triangle measures 16 ft., and the sum of the other two sides is 32 ft. Required the length of those sides? Ans. Hypoth. 20 ft. Per. 12 ft.

18. The foot of a ladder was moved out 12 ft., and the top fell 6 ft. Required the length of the ladder, supposing it to have stood perpendicularly against a wall at first? Ans. 15 ft.

19. A pole 90 ft. high was broken off by the wind, and the two parts held together. The top fell 30 ft. from the foot of the pole.—Required the length of each part? Ans. 40 ft. and 50 ft.

To measure standing timber:

Obs. 8. *Place two stakes so that they will range with the top of the timber; then multiply the distance from the short stake to the tree, by the difference between the length of the stakes, and divide the product by the distance between the stakes. The result will be the length of the timber above the shorter stake.*

20. I wish to find a stick of timber 36 ft. long, and find one that has a knot that will spoil it above a certain height. I therefore place a stake 10 ft. long, 30 feet from the tree, and 6 ft. from this I place

another stake $4\frac{1}{2}$ ft. long, so that the top of the two stakes will range with the knot. Will the stick answer my purpose by allowing $2\frac{1}{2}$ ft. for the stump?

Ans. It will not answer, it being but 35 ft. long.

To find the mean temperature of any day, reckoning from sunrise to sunrise:

Obs. 9. *Add together the morning observation, twice the afternoon observation, twice the evening observation, and the next morning observation, and divide the sum by* 6.

REMARK 1. This is called DeWitt's Rule. It gives the correct mean temperature, on the supposition that the thermometer has risen and fallen regularly between the observations. The observations are taken at sunrise, at 2 P. M., at 9 P. M., and at sunrise the next morning.

2. When any of the observations are below zero, that quantity, or its double, (as the case may me,) must be subtracted, instead of added.

3. The mean temperature for any number of days is found by dividing the sum of the mean temperature of all the days by their number.

21. On a certain day the thermometer stood, at sunrise 38° Fahrenheit, at 2 P. M. 50°, at 9 P. M. 42°, and at sunrise the next morning 30°. Required the mean temperature of that day?

Ans. 42°.

22. On a certain day the thermometer stood at sunrise —6°, at 2 P. M, 10°, at 9 P. M., 4°, and at sunrise the next morning at —10°; what was the mean temperature of that day? Ans. 2°.

This sign (—) signifies that the observation was below zero.

23. A square field is to be enclosed by a fence 8 rails high, and having two lengths to the rod. How large must this field be that it may contain as many acres as there are rails around it, or that each rail may fence an acre?

Obs. 10. *Multiply* 4 *times the number of rails on one rod by* 160; *this will give the number of rods on one side of the field.*

Ans. 655360 acres = 1024 sq. miles = 32 miles sq.

The sum and product of two numbers being given to find the numbers:

Obs. 11. *From the square of half their sum subtract their product, and extract the square root of the remainder; to the root add the half sum, and the result will be the greater number.*

24. The sum of two numbers is 56, and their product 768; required the numbers? Ans. 32 and 24.

25. A man bought a certain number of acres of land for $4375; if the number of dollars he paid per acre were added to the num-

ber of acres bought, the sum would be 200. How many acres did he buy, and what did he pay per acre?

Ans. He bought 175 acres, at $25 per acre.

26. Two men, A and B bought 150 acres of land for $375, of which A paid $208, and B $170; on account of a difference in the quality of the land, they divided it so that A paid $1.20 per acre more than B. How many acres did each get, and what did he pay per acre?

Ans. { A got 65 acres at $3.20 per acre.
B got 85 acres at $2.00 per acre. }

To find the number of acres obtained by him who paid least per acre:

Obs. 12. *Multiply the whole number of acres by the difference between the prices per acre, subtract the product from the price paid for the whole land, and divide the remainder by twice the difference between the prices per acre. Square the quotient, and to this square add the product of the whole number of acres multiplied by the sum paid by him who paid the least per acre, and divided by the difference between the prices per acre; extract the square root of the sum, and from this root subtract the number that was squared; the remainder will be the number of acres obtained by him who paid the least per acre.*

Solution of the last example.

150 = No. acres. $378 = Amt. pd. for the [whole.
$1.20 = Diff. in the price per acre. 180 = Prod. to be sub- [tracted.
———
180.00 Diff. 1.20 × 2 = 2.40)198.00(825 = quotient to [be squared.
1920
———
600
480
———
1200
1200
———
0000

150 = No. acres.
$170 = Sum paid by him who paid least per acre.
———
105
15
———
1.2|0)25500.0|0
———
21250 = Number to be added to the square of 82.5.

[Solution continued on next page.]

```
                                   82.5
                                   82.5
                                   ----
                                   4125
                                  1650
                                 6600
                                 ------
                                 6806.25
                                21250
                                -------
                                28056.25(167.5
150-85 =65 =Acres obtained by A. 1              82.5=No.sqrd.
$208÷65=$3.20=price per acre }  --             -----
              paid by A      } 26)180           85= Acres ob-
                                  156              tained by B.
                                  ---
                              327)2456
$170÷85 = $2.00= price per }       2289
            [acre paid by B. }     ----
                              3345)167 25
                                   167 25
                                   ------
                                   000 00
```

27. A and B bought 280 acres of land for \$738. of which A paid \$326.25, and B paid \$411.75. The land was divided so that B paid \$0.80 per acre more than A· How much land did each get and what did he pay per acre?

Ans. { A got 145 acres at \$2.25 per acre. B got 135 acres at \$3.05 per acre. }

The following questions are solved chiefly by *analysis*, without reference to any particular rules. They are easy enough to be understood if the pupil thoroughly understands the principles previously explained.

28. A, B, and C agreed to divide \$50 between them giving A $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{4}$. What was the share of each in this proposition?

Solution.—$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$; $50\div 2=25$; $50\div 3=16\frac{2}{3}$; $5\div 4=12\frac{1}{2}$.

Then $13 : 12 : \$25 : \$23\frac{1}{3}$ A's share. $13 : 12 :: \$16\frac{2}{3} : \$15\frac{5}{13}$ B's share. $13 : 12 :: \$12\frac{1}{2} : \$11\frac{7}{13}$ C's share.

29. \$335 is required to be divided between A, B and C, in such a manner, that as often as A gets \$3 B gets \$5, and C gets \$7 as often as B gets \$4.

Solution.—Since C gets \$7 as often as B gets \$4, it is evident that as often as B gets \$5 C gets $\frac{7}{4}$ of \$5 or $\$8\frac{3}{4}$. Hence, their shares are in the proportion of 3, 5 and $8\frac{3}{4}$; or 12, 20, and 35.

Ans. \$60 A's share. \$100 B's share. \$175 C's share.

30. Suppose A, B, and C start at the same point and travel in the same direction about an island 53 miles in compass, A traveling at the rate of 3, B 5, and C 7 miles an hour, in what time will they next come together?

Solution.—$5-3=2$ miles B's gain per hour on A.
$7-3=4$ " C's " " on A.

Dividing the distance about the island (53 miles) by the greatest common divisor of 2 and 4, the gains of B and C per hour on A, we find that they will all meet again in $53\div2=26\frac{1}{2}$ hours.

Ans. $26\frac{1}{2}$ hours.

31. A hare is 60 leaps before a greyhound, and takes 6 leaps while the greyhound takes 5, but 3 leaps of the hound are equal to 4 of the hare. How many leaps must the greyhound take before he catches the hare?

Solution.—As 3 leaps of the greyhound are equal to 4 of the hare, 5 leaps of the greyhound are equal to $\frac{5}{3}$ of $4=6\frac{2}{3}$ leaps of the hare; therefore he gains $\frac{2}{3}$ of a leap on the hare for every 5 leaps he takes, and gains 1 leap for every $(5\div\frac{2}{3}=)\ 7\frac{1}{2}$ leaps he takes: but he must gain 60 leaps to overtake the hare, therefore he must take $60\times7\frac{1}{2}=450$ leaps to overtake her. Ans. 450.

32. A person bought a certain number of apples at the rate of two for a cent, and afterwards bought as many more at the rate of 3 for a cent: he sold them all at the rate of 5 for 2 cents, and lost $2\frac{1}{2}$ cents. How many apples did he buy in all?

Solution.—2 apples for 1 cent is $\frac{1}{2}$ a cent apiece; also:
3 apples for 1 cent is $\frac{1}{3}$ of a cent apiece. Then 2 apples (that is 1 of each lot,) cost $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ of a cent, and 1 apple cost $\frac{5}{6}\div2=\frac{5}{12}$ of a cent.

Again, as he sold 5 apples for 2 cents, he sold 1 apple for $2\div5=\frac{2}{5}$ of a cent. Therefore he lost $\frac{5}{12}-\frac{2}{5}=\frac{1}{60}$ of a cent on each apple he sold; and 1 cent on 60 apples; and since he lost $2\frac{1}{2}$ cents, he must have sold $60\times2\frac{1}{2}=150$ apples. Ans. 150.

33. A, B, and C can do a piece of work in 5 days; B, C, and D in 6 days; C, D and A, in 7 days; and D, A and B, in 8 days; in what time would they all do it, working together, and in what time would each one do it alone?

Solution.—A, B, and C, can do $\frac{1}{5}$ of the work in 1 day.
B, C, and D can do $\frac{1}{6}$ " " "
C, D, and A can do $\frac{1}{7}$ " " "
D, A, and B can do $\frac{1}{8}$ " " "

Therefore they all can do $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}=\frac{533}{840}$ of the work in 3 days, or $\frac{533}{840}\div3=\frac{533}{2520}$ of it in 1 day.

Then $1\div\frac{533}{2520}=4\frac{388}{533}$ days in which they all would it.

Also: $\frac{533}{2520} - \frac{1}{5} = \frac{29}{2520}$ of the work done by D in one day, and $1 \div \frac{29}{2520} = 86\frac{26}{29}$ days, in which D would do it.

In the same manner it can be found that A can do it in $22\frac{34}{113}$ days; B can do it in $14\frac{98}{173}$ days; and C can do it in $11\frac{61}{109}$ days.

34. A certain piece of work is to be performed, requiring a framer, a carpenter, and a mason; and no two can work at the same time. The mason receives \$1.25 per day; the carpenter receives \$2 per day; and the framer receives \$2.50 per day; the work is completed in 100 days, and each receives the same amount of money. How many days did each work, and how much did each receive?

Solution.—As the framer receives \$2.50 per day, whilst the mason receives but \$1.25 per day, it would take $\frac{2}{1}:\frac{50}{25} = 2$ days of the mason to amount to 1 of the framer's; likewise, it would take $\frac{2}{2}:\frac{50}{00}$ $=1\frac{1}{4}$ days of the carpenter to amount to 1 of the framers. Therefore, the days they worked are to each other as 1, $1\frac{1}{4}$, and 2. Then $1+1\frac{1}{4}+2=4\frac{1}{4}$; and $4\frac{1}{4} : 1 :: 100 : 23\frac{9}{17}$ days worked by the framer. $4\frac{1}{4} : 1\frac{1}{4} :: 100 : 29\frac{7}{17}$ days worked by the carpenter.— And, $4\frac{1}{4} : 2 :: 100 : 47\frac{1}{17}$ days worked by the mason.

$$\left.\begin{array}{l} \text{And } 23\frac{9}{17} \times 2.50 = \$58\frac{14}{17}. \\ 29\frac{7}{17} \times 2 = \$58\frac{14}{17}. \\ 47\frac{1}{17} \times 1.25 = \$58\frac{14}{17}. \end{array}\right\} \text{the sum received by each.}$$

And 100 = the number of days worked by all.

35. A gentleman died leaving a son and daughter in foreign countries, and directed in his will, that if the son only returned, he should receive $\frac{2}{3}$ of the estate and the widow the balance; but if the daughter only should return, the widow should receive $\frac{2}{3}$, and the daughter the balance: as it happened both returned, and thus the widow lost in equity \$1600 dollars more than she would, had the daughter only returned. How much was the share of each, and how much would the widow have received had the son only returned?

Solution.—By the question, the son receives twice as much as the widow, and the widow twice as much as the daughter; therefore the daughter receives 1 part, the widow 2 parts, and the son 4 parts, and $4+2+1=7$ parts into which the estate is to be divided.

Had the daughter only returned, the widow would have received $\frac{2}{3}$ of these 7 parts, or $4\frac{2}{3}$ parts of the whole estate; but both returning, she receives $4\frac{2}{3}-2 = 2\frac{2}{3}$ parts less than she would, had the daughter only returned. But these $2\frac{2}{3}$ parts = \$1600; 1 part therefore, must be $\frac{1600}{2\frac{2}{3}} = \$600$; hence, the estate is $\$600 \times 7$ $= \$4200$; and $\$4200 \div 3 = \$1400$, the widow's share had the son

only returned. But as it is their shares are to each other as the numbers 1, 2, and 4; then

$$\begin{cases} 7 : 1 :: \$4200 : \$600 \text{ the daughter's share.} \\ 7 : 2 :: \$4200 : \$1200 \text{ the widow's share. And} \\ 7 : 4 :: \$4200 : \$2400 \text{ the son's share.} \end{cases}$$

SECTION XVIII.

MISCELLANEOUS EXERCISES FOR THE SLATE.

1. A sum of money is to be divided between five men; A receives \$130; B \$170; C as much as A and B; D as much as B and C; and E as much as all the rest. Required the sum divided? Ans. \$2080.

2. A man was 21 years old when he left college; he studied 3 years for a profession; then traveled 15 years; after which he married and lived with his wife 30 years, when she died, leaving him a son 15 years of age; the son was 26 years old when the father died. How old was the father at his death? Ans. 80 years.

3. Add together 4 quadrillions, 127 trillions, 9 billions, 18 millions, 102 thousand 407; 39 trillions, 7 millions, and 1; 18 trillions, and 17; 204 billions, 10 thousand; 7 millions, 4 thousands, 303; and 199. Ans. 4184213032116927.

4. A man's property was worth \$75000: he lost a house worth \$3000 by fire, and a vessel worth \$15000 in a storm. Required the value of the remaining property. Ans. \$57000.

5. From 100 quadrillions, 100 billions, and 1, take 101 billions, 101. Ans. 99999998999999900.

6. A farmer sold as follows: a quantity of wheat for \$125; corn for \$200; hay for \$75; oats for \$50; butter and cheese \$40; and some other things for \$25; he received in return a piece of broadcloth worth \$30; \$75 worth of satinet; \$90 worth of groceries; \$15 worth of muslin; \$40 worth of books, and the balance in money. How much money did he receive? Ans. \$265.

7. Multiply 146 millions, 201 thousand, 111, by 210 millions, 12 thousand, 222. Ans. 30704020179978642.

8. There two numbers; the lesser number is 1476 times 28921, and their differerence is 1492 times 1728. Required their product? Ans. 1932269397130512.

9. Divide 1 sextillion by 333.
Ans. $3003003003003003003\frac{1}{333}$.

10. Take 1 from 1 sextillion and divide the remainder by 999.
Ans. 1001001001001001001.

11. How long would it take to count a trillion by counting 57600 per day? Ans. $17361111\frac{1}{9}$ days = 47532 yrs. $48\frac{1}{9}$ days.

12. The bible contains 31173 verses: by reading 74 verses per day, how long will it take to read it through?

Ans. $421\frac{19}{74}$ days.

13. The sum of two numbers is 18261, and one of the numbers is 4927; what is the other number? Ans. 13334.

14. Two men, A and B have together $15000; A has $8976; how much has B? Ans. $6024.

15. C has $1000, which is $496 dollars more than D has; how much has D? Ans. $504.

16. Two men together have 150 cattle, and one has 8 more than the other; how many have each? Ans. One 71; the other 79.

17. What number multiplied by 144 will produce 34272?

Ans. 238.

18. What number divided by 216 will produce 383?

Ans. 82728.

19. If the dividend is 23188, and the quotient is 124, what is the divisor? Ans. 187

20. There is a certain number, to which if 460 be added, and from the sum 300 be subtracted. and the remainder divided by 8, the quotient will be equal to 9900 divided by 132. Wha is the number? Ans. 440.

21. A lad being asked how much he gave for a book, replied; if you divide the price by 2, to the quotient add 11, multiply the sum by 6 and from the product subtract 52, the remainder will be $20 Required the cost of the book? Ans. $2.

22. A peddler made three trips; the second trip he lost $25; but the third trip he gained $40; in the three trips together he gained $25. Did he gain or lose the first trip, and how much?

Ans. He gained $10.

23. A travels 30 miles per day, and B 40; if both travel the same road, and A has 5 days the start of B, how many days will it take B to overtake A? Ans 15.

24. Multiply 288 by 144, 216, 137. 2625, 1728 and 256, and divide the product by 576, 72, 274, 864, 1644, 135, and 512.

Ans. $1\frac{38}{137}$.

25, A owns 580 rods of land; B owns 796 rods; C owns 848 and D owns 1232 rods they agree to divide it into equal lots fixing on the greatest number of rods for a lot, that will allow each owner to lay out all his land, How many rods in each lot?

Ans. 4.

26. A gentleman has casks holding 28, 36, 42, and 63 gallons respectively; what is the smallest number of gallons that will just fill same number of casks of either kind? Ans. 252.

27. What fraction is that, to which if you add $\frac{1}{3}$ the sum will be $\frac{2}{5}$? Ans. $\frac{1}{15}$.

28. What fraction is that, from which if you subtract $\frac{2}{7}$, the remainder will be $\frac{4}{9}$? Ans. $\frac{46}{63}$.

29. What number is that which being multiplied by 12, produces $\frac{4}{5}$? Ans. $\frac{1}{15}$.

30. What number is that which being multiplied by 9, produces $\frac{3}{8}$? Ans. $\frac{1}{24}$.

31. What number is that which being divided by 6 produces $\frac{2}{3}$? Ans. 4.

32. What number is that which being multiplied by $\frac{2}{3}$, gives $\frac{3}{4}$? Ans. $1\frac{1}{8}$.

33. What number is that which being divided by $\frac{4}{5}$ gives $\frac{1}{2}$? Ans. $\frac{2}{5}$.

34. What number is that, from which, if you take $\frac{3}{7}$ of itself, the remainder will be 12? Ans. 21.

35. What number is that to which if you add $\frac{2}{3}$ of $\frac{3}{8}$ of itself, the sum will be 30? Ans. 24.

36. What number is that to which if you add $\frac{3}{4}$ and $\frac{5}{6}$ of itself the sum will be 93? Ans. 36.

37. What fraction is that to which if you add $\frac{1}{2}$ and $\frac{1}{3}$, the sum will be 1? Ans. $\frac{1}{6}$.

38. $\frac{1}{2}$ of a pole is in the air, $\frac{2}{7}$ of it in the water, and 6 feet is in the mud; required the length of the pole? Ans. 28 feet.

39. $\frac{3}{4}$ of a certain number exceeds $\frac{2}{3}$ of it by 4; what is the number? Ans. 48.

40. What number is that from which if you take $\frac{2}{3}$ of $\frac{4}{5}$ of itself and to the remainder and $\frac{5}{6}$ of $\frac{3}{8}$ of itself, the sum will be $93\frac{1}{2}$? Ans. 120.

41. What number is that to which if 12 be added, $\frac{4}{9}$ of the sum will be 16? Ans. 24.

42. A young man spent $\frac{5}{9}$ of his property in 18 months, and $\frac{5}{8}$ of the remainder in 12 months after; he then had \$3000 left. How much was his fortune at first? Ans. \$18000.

43. A grocer bought a number of boxes of tea, each containing $56\frac{16}{39}$ pounds, paying at the rate of \$5 for 8 pounds, and sold it at the rate of 11 pounds for \$8, and gained \$300. How many boxes were there? Ans. 52.

44. If $8\dfrac{1}{1\frac{1}{4}}$ English Guineas are worth $\$44\dfrac{11}{16\frac{2}{3}}$ and $16\dfrac{2\frac{2}{3}}{5}$ English Sovereigns are worth $\$80\frac{13}{125}$, how many Sovereigns are worth $19\dfrac{1}{10\frac{13}{19}}$ Guineas? Ans. 20.

45. If A does as much work in $1\frac{1}{4}$ days, as B does in $2\frac{1}{3}$ days,

and C does as much in $3\frac{2}{5}$ days, as A does in $2\frac{7}{8}$ days, how many days work of B are equal to $7\frac{2}{7}$ days work of C? Ans. $11\frac{1}{2}$.

46. A younger brother received \$1560, which was just $\frac{7}{12}$ of his elder brother's fortune, and $5\frac{3}{8}$ times the elder brother's fortune was $\frac{3}{8}$ as much again as the father was worth. Required the value of the estate? Ans. $\$19165\frac{5}{7}$.

47. A person spent all his income and $\frac{1}{5}$ as much more one year, but after that he saved $\frac{1}{20}$ of his income, and in 5 years made good his deficiency and had \$50 left. How much was his income?

Ans. \$1000.

48. What number multiplied by 16 will produce .96?

Ans. .06.

49. What number divided by .15 will produce 18?

Ans. 2.7.

50. To what decimal is $\frac{1}{45}$ equivalent? Ans. .0222+.

51. What is the value of the decimal .3625? Ans. $\frac{29}{80}$.

52. A farmer sold

150 bu. wheat	at	\$1.12½	per bu.
300 bu. corn	at	.37½	per bu.
250 bu. oats	at	.31¼	per bu.
175 lbs. butter	at	.12½	per lb.
412 lbs. cheese	at	.10	"
1200 lbs pork	at	.06¼	"

In return he received

150 lbs. sugar	at	\$0.10	per lb.
201 lbs. coffee	at	.08⅓	"
30 galls molasses	at	.44	per gal.
3 barrels salt	at	1.75	per bl.
2 quintals fish	at	.06¼	per lb.
30 yds. cloth	at	5.75	per yd.
24 yds. satinet	at	1.50	per yd.

And the balance in money.

How much money did he receive? Ans. \$224.75.

53. If 18 grains of silver will make a thimble, and 12 pwts. a teaspoon, how many timbles and teaspoons, of each an equal number can be made from 15 oz. 6 pwts. of silver? Ans. 24.

54. Sound moves at the rate of 1142 feet in a second of time: now I saw the flash of a cannon, fired from an eminence, just 1 minute before I heard the report. How far distant was the cannon?

Ans. 12 M. 7 fur. 32 rds. 4 yds.

55. I saw several men upon an eminence shooting at the rate of 20 shots per minute; I saw the flash of three rifles before I heard the first report. How far distant were they?

Ans. 1 M. 2 fur. 15 rds. 4 ft. 6 in.

56. The atmosphere presses upon all surfaces at the rate of about 15 lbs. to the square inch: how much pressure then is sustained by a surface measuring 15 sq. ft.? Ans. 16 T. 4 cwt.

57. I wish to cut off just 1 acre of land from the end of a lot 22 rods wide. How long a piece must I cut off?

Ans. $7\frac{3}{11}$ rods.

58. Some very important news having arrived at New York 71° 1′ 8″ W. lon., in 4 minutes it was communicated to St. Louis, Mo., 90° 15′ 16″ by magnetic telegraph. At what time was it known in St. Louis, it being known in N. Y. at half past 2 o'clock, P. M.?

Ans. 29 min. 4 sec. past 1 o'clock, P. M.

59. A traveler who had set a perfectly accurate watch by the sun in Augusta, Me., 69° 50′ W. lon., being in Cincinnati, 84° 27′, a few days after, was surprised to find it wrong when compared with the sun. Was it too fast or too slow? How much, and why?

Ans. It was 58 min. 28 sec. too fast, because the time is earlier at Augusta than at Cincinnati.

60. If a man travel 376 miles in 18 days, how far can he travel in 27 days? Ans. 564 miles.

NOTE. This, and the 47 following examples are to be solved by Analysis.

61. If \$38.25 pay for 17 barrels of salt, how many barrels may be bought for \$24.75? Ans. 11.

62. If a barrel of flour lasts 17 persons $5\frac{1}{2}$ weeks, how long will it last 23 persons? Ans. $4\frac{3}{46}$ weeks.

63. If $16\frac{2}{5}$ acres of land costs $\$325\frac{3}{4}$, what will $26\frac{4}{5}$ acres cost?

Ans. \$523.806.

64. A has 120 acres of land; $\frac{3}{4}$ of A's is equal to $\frac{6}{11}$ of B's. How many acres has B? Ans. 165.

65. A man owning $\frac{5}{7}$ of a store, sold $\frac{12}{17}$ of his share for \$2928; required the worth of the store? Ans. \$5807.20.

66. Two expresses start at the same time from two places 250 miles apart, and ride towards each other; A rides $8\frac{1}{4}$ miles per hour, and B rides $11\frac{3}{8}$ miles per hour. How far does each ride before they meet?

Ans. A rides $105\frac{15}{157}$ miles: B rides $144\frac{142}{157}$ miles.

67. A man-of-war in pursuit of a smuggler sails 8 miles whilst the smuggler sails 5; the ship passes a certain island when the smuggler is 24 miles beyond? How far must the man-of-war sail to overtake the other? Ans. 64 miles.

68. "As I was hunting on the forest grounds,
Up starts a hare before my two greyhounds;
The dogs being light of foot did fairly run,
Unto her fifteen rods, just twenty-one;
The distance that she started up before,
Was four score sixteen rods, just, and no more.
Now this I'd have you unto me declare.
How far they ran before they caught the hare."

Ans. 336 rds.

69. A set out from Boston for Hartford precisely at the time when B at Hartford set out for Boston, distant 100 miles; after seven

hours they met when it appeared that A had traveled $1\frac{1}{2}$ miles per hour more than B. At what rate per hour did each travel?

Ans. A, $7\frac{25}{28}$ miles, and B $6\frac{11}{28}$ miles per hour.

70. A cistern has two pipes, one of which will fill it in 24 hours, and the other in 30 hours; it has also a discharging pipe which will empty in 18 hours. If all are left open, how long will it take the cistern to fill? Ans. $51\frac{3}{7}$ hours.

71. A lion of bronze, placed upon the basin of a fountain, can spout water into the basin through his throat, his eyes. and his right foot. If he spouts through his throat only, he will fill the basin in 6 hours; if through his right eye only, he will fill it in 2 days; if through his left eye only, in 3 days; and if through his foot only, he will fill it in 4 hours. In what time will the basin be filled if the water flow through all the apertures at once?

Ans. $2\frac{14}{65}$ hours.

72. A can mow 10 acres in 6 days; B can mow 13 acres in 9 days; and C can mow 17 acres in 12 days: in what time can they all working together mow $108\frac{2}{3}$ acres? Ans. 24 days.

73. A man left \$13000 to be divided between his wife, son, and daughter; he left \$1500 more to the son than to the daughter. and \$2500 more to his wife than to his son. Required the share of each?

Ans. Wife's share \$6500; Son's share \$4000; Daughter's share \$2500.

74. Thomas sold 150 pine apples at $\$0.33\frac{1}{3}$ apiece, and received no more money than Harry did for a certain number of watermelons at \$0.25 apiece. How much money did each receive, and how many melons had Harry?

Ans. Each receives \$50, and Harry had 200 melons.

75. A farmer sold 50 bushels of wheat, at $\$1.06\frac{1}{4}$ per bushel, and took his pay in cloth at $\$2.37\frac{1}{2}$ per yard. How many yards did he receive? Ans. $22\frac{7}{19}$.

76. A goldsmith mixed 3 oz. of silver 14 carats fine, with 4 oz. 17 carats fine, 5 oz. 20 carats fine, and 6 oz. 23 carats fine. Required the fineness of the mixture? Ans. $19\frac{3}{9}$ carats.

77. A's age is double B's, and C's is 3 times A's; the sum of all their ages is 135 years. Required the age of each?

Ans. A's age 30 yrs.; B's 15 yrs.; C's 90 yrs.

78. A man bought a chaise, horse and harness for \$300; the horse cost twice as much as the harness, and the chaise cost 3 times as the horse and harness together. Required the cost of each?

Ans. Harness \$25; horse \$50; chaise \$225.

79. A man has \$24 to pay 20 laborers; he pays each boy 3 cents, as often as he pays each woman 5 cents, and each man 7 cents; for

every boy there were 3 women, and for every woman 2 men. How many were there of each, and how much did each receive?

Ans. $\begin{cases} \text{There were 2 boys, 6 women, and 12 men.} \\ \text{The boys received \$0.60 each; the women \$1 each; and} \\ \quad \text{the men \$1.40 each.} \end{cases}$

80. If 18 workmen do a piece of work in 20 days, working 10 hours per day, how many workmen will it take to do the same work in 15 days working 12 hours per day? Ans. 20.

81. In an orchard, $\frac{1}{3}$ of the trees bear apples, $\frac{1}{4}$ bear peaches, $\frac{1}{6}$ bear plums, and 30 bear cherries. How many trees in the orchard? Ans. 120.

82. A daughter's portion is $\frac{4}{5}$ of a son's portion; both their portions amount to \$7200; what is the portion of each?

Ans. Son's portion \$4000; Daughter's portion \$3200.

83. What sum of money is that whose $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ parts together amount to \$141? Ans. \$180.

84. What number is that to which if 25 be added, $\frac{3}{10}$ of the sum will be 21? Ans. 45.

85. What number is that which increased by $\frac{1}{4}$ of itself, is equal to $\frac{7}{8}$ of itself increased by 9? Ans. 24.

86. Out of a cask of wine, from which $\frac{1}{3}$ part had leaked 21 gallons were drawn, when the cask was found to be half full. How much did the cask hold? Ans. 126 gallons.

87. A person having spent $\frac{3}{8}$ of his income, found he wanted \$25 of having $\frac{2}{3}$ of it remaining. Required his income?

Ans. \$600.

88. A man bought a quantity of beef for \$7; he used 25 lbs., and then sold $\frac{1}{5}$ of the remainder for \$1.20 which was just what it cost. How many pounds did he buy at first? Ans. 175.

89. A father divided his estate between his children, giving the first $\frac{1}{2}$ wanting \$300, the second $\frac{1}{3}$ wanting \$150. and to the third the balance, which was $\frac{1}{4}$ wanting \$50. How much was the estate?

Ans. \$6000.

90. A general after a battle found that he had left 500 men more than $\frac{1}{2}$ of the whole army fit for action; 250 more than $\frac{1}{4}$ of the whole being wounded; 125 more than $\frac{1}{8}$ of the whole being killed; and the remainder, which was $\frac{1}{15}$ of the whole, missing. Of how many men did the army consist at first? Ans. 15000.

91. A hare starts 12 rods before a greyhound, but is not perceived by him until she has been up $1\frac{1}{4}$ minutes; she scuds away at the rate of 36 rods a minute, and the dog in view makes after at the rate of 40 rods a minute. How long will the course last, and how far will the dog run?

Ans. The course lasts $14\frac{1}{4}$ minutes, and the dog runs 570 rods.

92. A and B have the same income; A contracts an annual debt equal $\frac{1}{12}$ of his, but B lives on $\frac{5}{6}$ of his, and in 8 years lends A

enough to pay his debts, and has $200 to spare. How much is the income of each? Ans. $300.

93. A man being asked the time of day, answered that the time past noon was equal to $\frac{3}{5}$ of the time till midnight; what time was it? Ans. 30 minutes past 4 o'clock..

94. A man performing a journey found that the distance he had traveled was equal to $\frac{4}{7}$ of the distance he yet had to go; how far had he traveled, and how long was his journey in all?

Ans. His journey was 165 miles, and he had traveled 60 miles.

95. Three men have a certain sum of money; A has $200; B has as much as A, and $\frac{3}{7}$ as much as C; and C has as much as A and B both. How much have they all together? Ans. $1400.

96. A lady has two silver cups and only one cover, which weighs 5 ounces: if the cover be put on the first cup its weight will be double that of the second cup; but if the cover be put on the second cup; its weight will be $\frac{4}{5}$ of the first cup. Required—the weight of each cup?

Ans. Weight of the first cup 25 oz.; of the second cup 15 oz.

97. At what time between 5 and 6 are the hands of a clock together. Ans. 27 min. $16\frac{4}{11}$ sec. past 5.

98. A harmless dove was soaring high,
To stretch her wings in space;
At length a hawk did her espy,
And gave the dove a chase.
Just forty rods was there between
These birds, as we could view—
And whilst the dove flew seventeen,
The hawk flew twenty-two.
The hawk pursued with all his strength,
As those who saw did say,
Then tell the rods he flew in length,
Before he caught his prey.

Ans. 176 rds.

99. At what time between 8 and 9 are the hands of a clock exactly opposite each other? Ans. 10 min. $54\frac{6}{11}$ sec. past 8.

100. A man bought a lot of wheat, and gave $\frac{1}{4}$ of it to some poor families, and then sold the remainder for 1.33\frac{1}{3}$ per bushel, and received what the whole cost him. How much did it cost per bushel? Ans. $1.00.

101. What number is that from which if you take $\frac{4}{5}$ of $\frac{7}{8}$ of itself, and to the remainder add $\frac{7}{9}$ of $\frac{3}{20}$ of the number, the sum will be 50? Ans. 120.

102. A man when he married was 3 times as old as his wife; 15 years after he was but twice as old as his wife. Required—the age of each when married?

Ans. Man's age, 45 yrs; Wife's age, 15 yrs.

103. Divide \$450 between A, B, and C, giving A \$50 more, and B \$50 less than C.

Ans. A \$200; B \$100; and C \$150.

104. There are two numbers whose difference is 8; $\frac{4}{5}$ of the less is equal to $\frac{2}{3}$ of the greater. Required—the numbers?

Ans. 40 and 48.

105. If one ship containing 150 hogsheads of wine, pays for toll, at the Sound, the value of two hogsheads, wanting \$6, and another ship containing 240 hogsheads pays at the same rate the value of two hogsheads, and \$18 besides, what is the value of the wine per hogshead? Ans. \$23.

106. The smaller of two numbers is 15, and if we add to this $\frac{1}{4}$ of both numbers, the sum will be the greater number. Required—the greater number? Ans. 25.

107. A man having \$50, spent a part of it; he afterwards received 6 times as much as he had spent, and then had twice as much as he had at first. How much did he spend? Ans. \$10.

108. If 2 men can do a piece of work in 25 days, and it takes 3 women the same time to do it, in what time will one man and one woman together perform it? Ans. 30 days.

109. A man left his daughter $\frac{4}{15}$ of his property, to his wife $\frac{6}{11}$ of the remainder, and to his son what was left. Required—the share of each, the shares of the son and daughter together amounting to \$4500.

Ans. { Daughter's share \$2000, Wife's share \$3000; Son's share \$2500.

110. If cloth worth \$0.25 cash, is worth \$0.31$\frac{1}{4}$ in trade, what is wheat worth in trade that is worth \$1.06$\frac{1}{4}$ cash?

Ans. \$1.32$\frac{13}{16}$.

111. At the rate of \$72 for 64 days' work, how many days must a man work to earn \$99? Ans. 88.

112. If the hind wheel of a wagon is 14 ft. 8 in circmuference, and the forward wheel 11 ft. 9 in, in circumference, how many times will the forward wheel turn over more than the hind wheel, in going 470 miles? Ans. 42000 times.

113. If $\frac{7}{8}$ of a farm is worth \$2100, what is $\frac{7}{12}$ of it worth?

Ans. 1400.

114. If 5$\frac{3}{7}$ acres of land are worth \$34$\frac{1}{5}$ what are 9$\frac{5}{9}$ acres worth? Ans. \$60.20.

115. If 15 men mow 250 acres of grass in 10 days, working 12 hours per day, how many men will it take to mow 300 acres in 6 days, working 10 hours per day? Ans. 36.

116. If 30 men, working 10 hours per day, dig a ditch 450 yards long, 5 feet wide, and 4 feet deep, in 15 days how many days will it take 45 men to dig a ditch 540 yards long, 4 feet wide, and 3 feet deep, working 11 hours per day, if the strength of the former party

is only $\frac{3}{4}$ of that of the latter, and the hardness of the ground in the latter case is $\frac{11}{9}$ times that in the former? Ans. 6 days.

NOTE.—Let this question also be solved by Analysis.

117. A, B, C, and D trade together: A puts in $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D the rest; they gain $5400. What is each one's share?

Ans. A's $1800; B's $1350; C's $900; and D's $1350.

118. If the third of 6 were 3,
What would the fourth of 20 be? Ans. $7\frac{1}{2}$.

119. If 6 and 4 just 9 had been,
Pray tell how much were 7 and 10?

Ans. $15\frac{3}{10}$.

120. A bankrupt owes as follows: to A $1000; to B $1200; to C. $800; and to D $1500; his property is worth $4000. How much mus each creditor receive?

Ans. { A, 888.888\frac{8}{9}$; B, 1066.666\frac{2}{3}$; C, 111.111\frac{1}{9}$; D, 1333.333\frac{1}{3}$.

121. In a storm at sea, a vessel loaded with flour was obliged to throw overboard 150 barrels; of the cargo, A owned 1000 barrels; B owned 1400; C owned 1600; and D owned 2000. What part of the loss must each sustain?

Ans. A, 25 bls.; B, 35 bls.; C, 40 bls.; and D, 50 bls.

122. A prize of $400 is to be divided between a Captain, two lieutenants, and 6 soldiers: the Captain receives a share and a half, the lieutenants a share each, and the soldiers a half share each.—How much does each receive?

Ans. Captain, 92\frac{4}{13}$; Lieut. 61\frac{7}{13}$ each; Soldiers, 30\frac{10}{13}$.

123. A, B, C, and D agree to trade together with $1200 capital, of which A was to furnish $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$; D withdraws, and C dies. What part of the stock must A and B furnish, to have the same proportion as before? Ans. A, 685\frac{5}{7}$; B, 514\frac{2}{7}$.

124. A commenced trading with $1000 capital; at the end of 3 months he took in B, with $1400; two months following, they took in C, with $1200; and 4 months after, they took in D, with $1500: at the end of the year they find they have gained $800. How much is each one's share?

Ans. A's, $256; B's, $268.80; C's, $179.20; and D's, $96.

125. A hired two horses and a carriage to go 30 miles and back, for $20: after proceeding 12 miles, he took in B to ride through and back again; when within 5 miles of the end of their journey, they took in C, who also rode back with them; and when half way back, they took in D, who rode the balance of the way. How much ought each to pay?

Ans. A, $7.50; B. $6.25, C, 4.37\frac{1}{2}$; and D, 1.87\frac{1}{2}$.

126. Sold, $1500 worth of goods for a friend. How much did my commission amount to, at $12\frac{1}{2}$ per cent? Ans. $187.50

127. What will be the cost of insuring $2500 worth of property, at $3\frac{5}{8}$ per cent? Ans. 90.62\frac{1}{2}$.

128. A merchant bought a lot of goods for $21500, and paid $150 for transportation. For how much must he sell them to gain 18 per cent? Ans. $25547.

129. Bought a quantity of goods on credit for $75, but for cash 12 per cent was deducted. How much did they cost? Ans. $66.

130. Bought goods for $1200, and sold them for $1400. What per cent was gained? Ans. $16\frac{2}{3}$.

131. Sold tea for 1.12\frac{1}{2}$ per pound, by which $28\frac{4}{7}$ per cent was gained. Required—its cost? Ans. 0.87\frac{1}{2}$.

132. Bought goods for $500, and sold them for $450. What per cent was lost? Ans. 10.

133. What is the interest and amount of $180 for 2 yrs. 4 mo. 23 da. at 6 per cent? Ans. Int. $25.89; Amt. $205.89.

134. What is the interest and amount of $360 for 3 yrs. 5 mo. 29 da. at 7 per cent? Ans. Int. $88.13; Amt. $448.13.

135. A man gave $300 for the use of $2500, 2 years. What was the rate per cent allowed? Ans. 6.

136. A gentleman gave $4500 for $3800 which he had used 3 years 6 mo. 15 da. What rate per cent did he allow? Ans. 5+.

137. In what time would $300 amount to $408, at 6 per cent? Ans. 6 years.

138. What principal would amount to $221.40 in 1 yr. 9 mo. 12 da., at 6 per cent? Ans. $200.

139. What sum would gain $55.30 in 2 yrs, 7 mo. 18 da., at 6 per cent? Ans. $350.

140. I have a note of $540.50, due me in 2 yrs. 6 mo. without interest: my creditor wishes to pay me now. What discount ought I to make? Ans. $70.50.

141. A certain person wished to sell his property, and a gentleman offered him $2100 for it at the present time, or $2350 payable in 2 yrs. 11 mo; he chose the latter. Did he gain or lose, and how much, the money being worth 6 per cent? Ans. He lost $100.

142. What is the difference between the interest of $1000 for 3 yrs. 7 mo. 6 da., at 7 per cent, and the discount of the same sum, for the same time, and at the same rate? Ans. 50.722.

143. What is the present worth of 6 per cent of $2400, payable, $\frac{1}{3}$ in 6 mo., $\frac{1}{3}$ in 9 mo., and the rest in 1 year. The discount?
Ans. Present worth, $2296.965; Discount, $103.035.

144. A broker subscribed for 40 shares in a bank, each share being $100: he paid in 40 per cent of his stock when he subscribed, 50 per cent of the remainder 4 months after, and the balance 3 months following. In 12 months after he subscribed there was a dividend of $4\frac{3}{4}$ per cent of the stock among the stockholders, and

a dividend of 3 per cent accrued each 6 months afterwards. At the end of 4 yrs. 6 mo. the broker sold his stock at $8\frac{1}{3}$ per cent advance. Now supposing he hired his money, and paid up his notes when he sold his stock, did he gain or lose by the speculation, and how much?

Ans. He gained $401.60.

145. A certain room measures 20 ft 10 in. in length, 16 ft. in width, and 7 ft. 10 in. in height: now deducting two doors, each measuring 6 ft. 7 in. by 3 ft. 6 in., and three windows, each measuring 5 ft. 4 in. by 3 ft. 4 in., what will it cost to plaster this room at $0.20 per square yard? Ans. 11.82\frac{2}{9}$.

146. A certain brick building measures 64 ft. 9 in. in length, 40 ft. 8 in. in width, and 30 ft. 6 in. in heighth, and the walls are 1 ft. thick. It has two partition walls through the length of the building, and 6 additional walls, each 15 ft. 2 in. in length; from the side of the building to these partitions, all of the same heighth and thickness as the outside walls. Now deducting two outside doors, each measuring 7 ft. 8 in. by 3 ft. 6 in. and 24 inside doors, each measuring 6 ft. 9 in. by 2 ft. 8 in., and 36 windows, each measuring 6 ft. 8 in. by 3 ft. 9 in., how many bricks did it require for this building, each brick being 8 in. in length, 4 in. in width, and 2 in. in thickness, and how much did they cost at $3.50 per M.?

Ans. It required 311.202 bricks, and they cost $1089.207.

147. What number added to $\frac{4}{15}$ of 120, is equal to $\frac{9}{32}$ of the square of 64? Ans. 1120.

148. What number is that, which substracted from 6 times the square root of 1849, leaves $\frac{1}{4}$ of 552? Ans. 120.

149. A company of men spent $51.84, each man spending 4 times as many cents as there were men in the company. How many men were there, and how much did each spend?

Ans. 36 men; each spent $1.44.

150. A General, forming his army into a square, found he had 76 men over and above a square; but by increasing each side by 1 man, he wanted 161 men to complete the square. How many men had he? Ans. 14000.

151. If A travels due north 24 miles, and B due east 32 miles, how far apart are they? Ans. 40 miles.

152. A certain triangle measures 16 feet on each side. Required—the length of a perpendicular from any angle to to the opposite side? Ans. 13.85+ feet.

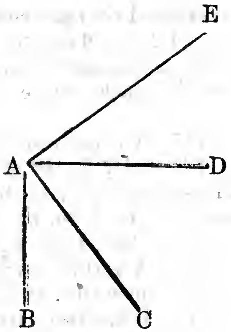
153. If a circle measures 9 inches in diameter, what is the diameter of one 4 times as large? Ans. 18 inches.

154. If the circumference of a circle is 36 feet, what is the circumference of one that contains but $\frac{1}{9}$ as much? Ans. 12 ft.

155. The distance between two places is such, that if you increase its cube by 16, from the sum subtract 30, and multiply the remainder by 2, the product will be 100. Required—the distance?

Ans. 4 miles.

156. In measuring a certain figure like the one in tbe margin, it was found that the length of the lines were such, that as often as 8 inches were measured from A to B, 15 inches were measured from A to E; and as often as 3 inches were measured from A to E, 2 inches were measured from A to C; and as often as 6 inches were measured from A to D, 5 inches were measured from A to C; the length of the four lines is 675 inches. Required—the length of each line, and their distance apart at their extremities?



Ans. Distance from A to B 120 inches.
" " A to C 150 inches.
" " A to D 180 inches.
" " A to E 225 inches.
" " B to C 90 inches.
" " B to D 216.333+ inches.
" " B to E 312.129+ inches.
" " C to D 150 inches.
" " C to E 270.416+ inches.
" " D to E 135 inches.

157. There are two columns in the ruins of Persepolis left standing upright; one is 64 feet high, and the other 50 feet. In a direct line between these, stands an ancient statue, the head of which is 97 feet from the top of the higher column, and 86 feet from the top of the lower column. The distance from the foot of the lower column to the foot of the statue is 76 feet. Required—the distance between the tops of the columns?

Ans. 169.96 ft. nearly. Or,
157.04 ft. nearly.

NOTE.—The learner will perceive that this question admits of two diffetent answers, according as we suppose the statue to be higher or lower than the columns.

158. How many cubical blocks, each measuring $\frac{1}{8}$ of an inch on a side, will it take to fill a cubical box, each side of which is 2 feet?

Ans. 4096.

159. In a cubical foot, how many cubes measuring 2 inches on each side? Ans. 216.

160. What is the difference between a solid half foot, and half a solid foot? Ans. 648 s. in.

161. Required—the length of one side of a solid body containing $\frac{1}{8}$ as much as another solid body measnring 6 inches on each side?

Ans. 3 inches.

162. Required, the area of a triangle whose base is 46 feet, and perpendicular heighth 32 feet? Ans. 736 sq. ft.

163. Required—the area of a triangle whose sides are $19\frac{1}{2}$, 21, and $22\frac{1}{2}$ ft. respectively? Ans. 189 sq. ft.

164. If the wheel of a gig are 4 ft. 7 in. in diameter, how many times will they revolve in going 1 mile? Ans. 366.692+.

165. If the propelling wheels of a locomotive are 3 ft. 4 in. in diameter, and if they make 400 revolutions per minute, how far will the engine move forward in an hour? Ans. 47 miles, 192 rods.

166. A circular meadow I have on my land.
Which contains just two acres and 3 tenths of ground,
How long must that line be, which fixed to a pole,
Will just let my horse graze around on the whole?

Ans. 10.823+ rods=178 ft., 69.54 in.

167. Required—the solid contents of a triangular prism, the sides of which are 4, 5, and 6 feet, and its heighth 12 feet?

Ans. 119.0588+ s. ft.

168. Required—the solid contents of a cylinder 15 ft. long, and 3 ft. in diameter? Ans. 106.029 s. ft.

169. Required—the area of the surface and base of a triangular prism, 15 ft. in length, and measuring 4 ft. on each side?

Ans. 110.851 sq. ft.

170. Required—the area of a curved surface of a cylinder 16 ft. in length, and 3 ft. in diameter? Ans. 150.7968 sq. ft.

171. What is the area of the surface of a triangular pyramid, 23 ft. in heighth, and each side of the base being 9 feet?

Ans. 337 sq. ft. 72 sq. in.

172. On the fourth of July a pole was erected,
Composed of three pieces all nicely connected;
Five feet and four inches it measured around,
At the plåce where it stood at the top of the ground.
Its shape w s a cone, its surface complete,
And the height of the same was twice 60 feet.
How many yards of blue ribbon, procured at the shop,
Will wind round this pole from bottom to top,
Laying smooth and plain to be seen,
By leaving a space of 6 inches between?

Ans. 214.268.

173. A gentleman has a vessel in the shape of a cone, and wishes to know how many gallons of wine it will contain, it being 25 inches deep, and 5 inches in diameter at the largest end. Can you tell the the number? Ans. $708\frac{1}{3}$=2 qts. 1 pt. $2\frac{2}{3}$ gi.

174. How many square inches of leather will it require to cover a ball 5 inches in diameter? Ans 78.54.

175. Required—the solid contents of a globe 15 inches in diameter? Ans. 1767.15 s. in.

176. A stone was put into a gallon measure, which was then filled with 1 qt. of water. Required—the solid contents of the stone? Ans. $173\frac{1}{4}$ s. in.

177. How many bushels will a box contain that is 5 ft. long, 4 ft. wide, and 4 ft. deep? Ans. $64\frac{2}{7}$.

178. If a cord that passes over 4 movable pulleys, be attached to an axle 3 inches in diameter, and the wheel is 6 feet in diameter, what weight can be raised by the pulleys, by applying 250 lbs. to the wheel? Ans. 48000 lbs.

179. What weight may be supported by a screw, the threads of which are 1 inch apart, and the length of the lever 9 feet, by a power of 180 lbs., supposing $\frac{1}{3}$ of the power to be lost in consequence of friction? Ans. 81430.272 lbs.

180. If a wheel containing 84 cogs runs into another containing 14 cogs, and at the other end of the shaft of the latter, a wheel of 64 cogs juts into one of 12 cogs, which is attached to a shaft, on the other end of which a wheel of 52 cogs runs into one of 8 cogs, that is connected by a rod with a band-wheel 32 inches in diameter, and the diameter of the cylinder where the band runs is 4 inches. How many times does the cylinder revolve whilst the master-wheel revolves once? Ans. 1664 times.

181. A and B commence trading with $1000 capital, of which A furnished a part, and B. the rest. They gained 25 per cent, and shared it according to the stock each furnished, when A said to B—"I have made a handsome speck." "Yes," said B, "but if I only had as many such sums as I now have as you have dollars, I should then have $375.000. What share of the capital did each furnish?
Ans. A, $600, B, $400.

182. A certain pole being broken off by the wind, the top fell 27 feet from the foot of the pole: the difference between the length of the two pieces was 9 feet. Required—the length of the pole?
Ans. 81 ft.

183. Two men, A and B, are to dig 100 rods of ditch for $100, of which each receives $50. A begins at one end, and B begins at the other end. Upon settlement, owing to a difference in the hardness of the ground, A receives $0.25 per rod more than B. How many rods does each dig, and at what price per rod?

Ans. { A digs 43.845 rods, at $1.14 per rod;
B digs 56.155 rods, at $0.89 per rod.

184. A, B, C, and D, agree to divide $120 between them, giving A $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$. Required—the share of each?
Ans. $A, $42\frac{2}{19}$; B, $$31\frac{11}{19}$; C, $$25\frac{5}{19}$; and D, $$21\frac{1}{19}$.

185. A gentleman wishing to distribute some money among some children, found he wanted 8 cents to give them 3 cents apiece; he therefore gave them 2 cents apiece, and had 3 cents left. How many children were there? Ans 11.

186. A sum of money is required to be divided between A, B, C, and D, in such a manner, that as often as A gets \$2, B gets \$3; and B gets \$4 as often as D gets \$5; and D gets \$9 as often as C gets \$8; C. gets \$400. Required—the share of the others, and the sum divided?

Ans. { A's share, \$240; | D's share, \$450;
B's share, \$360; | Sum divided, \$1450.

187. A farmer observed that $\frac{1}{4}$ of his land was sowed with wheat, $\frac{1}{5}$, of the remainder with oats, and $\frac{5}{12}$ of the remainder was planted with corn: likewise, he observed that his wood-land was but $\frac{2}{5}$ of his meadow land, which together amounted to 70 acres, and this completed his farm. How many acres had he in all, and how many acres had he of each crop?

Ans. { Entire farm, 200 acres. | Of corn, 50 acres.
Of wheat, 50 " | Of meadow, 50 "
Of oats, 30 " | Of timber, 20 "

188. In turning a gig within a certain diameter, it was discovered that the outer wheel turned thrice, whilst the inner wheel turned but twice: now supposing the axletree to be 4 ft. long, and the wheels of an equal size, the circumference described by each wheel is required?

Ans. Outer circumference, 75.3984 ft; Inner cir. 50.2656.

189. A father left \$2000 to be divided between his three sons, aged 14, 16, and 18 years, in such a manner that the share of each, being placed at simple interest, at 6 per cent, until he arrived at the age of 21 years, should amount to the same sum. How much was the share of each?

Ans. { Share of the youngest, \$606.851+.
" " " second, \$662.868+.
" " " eldest, \$730.279+.

190. Four persons, A, B, C, and D, start from the same point, to travel in the same direction around an island 65 miles in compass: A goes 2 miles, B 4 miles, C 6 miles, and D 8 miles an hour. How long before they next come together?

Ans. 1 day, 8 hours, 30 minutes.

191. In performing a journey, A has 15 days the start of B, and travels 7 days per week, whilst B, stopping Sundays, travels but 6; but B travels as far in 4 days as A does in 5. How many days before B overtakes A? Ans. 60.

192. A, B, and C can do a piece of work in 6 days; B, C, and D, in 7 days; C, D, and A, in $7\frac{1}{2}$ days; and D, A, and B, in 9 days. In what time can it be done by all of them together, and by each of them singly?

Ans. { By all in $5\frac{145}{349}$ da.; By A in $23\frac{73}{79}$ da.; By B in $19\frac{47}{97}$ da.;
By C in $13\frac{83}{139}$ da.; By D in $55\frac{10}{17}$ da.

193. A certain cellar was dug, the width of which was $\frac{4}{5}$ of the

length, and the depth was $\frac{4}{4}$ of the width: 160 cubic yards of earth was removed. Required—the length of the ditch? Ans. 30 ft.

194. A in a scuffle seized on $\frac{2}{3}$ of a parcel of sugar plums; B catched $\frac{3}{8}$ of them out of his hands, and C held on to $\frac{3}{10}$ more; D ran off with all that A had left, except $\frac{1}{7}$, which E afterwards secured slyly for himself: next A and C jointly set upon B, who in the conflict let fall $\frac{1}{2}$ he had, which was equally picked up by D and E: B then knocked down C's hat, and to work they went anew for what it contained, of which A got $\frac{1}{4}$, B, $\frac{1}{3}$, D, $\frac{2}{7}$, and C and E equal shares of what was left of that stock: D then struck $\frac{3}{4}$ of what A and B last acquired out of their hands, when they with difficulty recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ apiece of the same. Upon this they called a truce, and agreed that the $\frac{1}{3}$ of the whole left by A at first should be equally divided among them. What share of the prize after this distribution remained with each of the competitors? And supposing the least common multiple of the denominators of the fractions expressing their shares to be the number of sugar plums divided, how many did each finally obtain?

Ans. to the last. { A, 2863; B, 6335; C, 2438; D, 10294; E, 4950.

195. A merchant bought some cloth at the rate of \$4 per yard, and afterwards bought $\frac{2}{3}$ as much more at \$5 per yard: he sold it all at the rate of 3 yds. for \$16, and gained \$140. How many yards did he buy in all? Ans. 150.

196. A man bought apples at 7 cents a dozen, $\frac{1}{3}$ of which he exchanged for lemons, at the rate of 7 apples for 3 lemons: he then sold all his apples and lemons at $1\frac{1}{4}$ cents apiece, and gained 36 cents. How many apples did he buy, and what did they cost?

Ans. He bought 7 doz., and they cost \$0.49.

197. The sum of two numbers is $812\frac{1}{2}$, and 4 times the less, is equal to $\frac{1}{3}$ of the greater. Required—the numbers?

Ans. 750 and $62\frac{1}{2}$.

198. A man bought 6 oranges and 4 lemons for 42 cents, and at the same rate, bought 5 oranges and 8 lemons for 49 cents. Required—the price of an orange, and of a lemon?

Ans. Price of an orange 5 cents, and of a lemon 3 cents.

199. Three gentlemen contribute \$675 to build a church, which is situated at the distance of 2 miles from the first, $2\frac{7}{8}$ miles from the second, and $3\frac{1}{2}$ miles from the third, and agreed that their shares should be reciprocally proportional to their distances from the church. Required—the amount each contributed?

Ans. { The first, \$289.80; the second, \$201.60; the third, \$165.60.

200. A teacher agreed to teach a certain time upon these conditions: If he had 20 scholars, he was to receive \$25; but if he had

30 scholars, he was to receive but $30. He had 29 scholars. Required—his wages. Ans. $29.725.

201. "Between Sing-Sing and Tarry-Town,
I met my worthy friend John Brown
And seven daughters, riding nags,
And every one had seven bags;
In every bag were thirty cats,
And every cat had forty rats,
Besides a brood of fifty kittens.
All *but* the nags were wearing mittens!
Mittens, kittens—cats, rats—bags, nags—Browns,
How many were met between the towns?
"Ans. 2184192."

NOTE.—This question is to be taken in its plain, literal, common-sense meaning, without any quibble, and the mittens are to be counted *separately*, and not in pairs.

202. A man spent $12 for apples and peaches, giving $1 for a bushel of apples, and $1 for 3 bushels of peaches: he afterwards sold $\frac{1}{4}$ of his apples, and $\frac{1}{3}$ of his peaches for $4, which wss $0.50 more than they cost him. How many bushels did he buy of each?
Ans. 24 bushels of apples; 18 bushels of peaches.

THE END.

If for 10 scholars increase is $5
" 9 " " 4.50
∴ His wages would be 27.50

Rate per pupil, at 20, — $\frac{5}{4}$
" " " 30 — 1
Difference for 10 — $\frac{1}{4}$
" " 9 — $\frac{9}{40}$

$\frac{5}{4} - \frac{9}{40} = \frac{41}{40}$; $\frac{41}{40} \cdot 29 = 29.725$

ERRATA

Page 5, Obs., 4th line from top, for "left," read "right."
" 46, Rule II., last line, for "of," read "as."
" 57, and 58, The remarks under Obs. 20 relate to "Practical," and not to "Pure Arithmetic."
" 122, Second line from bottom, for "$1\frac{60}{60}$," read "1 or $\frac{60}{60}$."
" 132, "*c*" should read, "*mills are reduced to cents by pointing off one figure to the right, and to dollars by pointing off three figures*," &c.
" 139, Third and fourth lines from the bottom, for "Art. 1," read "Art. 7."
" 143, Fourth line from the bottom, for "Art. 1," read "Art. 7."
" 155, Rem. 3, for "8 lbs.," read "80 lbs."
" 156, Rem., for "Liquor," read "Liquid."
" 158, Note at the bottom of the page, for "$69\frac{1}{2}$," read "$69\frac{1}{12}$."
" 161, Table, for "160 Sq: Yds.," read "160 Sq. rds."
" 172, Ex. 46, for "post," read "foot."
" 173, Ex. 57, for "area," read "arc."
" 182, Ex. 14, for "Sq. ft," "Sq. in.," read "s. ft. s. in."
" 185, Bottom of the page, for "2d," read "22d."
" 186, Obs. 1, for for "Sect. XIV.," read "Sect. XIII.;" and Ex. 25, for "Area," read "Arc."
" 223, Obs. 14, for "Ratio of Inequality," read "Ratio of Lesser equality."
" 226, Obs. 11, Illustration, for "two Means," read "two Extremes;" and for "two Extremes," read "two Means."
" 227, Obs. 14, Rem., for "first terms," read "first three terms."
" 230, Fourth line above Obs. 20, for "Obs 4," read "Obs. 14."
" 237, Top line, for "17," read "19."
" 263, Obs. 8, Rem. 5, for "Obs. 21," read "Obs. 5."
" 272, Case 4, for "any number," read "any sum;" and General Rule, for "given number," read "given time."
" 297, Note, for "Obs. 30," read "Obs. 31."
" 298, Obs. 33, for "Obs. 30," read "Obs. 31;" and Ex. 12, for "Obs. 31," read "Obs. 39."

" 299, Obs. 2, for "Obs. 15," read "Obs. 17," and for "Obs. 18," read "Obs. 20."
" 301, Rem. 1, for "Obs. 15," read "Obs. 17."
" 303, Second line from top, for "Obs. 18," read "Obs. 20."
" 309, Third line from the top, for "cube of 8," read "cube root of 8."
" 311, Obs. 15, for "their Factors are," read "their Squares are."
" 321, Fourth line from bottom, for "Obs. 15," read "Obs. 17."
" 322, Operation, for "Obs. 15," read "Obs. 17."
" 324, Definition, for "Obs. 14," read "Obs. 16;" and Obs. 2. for "Obs. 15," read "Obs. 17."
" 326, Operation, for "Obs. 24," read "Obs. 23."
" 331, Second line from top, for "Obs. 8," read "Obs. 10."
" 338, Operation, for "$\sqrt{512}$," read "$\sqrt[3]{512}$."
" 339, Operation, for "Art. 1, Obs. 2," read "Art. 1, Obs. 4."
" 340, Obs. 2, Demonstration, for "Obs. 9," read "Obs. 11."
" 342, Obs. 10,, Rem., for "Obs. 17," read "Obs. 19;" and Obs. 12, for "Obs. 18," read "Obs. 20."
" 344, Obs. 17, Demonstration, for "Obs. 12," read "Obs. 14."
" 349, Bottom line, for "Obs. 11," read "Obs. 14, *a*."
" 359, Note 3, should be signed "Peter Trueman, "Thomas Jones."

These are the most prominent errors detected. Some few typographical errors have been noticed, which it is not necessary to mention here. There may perhaps be a few errors of importance that have escaped notice. All such will be corrected in future editions.

186017

RETURN TO: CIRCULATION DEPARTMENT
198 Main Stacks

LOAN PERIOD 1 Home Use	2	3
4	5	6

ALL BOOKS MAY BE RECALLED AFTER 7 DAYS.
Renewals and Recharges may be made 4 days prior to the due date.
Books may be renewed by calling 642-3405.

DUE AS STAMPED BELOW.

SENT ON ILL		
APR 22 2003		
U. C. BERKELEY		

FORM NO. DD6
50M 5-02

UNIVERSITY OF CALIFORNIA, BERKELEY
Berkeley, California 94720–6000

U. C. BERKELEY LIBRARIES

C061347477